REDUCED-RANK BLIND ADAPTIVE FREQUENCY-SHIFT FILTERING FOR SIGNAL EXTRACTION

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ABSTRACT

In this paper, we first illustrate that a blind adaptive frequency-shift (BA-FRESH) filter can be represented as a generalized sidelobe canceler (GSC). Since the computational power of the BA-FRESH filter is quite high, a reduced-rank implementation is thus proposed and achieved by using the eigen-subspace method. To avoid under representation, a rule for choosing the rank/dimension of the signal subspace is introduced by looking at the eigenvalue spread of the signal covariance matrix. The proposed PCA-based reduced-rank BA-FRESH filter not only has a lower computational complexity, but is also more efficient in signal extraction when compared with the conventional, CSP-based and Krylov subspace-based BA-FRESH filters. The performance of this new method in reducing the spectrally overlapped interference of BPSK signals has been examined rigorously.[1]

1. INTRODUCTION

The cyclostationary properties of different signals can be employed to design a frequency-shift (FRESH) filter to separate the spectrally overlapped signals in a multiuser communication environment [1]. Such a filter will achieve the Wiener solution when the desired signal is known *a priori*. However, for most practical applications, the blind adaptive FRESH (BA-FRESH) filter is preferred because information of the desired signal is usually not available [2]. It has been shown that the BA-FRESH filter is capable to converge asymptotically to the optimum Wiener solution, however, the computational requirement is also substantial. On the other hand, it is easy to verify that the expression of the optimum BA-FRESH filter is consistent with that of the generalized sidelobe canceler (GSC) which has been widely studied in array signal processing [3]. By making use of this result, we reformulate the BA-FRESH filter into the mean-square error (MSE) domain that leads to a reduced-rank implementation of the BA-FRESH filter based on the eigen-subspace method.

When using subspace method, the dimension of the model is an consideration. In [4], the Akaike's information theoretic criterion (AIC) has been applied to determine the dimension of the signal subspace based on the maximum likelihood estimates and the minimum description length (MDL) criterion has been proposed in [5]. However, the performance of these two methods varies greatly with the signal-to-noise ratio. In this paper, we propose a simpler and straightforward method to select the desired signal subspace dimension. It is determined by the spread of the normalized eigenvalues of the signal covariance matrix. D is selected to be the signal subspace rank if the $(D+1)^{th}$ normalized eigenvalue is dropped to less than one-fourth of its preceding eigenvalue or it is lower than 20% of the overall eigenvalue spread. This can avoid under or over modelling of the signal subspace.

Simulation results are used to evaluate the performance of the proposed eigen-based reduced-rank (RR) BA-FRESH filter. It is shown that an appropriate reduced-rank can always be obtained and the RR BA-FRESH filter is capable to suppress the spectrally overlapped interfering signals energy, which outperforms the Krylov subspace-based one.

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2. FORMULATION OF REDUCED-RANK BA-FRESH FILTER

Given a real input $x(n)$, the output of a FRESH filter [1] is governed by

$$y(n) = \sum_{m}^{M} h_m(n) \otimes [x(n)e^{j2\pi\alpha_m n}] = \mathbf{h}^{\dagger}\mathbf{x}(n) \qquad (1)$$

where $\otimes$ denotes the convolution operation, α_m is the cycle frequency parameter, † represents the Hermitian conjugate of a vector or a matrix, $\mathbf{h}$ is the impulse response of the FRESH filter with length L, and $\mathbf{x}(n)$ is an ML order vector whose components contain the frequency-shifted versions of $x(n)$. Here $x(n)$ is assumed to consist of the desired signal $s(n)$, the interfering signal $i(n)$ which is spectrally overlapped with $s(n)$ and the white Gaussian noise $v(n)$. Similar to [2], we construct a reference signal $d(n) = x(n)e^{j2\pi\alpha^o n}$ for $\alpha^o \neq \alpha_m, m = 1, 2, \cdots, M$. Of course, this reference signal also contains interferences that are spectrally overlapped with the desired signal plus background noise.

We assume all the signals, including the desired and interfering signals, are real and cycloergodic in mean and in autocorrelation and are independent from sample to sample [2]. In addition, we assume that the cycle frequencies of the desired signal are different from that of the interferer. We will blindly adapt the FRESH filter with priori knowledge of only its modulation type, carrier frequency and baud rate.

Our goal is to adjust the coefficients of $\mathbf{h}$ so as to reduce the interferences embedded in $d(n)$ to minimum. This can be attained by minimizing the mean-square error (MSE) between $d(n)$ and $y(n)$ which is given by

$$E\{|d(n) - y(n)|^2\} = \sigma_d^2 - 2\mathbf{h}^\dagger \mathbf{r} + \mathbf{h}^\dagger \mathbf{R}\mathbf{h} \tag{2}$$

where $\sigma_d^2 = E\{|d(n)|^2\}$, $\mathbf{r} = E\{\mathbf{x}(n)d^*(n)\}$ and $\mathbf{R} = E\{\mathbf{x}(n)\mathbf{x}^\dagger(n)\}$. It is straightforward to find the optimum coefficient vector to be equal to

$$\mathbf{h}_{opt} = \mathbf{R}^{-1}\mathbf{r} \tag{3}$$

Substituting (3) into (2) yields the minimum MSE (MMSE):

$$MMSE = \sigma_d^2 - \mathbf{r}^\dagger \mathbf{R}\mathbf{r} = \sigma_d^2 - \sum_{i=1}^{ML} \frac{|\mathbf{v}_i^\dagger \mathbf{r}|^2}{\lambda_i} \tag{4}$$

where λ_i and $\mathbf{v}_i$ are the i^{th} eigenvalue and its corresponding eigenvector of $\mathbf{R}$ respectively. From the standpoint of subspace analysis, the filtering process is the same as projecting the received data vector $\mathbf{x}(n)$ onto the signal subspace whose dimension, say *D*, is in general much smaller than that of the received data vector, i.e. $D \ll ML$. The basic vectors that span the desired signal subspace can be obtained by using the principal component analysis (PCA) [6], the cross-spectral (CSP) metric [7] or the Krylov subspace method [8].

2.1. CSP Method

To minimize (4), the CSP method chooses the *D* eigenvectors to associate with the largest *D* values of the $|\mathbf{v}_i^\dagger \mathbf{r}|^2/\lambda_i$. Thus, a different choice of these eigenvalues will minimize the mean-square error as a function of the rank. It is desirable that the steady-state performance of the RR BA-FRESH be as close as possible to the optimal BA-FRESH filter for each value of the rank. Although the CSP method is the optimal eigen-based subspace selection rule of Gaussian signals in an information-theoretic sense, it requires a huge computational complexity, as much as $(ML)^3$ multiplication operations is needed, to search for the full eigen-subspace in order that the cross-spectral components can be determined.

2.2. PCA Method

On the other hand, the PCA method only uses those eigenvectors corresponding to the first *D* largest eigenvalues of $\mathbf{R}$ to form the rank *D* signal subspace. The computational complexity of this method can be greatly reduced to $\{D \cdot (ML)^2\}$ multiplications and it can be further reduced by fast PCA algorithm implementation [10].

Appropriate Choice of Rank

Since the eigenvectors with the first D largest eigenvalues form the rank *D* signal subspace, the performance of BA-FRESH filtering also depends on the rank. Thus, D should be carefully chosen. In our work, we use the eigenvalue spread as an indicator to select a suitable D. Without loss of generality, let $\hat{\lambda}_1 \geq \hat{\lambda}_2 \geq ... \geq \hat{\lambda}_{ML}$ be the normalized eigenvalues in descending order with $\hat{\lambda}_1 = 1$. The eigenvalue spread is given by,

$$\lambda_S = 1 - \hat{\lambda}_{ML} \tag{5}$$

The first D eigenvectors will be used if the $(D+1)^{th}$ normalized eigenvalue is decreased to less than one-fourth of the D^{th} eigenvalue or smaller than 20% of the eigenvalue spread. That is

$$\hat{\lambda}_{D+1} < 0.25\hat{\lambda}_D \ \ or \ \ \hat{\lambda}_{D+1} < 0.2\lambda_S \tag{6}$$

In order to ensure signal information is adequately represented while the computational cost remains affordable, we apply the following constraint: $3 \leq D < ML/3$. Under these criteria, although empirical, the eigen-based RR BA-FRESH filter almost always gives a better performance with lower computational complexity than that of using an arbitrary fixed rank D, which very often led to under or over modelling.

Let $\mathbf{V}_D$ denote the transformation matrix whose columns consist of the *D* eigenvectors obtained by the CSP or PCA method. Then, introduce the transformed data vector as

$$\tilde{\mathbf{x}}(n) = \mathbf{V}_D^\dagger \mathbf{x}(n) \tag{7}$$

such that

$$y(n) = \tilde{\mathbf{h}}^\dagger \tilde{\mathbf{x}}(n) \tag{8}$$

where $\tilde{\mathbf{h}}$ denotes the *D*-dimensional coefficient vector of the RR BA-FRESH filter and it is given by

$$\tilde{\mathbf{h}} = \tilde{\mathbf{R}}^{-1}\tilde{\mathbf{r}} \tag{9}$$

where $\tilde{\mathbf{R}} = \mathbf{V}_D^\dagger \mathbf{R}\mathbf{V}_D$ and $\tilde{\mathbf{r}} = \mathbf{V}_D^\dagger \mathbf{r}$. Fig. 1 illustrates the block diagram of the proposed reduced-rank BA-FRESH filter. Clearly, the computational requirement of $\tilde{\mathbf{h}}$ is greatly reduced when compared to the conventional BA-FRESH filter $\mathbf{h}$ due to the condition that $D \ll ML$. Furthermore, a lower-rank filter will converge faster because the adaptation characteristic of the least squares class of adaptive algorithm is a function of the filter order [2].

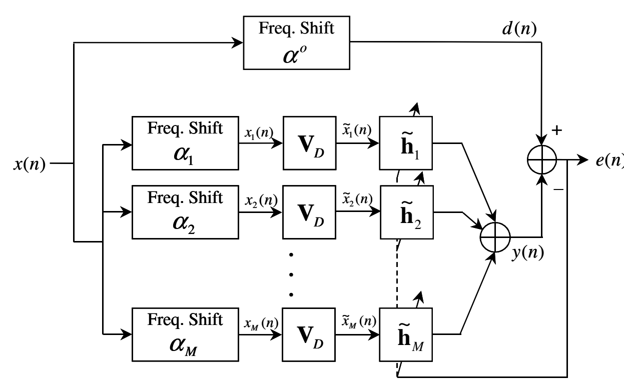

Fig. 1. Block Diagram of The Proposed RR BA-FRESH Filter

3. RANK REDUCTION ON KRYLOV SUBSPACE

Besides the eigen-subspace rank reduction method, Krylov subspace method is also a useful and popular tool for solving large sets of linear equations, e.g. the multistage Wiener filter (MSWF), the conjugate gradient reduced-rank filter (CGRRF) and the powers of $\mathbf{R}$ (POR) method [8], [9]. Krylov subspace is generated by taking the powers of the covariance matrix of observations on a cross-correlation vector, where

$$K^D(\mathbf{R}, \mathbf{r}) = span\{\mathbf{r}, \mathbf{Rr}, \mathbf{R}^2\mathbf{r}, \cdots, \mathbf{R}^{D-1}\mathbf{r}\} \quad (10)$$

is referred to a D^{th} Krylov subspace. Computation of Krylov subspace requires $\{(D-1)(ML)^2\}$ multiplication operations, which is slightly less than that of the conventional PCA implementation. Rank reduction by Krylov subspace can be applied to the BA-FRESH filter by simply putting $K^D(\mathbf{R}, \mathbf{r})$ as the transformation matrix $\mathbf{V}_D$ in (8).

We intend to investigate the rank reduction effectiveness of our proposed PCA-based RR BA-FRESH filter versus the one using Krylov subspace, in terms of the computational and signal extraction efficiency in the following simulation section.

4. SIMULATION RESULTS

Hereunder, we examine the performance of the proposed PCA-based RR BA-FRESH filter in comparison with the conventional BA-FRESH filter, CSP-based and Krylov subspace-based RR BA-FRESH filters respectively. We generated the desired signal and the interfering signal from [2], which are both BPSK signals, with

$$s(nT_s) = \sum_{k=-\infty}^{\infty} b_k p(nT_s - kT_{b1}) \cos(2\pi f_1 nT_s) \quad (11)$$

$$i(nT_s) = \sum_{k=-\infty}^{\infty} \beta_k p(nT_s - kT_{b2}) \cos(2\pi f_2 nT_s) \quad (12)$$

where T_{b1} and T_{b2} denote the baud periods, f_1 and f_2 denote the carrier frequencies of $s(nT_s)$ and $i(nT_s)$ respectively, and T_s is the sampling period. $\{b_k\}$ and $\{\beta_k\}$ are stationary random binary sequences. The pulse shaping filter $p(nT_s)$ is the raised cosine filter with a 100% rolloff factor.

In the following two simulations, the baud rates of both the desired and interfering signals are equally set as 2kHz. The carrier frequency of the desired signal is fixed at f_1 = 10kHz, whereas the carrier frequency of the interference f_2 changes from 11kHz to 14kHz, which produced a spectral overlap of the two signals, that is clearly shown in Fig. 2. Both the input signal-to-interference ratio (SIR) and the signal-to-noise ratio (SNR) are 0dB. The cycle frequencies are set as α_1 = 20kHz and α^o = 0Hz. The filter order is fixed at ML = 30. A total of 50 independent trials of problem were performed for each simulation.

4.1. Simulation 1: A Fixed Rank

Fig. 2 shows the normalized power spectral density (PSD) of the received signal and the filtered signal for D is arbitrarily chosen to be 3. It can be observed that the spectrum of the desired signal can be separated from the received data with spectrally overlapped interference plus white Gaussian noise by using the RR BA-FRESH filter. It can also be seen that the proposed PCA-based RR BA-FRESH filter outperforms the full-rank BA-FRESH filter (i.e. the signal subspace rank is chosen to be the filter order ML) and the CSP-based one.

However, there is a spurious peak occurs in the low frequency band which has an amplitude comparable to the desired signal. This undesirable effect is probably due to an inappropriate choice of rank. If a larger D is chosen, noise would be wrongly projected to the signal subspace. On the other hand, if a smaller D is used, part of the signal would not be represented. So, spurious peaks may appear in the signal spectrum as shown in Fig. 2. Thus, an adaptive method for choosing rank D is needed as described in Section 2.2.

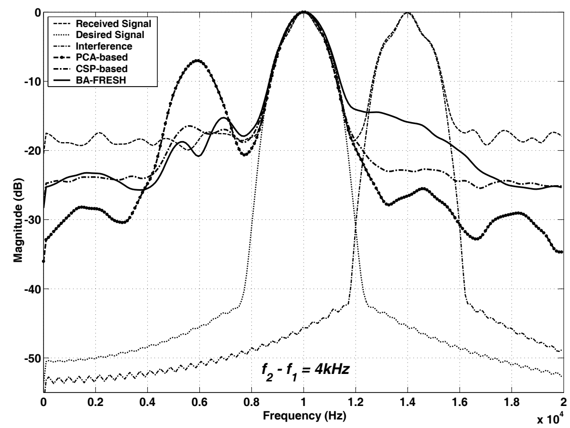

Fig. 2. PSD with 4kHz Frequency Difference ($D = 3$)

4.2. Simulation 2: A Variable Rank

In simulation 2, we use (6) to choose the rank of the signal subspace D. The updating process of D is done from frame to frame adaptively with frame length equals to 30 time samples, i.e. the filter order. Its performance is also compared with the Krylov subspace-based RR BA-FRESH filter with rank D being arbitrarily set as 3. From Fig. 3, we found that the PCA-based RR BA-FRESH filter with the proposed rank selection method can reduce the interfering signal energy effectively even when the interferer is spectrally close to the desired signal, with a difference of 1kHz only. As shown in Fig. 4, the proposed PCA-based RR BA-FRESH filter still outperforms the full-rank BA-FRESH filter and the CSP-based one for different spectrally overlapped interfering signals.

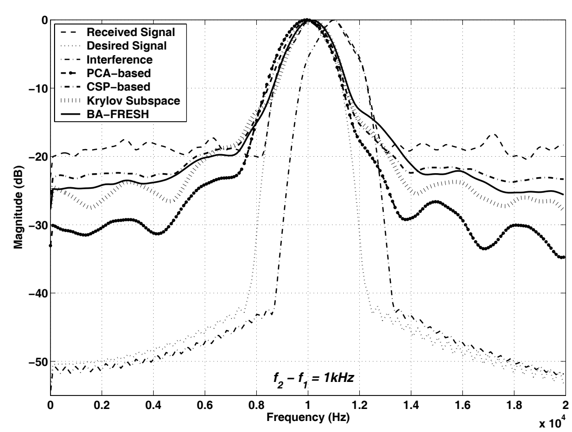

Fig. 3. PSD with 1kHz Frequency Difference (Variable D)

When comparing with the one using a fixed rank in Section 4.1, the performance improvement of the proposed filter is significant. By applying (6), a smaller signal subspace rank D is used when the interferer is closer to the signal, since the eigenvalue spread is greater. Thus, the unwanted peaks in the low frequency ranges can be greatly reduced in this case. In addition, the performance of the RR BA-FRESH filter on Krylov subspace is also not as good as that of the proposed one, although carrying out rank reduction on Krylov subspace has a slightly lower computational cost.

The results show that in a multiuser environment, the energy of the interfering signal is mainly contained within the PCA subspace which enables us to implement effectively the RR BA-FRESH filter. Unlike the CSP method, the PCA method only requires the calculation of the first D principal eigenvectors of $\mathbf{R}$. Furthermore, many fast PCA algorithms are available. Therefore, the PCA-based RR BA-FRESH filter is preferred for practical applications due to its computational simplicity and better signal extraction performance.

5. CONCLUSION

A reduced-rank blind adaptive frequency-shift (RR BA-FRESH) filter has been proposed. The PCA-based implementation of the BA-FRESH filter is able to separate effectively the desired signal from spectrally overlapped interfering signal. An adaptive method to select an appropriate rank D signal subspace is introduced. It is based on the normalized eigenvalue spread of the signal covariance matrix and simulation results show that the performance is greatly improved than that of using an arbitrarily chosen fixed rank. The fast convergence and computational simplicity of the PCA algorithms can be employed to implement the RR BA-FRESH filter, and it is expected to find practical applications in multiuser communication environment as it can reduce the spectrally overlapped interfering signals.

6. REFERENCES

[1] Gardner W.A.: "Cycle Wiener Filter: Theory and Method", *IEEE Trans. Commun.*, 1993, 41, pp. 151-163.

[2] Zhang J., Wong K.M., Luo Z.Q. and Ching P.C.: "Blind Adaptive FRESH Filtering for Signal Extraction", *IEEE Trans. Signal Processing*, 1999, 47(5), pp. 1397-1402.

[3] Griffith L.J. and Jim C.W., "An Algorithm for Linearly Constrained Adaptive Array Processing", *Proc. IEEE AP*, Aug 1972, 60(8), pp. 929-935.

[4] Akaike H., "Information Theory and an Extension of the Maximum Likelihood Principle", *Proc. 2nd Int. Symp. Information Theory, Supp to Problems of Control and Information Theory*, 1972, pp. 267-281

[5] Schwartz G., "Estimating the Dimension of a Model", *Ann. Stat.*, 1978, Vol. 6, pp.461-464

[6] Haimovich A.M. and Ness Bar Y.: "An Eigenanalysis Interference Canceler", *IEEE Trans. Signal Processing*, 1991, 39(1), pp. 76-84.

[7] Goldstein J.S. and Reed I.S.: "Reduced-Rank Adaptive Filtering", *IEEE Trans. Signal Processing*, 1997, 45(2), pp. 492-496.

[8] Burykh S. and Abed-Meraim K.: "Reduced-Rank Adaptive Filtering Using Krylov Subspace", *EURASIP Journal on Applied Signal Processing*, Dec 2002, No. 12, pp.1387-1400.

[9] Burykh S. and Abed-Meraim K.: "Multi-Stage Reduced-Rank Filter with Flexible Structure", *Proc. EUSIPCO*, 2002, Vol. III, pp. 221-224.

[10] Ouyang S., Bao Z. and Liao G.S.: "Robust Recursive Least Squares Learning Algorithm for Principal Component Analysis", *IEEE Trans. Neural Networks*, 2000, 11(1), pp. 215-221.

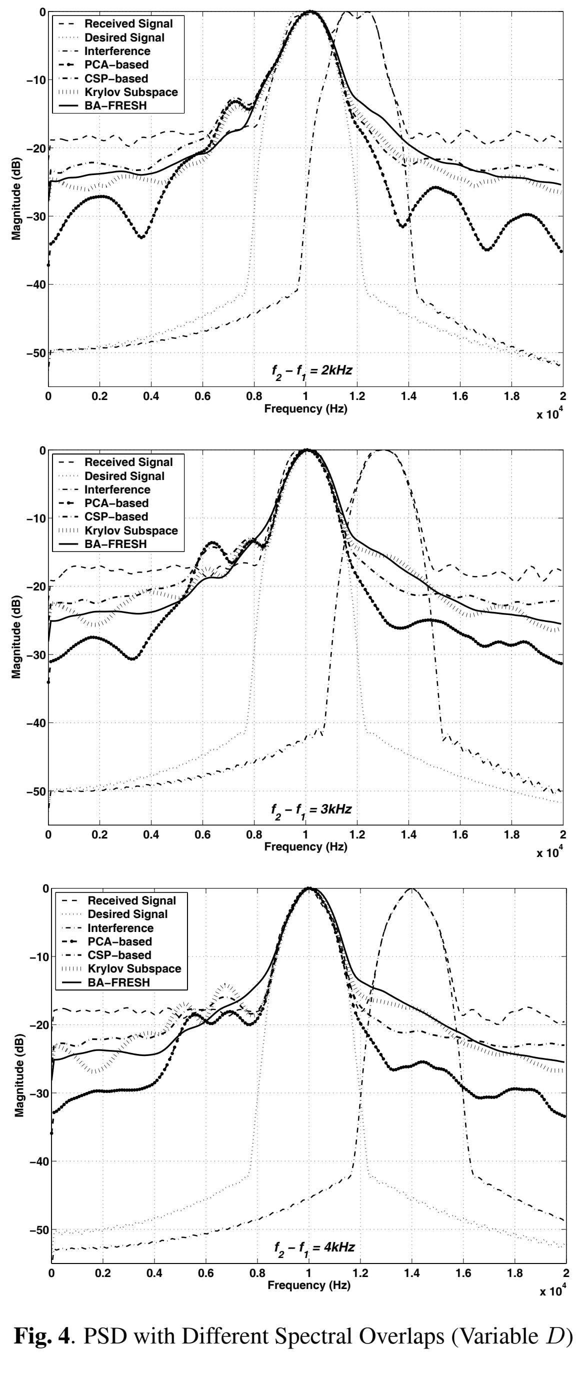

Fig. 4. PSD with Different Spectral Overlaps (Variable D)