FACTORIZED ALL-PASS IIR ADAPTIVE NOTCH FILTERS

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ABSTRACT

A new family of IIR adaptive notch filters is presented. The family proposed is based on a second-order factorization of the all-pass transfer function that forms the multiple notch filter. These realizations represent an extension of a previous ad-hoc scheme for adaptive notch filtering, that avoid a high-order polynomial root finding in order to obtain the unknown frequencies of interest. An interesting aspect related to these novel algorithms is the fact that they introduce a different compromise between bias and SNR if compared with previous realizations available in the literature. Specifically, lower bias than in other approaches for low SNR can be achieved using the new realizations. This property is particularly attractive for multiple sinusoids estimation and tracking. In addition to the algorithm presentation, a discussion of the different properties and characteristics (stationary points, convergence) is also included. Also, computer simulations are presented in order to illustrate the expected performance of the adaptive filters proposed.

1. INTRODUCTION

The classical problem of multiple sinusoid frequency estimation can be traced back to the *Adaptive Line Enhancer* [13] where MSE minimization using a k-step FIR prediction filter was the basic structure. Computation of unknown frequencies require the calculation of the roots of the associated polynomial. FIR solutions have proven to be inefficient to recover sinusoids in noise, mainly because a high-order filter is required to model a deep notch filter.

Due to its natural efficiency IIR based adaptive notch filters (ANF) or their dual, narrow pass-band filters with a very selective frequency characteristic, are increasingly used in practical cases. In spite that an exact solution is not possible, nice and efficient approximations can be obtained using IIR ANF realizations of adequate order. A popular IIR ANF, proposed in [7], contemplates a canonical (minimum number of parameters corresponding to each of M unknown frequencies to estimate) direct-form realization of order $2M$. The zeros of the ANF are located on the unit circle and the module of the poles (at the same radius but, logically, inside the unit circle) is a user defined parameter. Properties and accuracy of this ANF have been extensively studied in the literature [8], [12]. Despite that classical estimation properties ([6]) can be related to this model, no direct availability of the estimated frequencies is obtained except by finding the roots of a high-order ($2M$) polynomial.

Alternative ANF using the same model but with different realizations were also studied in the past, most remarkable the approaches of [5] or [2] (see also [14]). In these cases a cascade of second-order ANF was proposed to overcome the problem of direct availability of the frequencies of interest. Two different realizations of the second-order ANF can be considered, quality factor (Q) constant or notch bandwidth constant. As concluded in [2], this kind of ANF are biased due to input measurement noise.

A different notch filter model based on a serial-sinusoid canceller strategy, using cascaded second-order all-pass lattice sections, was presented in [9]. Zeros and poles for each section in this model are, respectively, outside and inside the unit circle. In this case, individual second order sections were based on the notch bandwidth constant realizations. An ad-hoc updating algorithm using local errors was proposed [10]. In spite of the low computational complexity obtained using the serial-sinusoidal cancelling strategy for multiple sinusoid estimation, the fact of using local errors in the ad-hoc updating algorithm does not leads to a robust behavior, even more when considering tracking applications.

We consider here a generalization of the model discussed in [9], i.e., different to what was proposed there, the overall all-pass filter is factorized in second-order sections. For the single-sinusoid case both algorithms coincide. This factorization leads to important differences with respect to the previous approaches as discussed in the following sections. The most interesting aspect of the new ANF proposed is that they introduce a different bias performance if compared with other alternatives. Specifically, a lower bias than with other algorithms for low SNR can be obtained.

The article presentation is organized in the following manner. Novel ANF algorithms using the all-pass factor-

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ization concept are presented in section 2. Some of the most important characteristics of the algorithms proposed are discussed in Section 3. A discussion of general properties and comparisons, by computer simulations, with other ANF available in the literature are presented in Section 4. Finally, some conclusions are included in Section 5.

2. THE FACTORIZED ALL-PASS IIR ADAPTIVE NOTCH FILTERS

The input signal $u(n)$ considered is formed by M sinusoids with unknown amplitude p_i and frequency w_{oi}, immerse in additive measurement noise $\nu(n)$, with variance σ_ν^2. The input signal is given by

$$u(n) = \sum_{i=1}^{M} p_i \sin(w_{oi} n + \eta_i) + \nu(n)$$

where η_i is the corresponding phase of sinusoid i. The notch filter $H(z)$, defines an output signal $y(n) = H(z)u(n)$. The output signal variance is given by

$$E[y^2(n)] = < H(z), H(z) >_{S_u} + < H(z), S_\nu(z)H(z) >$$

where the first term describes the inner product induced by the sinusoidal components, i.e., $< H(z), H(z) >_{S_u} = \sum_{i=1}^{M} p_i^2 |H(e^{jw_i})|^2$. $S_\nu(z)$ is the power spectral density of $\nu(n)$, and the second term is the standard inner product in L_2, $< H(z), S_\nu(z)H(z) >= \frac{1}{2\pi}\int_{-\pi}^{\pi} S_\nu(e^{jw})|H(e^{jw})|^2 dw$.

The factorized all-pass notch filter $H(z)$ is defined by

$$H(z) = \frac{1}{2}\left[1 + V(z)\right] = \frac{1}{2}\left[1 + \prod_{i=1}^{M} V_i(z)\right]$$

where

$$V_i(z) = \frac{\overline{D}_i(z)}{D_i(z)} = \frac{s_{2i} + s_{1i}(1 + s_{2i})z^{-1} + z^{-2}}{1 + s_{1i}(1 + s_{2i})z^{-1} + s_{2i}z^{-2}}$$

$s_{1i} = \sin\theta_{1i}$ and $s_{2i} = \sin\theta_{2i}$. Note that using this realization the independence between notch frequencies and the corresponding 3-dB bandwidths B_i for each pole pair is maintained. Note also that, on the unit circle, and in terms of their phase $\phi_i(w)$, $V_i(e^{jw}) = e^{j\phi_i(w)}$ can be described by $\cos(\phi_i(w)/2) = \frac{(1+s_{2i})(s_{1i}+\cos w)}{|D_i(w)|^2}$, and also $\sin(\phi_i(w)/2) = -\frac{(1-s_{2i})\sin w}{|D_i(w)|^2}$ and $|D_i(w)|^2 = (1+s_{2i})^2(s_{1i}+\cos w)^2 + (1-s_{2i})^2\sin^2 w$.

Using this new model, a family of ANF can be developed [11]. Specifically, one kind of algorithms can be designed using a *Recursive Prediction Error* approach (similar to that in [7]). For space reasons we discuss here only a different, not conventional, approach.

Using the factorized all-pass model (i.e., avoiding the root-finding requirement for a direct-form realization as in [1]), an a posteriori off-line error linear in the parameters is defined to minimize the output signal variance. The minimization of the following error is considered

$$e(n+1) = \frac{1}{2}\left(\prod_{i=1}^{M}\frac{\overline{D}_i^{n+1}(z)}{D_i^n(z)} + \prod_{i=1}^{M}\frac{D_i^{n+1}(z)}{D_i^n(z)}\right)u(n) \quad (1)$$

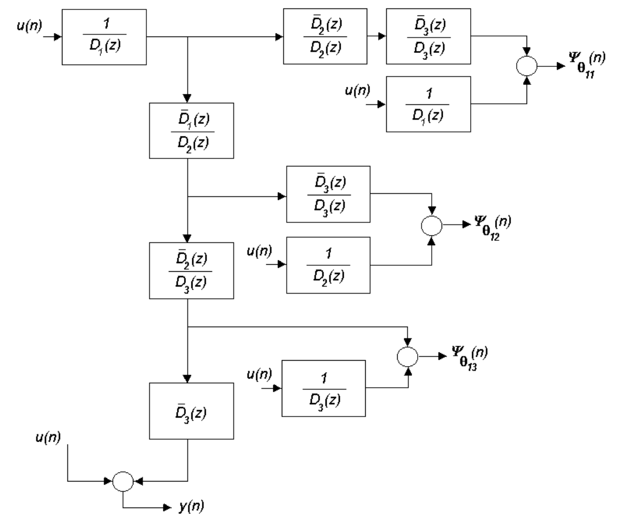

Fig. 1. Gradient and filter realization for the Factorized all-pass based adaptive notch filter, sixth order example.

In order to obtain an on-line algorithm, suitable for adaptive notch filtering, the a priori error obtained from (1) can be written as

$$e(n) = \frac{1}{2}\left[1 + \prod_{i=1}^{M}\frac{\overline{D}_i^n(z)}{D_i^n(z)}\right]u(n)$$

Then, following the instantaneous gradient of the on-line mean-squared-error, the regressor is given by

$$\Psi_{\theta_{1i}}(n) \quad = \quad -\left[\frac{z^{-1}}{D_i(z)} + F_i(z)\right]u(n)$$

where $F_i(z) = \frac{z^{-1}}{D_i(z)}\prod_{k\neq i}\frac{\overline{D}_k(z)}{D_k(z)}$ defines an ortogonal basis $\in L_2$, useful not only for analysis but also to describe the realization of the proposed ANF [4]. Finally, the following update equations for the *Factorized all-pass adaptive notch filter* (FANF) results

$$\begin{aligned} \theta_{1i}(n+1) &= \theta_{1i}(n) + \frac{\mu}{r(n)}e(n)\Psi_{\theta_{1i}}(n) \qquad (2)\\ r(n+1) &= (1-\lambda)r(n) + \mu|\Psi_{\theta_{1i}}(n)|^2 \end{aligned}$$

where $0 < \lambda < 1$ is the forgetting factor and μ ($\cong 1 - \lambda$) is the step size. An efficient realization of the FANF is illustrated in Figure 1, where the orthogonal functions $F_i(z)$ were used. A justification for the normalization factor $r(n)$ will be given in the next section.

3. CHARACTERIZATION AND PROPERTIES

In order to study FANF stationary points and convergence (for $\nu(n)$ white noise) we consider, for the moment, only (2). Then, using the inner product notation, we obtain

$$\begin{aligned}\frac{\partial E[e^2(n)]}{\partial \theta_{1i}} &= -\left\langle \left[\frac{z^{-1}}{D_i(z)} + F_i(z)\right], [1+V(z)] \right\rangle_{S_u} \\ &\quad -\sigma_\nu^2 \left\langle \left[\frac{z^{-1}}{D_i(z)} + F_i(z)\right], [1+V(z)] \right\rangle \end{aligned} \tag{3}$$

that is formed by a signal induced part and a noise induced part, respectively. Considering only the noise induced part, and using the fact that $F_i(z)$ is an ortogonal basis related to $V(z)$ [4] [10], it follows that

$$\begin{aligned}&\left\langle \frac{z^{-1}}{D_i(z)}, 1 \right\rangle + \left\langle z^{-1}F_i(z), 1 \right\rangle \\ &\quad + \left\langle 1, z^{-1}F_i(z) \right\rangle + \left\langle 1, z^{-1}F_i(z) \right\rangle = 0\end{aligned}$$

since all these inner products involve projections of strictly causal functions on a constant. This indicates that the noise induced term does not have any influence on the stationary points.

On the other hand, using the second-order section all-pass phase $\phi_i(w)$ and after some straightforward (but tedious) algebraic manipulations, the signal induced part can be written as

$$\begin{aligned}\frac{\partial E[e^2(n)]}{\partial \theta_{1i}} &= -\sum_{k=1}^{M} \frac{p_k^2}{|D_i(w_{ok})|} \left[\cos(\frac{\phi_i(w_{ok})}{2}) \right. \\ &\quad \left. + \cos(\frac{\phi_i(w_{ok})}{2} + \phi^i(w_{ok}))\right]\end{aligned}$$

where $\phi^i(w) = \sum_{m=1, m\neq i}^{M} \phi_i(w)$. After some reordering this can be written as

$$\frac{\partial E[e^2(n)]}{\partial \theta_{1i}} = B_{1i} - B_{2i} \tag{4}$$

where

$$B_{1i} = -\sum_{k=1}^{M} \frac{2p_k^2(1+s_{2i})}{|D_i(w_{ok})|^2} \left[\cos^2(\frac{\phi^i(w_{ok})}{2})\right] (s_{1i} + \cos w_{ok})$$

$$B_{2i} = \sum_{k=1}^{M} \frac{2p_k^2(1-s_{2i})}{|D_i(w_{ok})|^2} \left[\cos(\frac{\phi^i(w_{ok})}{2}) \sin(\frac{\phi^i(w_{ok})}{2})\right] \sin w_{ok}$$

Remarks:

i) Following a local analysis, it is not hard to note that close to w_{ok}, $\phi^i(w_{ok}) \cong 2\pi q$, for some integer q, $1 < q < M-1$ (all sections converged except the i), then $B_{2i} \cong 0$ and $B_{1i} \cong 0$ at $s_{1i} = -\cos w_{ok}$, for $k = 1, \cdots, M$.

ii) B_{2i} represents a deterministic bias (for constant w_{oi}) that can be asymptotically eliminated using $s_{2i} \to 1$, in general.

iii) Due to the term $|D_i(w_{ok})|^2$ at w_{oi}, for $s_{2i} \to 1$, the dominant term in B_{1i} results in

$$\frac{\partial E[e^2(n)]}{\partial \theta_{1i}} \cong -\frac{2p_i^2(1+s_{2i})}{|D_i(w_{oi})|^2}(s_{1i} + \cos w_{oi}) \tag{5}$$

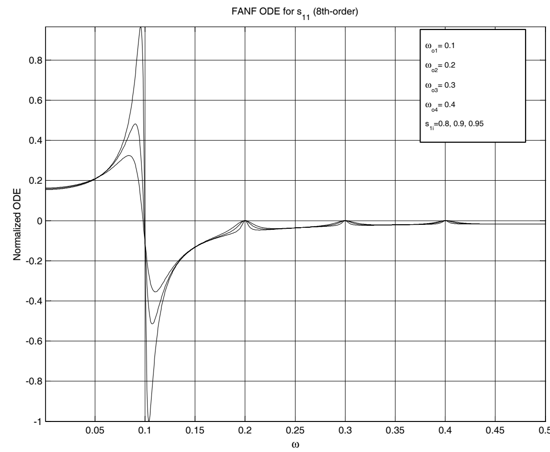

Fig. 2. ODE associated to FANF for an 8th-order example.

then convergence can be proved (see [10], Chapter 10).

iv) As expected, for a cascade realization, there are multiple equivalent stationary points, in the same way that in [5] or [2].

v) Since only s_{1i} intervenes in (5) (i.e., at the stationary points the gradients are approximately orthogonal), then a normalized stochastic gradient algorithms will have similar convergence behavior that a complete Gauss-Newton algorithm. This justify the choice of the normalization factor $r(n)$.

4. EVALUATION AND COMPARISONS

In order to illustrate the behavior of FANF, stationary points can be characterized evaluating (3). Figure 2 depicts the frequency evaluation (using $s_{1i} = -\cos w$) for a four-sinusoids example of $\partial E[e^2(n)]/\partial\theta_{11}$, where $s_{2i} = 0{,}8, 0{,}9$ and 0,95 were used. As can be observed, there are multiple stationary points and when $s_{2i} \to 1$ the ODE is dominated by $s_{11} \cong -\cos w_{o1}$.

As advanced, an interesting aspect of FANF bias (in addition that it can be reduced with $s_{2i} \to 1$) is the fact that it is not related to the input SNR, as is the case of [5] or [2]. In order to illustrate this, the ODE associated to both algorithms: FANF and [2], for a 4-th order example, was evaluated in terms of the SNR. This is depicted in Figure 3 where, different from the fixed bias related to FANF, the solution of [2] has a SNR-related bias. As can be noted, lower bias for lower SNR is obtained with FANF.

More interesting is the performance of the FANF when tracking is the main objective. A study of the performance of the FANF for the same frequencies than in the first example is illustrated in Figure 4. In this case, the SNR is 0 dB and the evaluation was made considering a linear variation (chirp rate $\beta = 10^{-4}$) in w_{o1} with $s_{2i} = 0{,}95$. Similar to

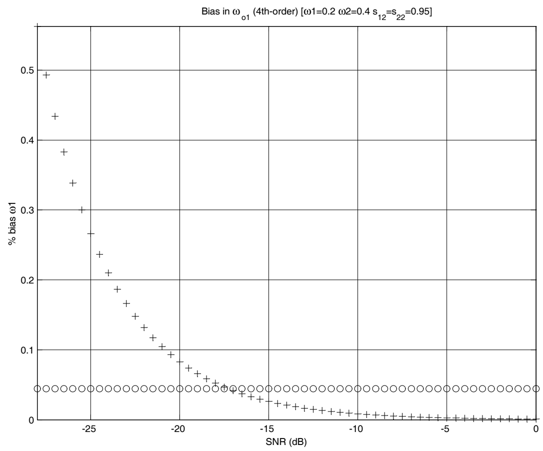

Fig. 3. Bias in w_{o1} using FANF ($\circ$) and the algorithm of [2] ($+$), for a 4th-order example, for different input SNR.

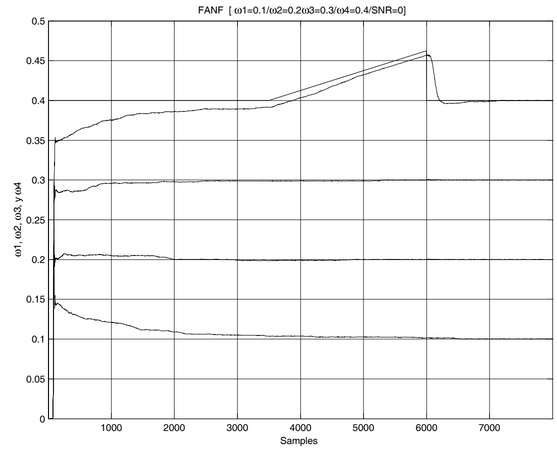

Fig. 4. Learning curves for FANF in a tracking application (8-th order example, SNR$=0$ dB, 100 averaged runs).

other extensive computer simulations performed, a constant tracking error can be observed. This behavior is analogous to that obtained in [3]. This was expected since both algorithms are similar near stationary points, although no formal proof is available yet.

5. CONCLUSIONS

A new family of IIR ANF in the context of a factorized all-pass based realization was presented. A study of the stationary points and local convergence analysis was also outlined. Evaluation and comparisons were used to illustrate the expected properties of the algorithms presented. Owing to their particular kind of bias, the new algorithms represents an interesting alternative for tracking applications. Further research is being performed to characterize formally their tracking performance, although preliminar result obtained using extensive simulations are promissory.

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