

POLE OPTIMISATION IN ADAPTIVE LAGUERRE FILTERING

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ABSTRACT

In Laguerre-based adaptive filtering, the pole can be used as an extra degree of freedom. An optimal pole is difficult to determine on the basis of a squared error criterion. A compaction criterion, on the other hand, does yield a simple explicit expression for the pole in terms of characteristics derived from the impulse response of the system. In this paper, it is shown how these characteristics can be calculated directly from a Laguerre expansion of the impulse response. For truncated Laguerre series, this implies a much faster algorithm than the one based on characteristics derived from the impulse response. Such an algorithm can be used to construct an adaptive Laguerre-based adaptive system, where the current estimated Laguerre expansion can be used to determine the pole in a next step. Simulation results on the behaviour of such a system for a synthetic signal are given in terms of signal cancellation and convergence speed.

1. INTRODUCTION

A general scheme of adaptive filtering is depicted in Fig. 1. The adaptive filter is shown in the dashed box and receives as input signal x and y and produces an output signal e . Internally, the adaptive filter consists of a filter F with adaptive filter coefficients and a control box C which tunes the coefficients of F . The tuning is done typically on basis of the signals x , y and e .

The signal y is assumed to consist of two components, one correlated and the other uncorrelated with x . The correlation is assumed to stem from an unknown, slowly time-varying linear system G . The adaptive system tries to remove the correlation between x and y in as far as the modelling capabilities in F allow this. Typically, the filter F is a tapped delay line. Other choices are possible as well. Here we consider Laguerre-based [1] adaptive filtering. In that case, an extra pole appears when compared to

a TDL. This is an advantage because this pole can be tuned to G in order to obtain a better compromise between the order of the filter F and the resulting performance. The question that arises then is whether this pole can be automatically tuned.

Tuning of the pole in a Laguerre filter has already drawn some attention. An expression for a necessary condition for pole according to a squared error criterion is given in [2]. Unfortunately, this condition does not lead to an explicit expression for the pole and, furthermore, it is order dependent. Optimisation of the pole according to a compaction criterion was considered by several authors for a real pole [3, 4, 5, 6, 7] as well as for a complex pole [8, 9]. The benefits from this approach are that it leads to a simple and order-independent explicit expression for the pole. Applications of tuning the pole in adaptive Laguerre-based filters have been considered in [10, 11] using a squared error criterion and [12] using a compaction criterion.

The problem that we address is that the optimal pole according to the compaction criterion is expressed in terms of the impulse response. This means that the impulse response of the estimated filter has to be determined and from that an estimate of the pole has to be derived. Since the impulse response is a linear function of the Laguerre expansion coefficients, it is clear that the optimal pole can be expressed directly in the expansion coefficients. The pertinent expressions are derived (Section 2), and have the advantage that the computational costs associated with determining the optimal pole are reduced to a large extent.

Such an approach was already followed in [12] for a real pole. In this paper, these results are extended to cover the case of a complex pole. Though it is no surprise that the results in [12] can be extended to cover the complex pole case, the derivations proved to be more involved than expected. In particular, where in the real pole case the results can be derived from the orthogonality, recurrence relation and difference equation, we need to introduce an additional relation (Section 2).

The behaviour of an adaptive Laguerre system including the pole optimisation is shown by an example (Section 3) where we consider the convergence and signal suppression. We conclude with a discussion (Section 4).

2. FAST POLE DETERMINATION

The discrete time Laguerre functions $\phi_k(\lambda, n)$ are defined [13] by their z -transform $\Phi_k(\lambda, z)$ according to

$$\Phi_k(\lambda, z) = \frac{\sqrt{1 - |\lambda|^2}}{1 - z^{-1}\lambda} \left\{ \frac{-\lambda^* + z^{-1}}{1 - z^{-1}\lambda} \right\}^k \quad (1)$$

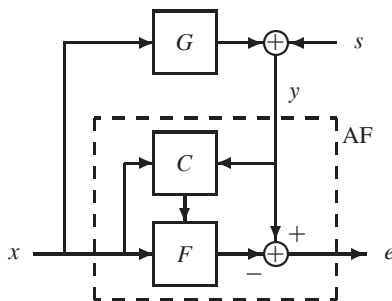


Fig. 1. A general adaptive filtering (AF) scheme.

with $k \in \mathbb{N}$, $\lambda \in \mathbb{C}$ and $|\lambda| < 1$. The Laguerre functions constitute an orthonormal set on the interval $n \in \mathbb{N}$. The Laguerre system can be implemented efficiently as a tapped all-pass line preceded by the section

$$A_0(z) = \frac{\sqrt{1-|\lambda|^2}}{1-z^{-1}\lambda}. \quad (2)$$

The allpass sections are denoted by A with

$$A(z) = \frac{-\lambda^* + z^{-1}}{1 - z^{-1}\lambda}. \quad (3)$$

The Laguerre functions are closely related to the Laguerre polynomials and have a number of attractive properties. From these we mention those which will be used later. Orthonormality:

$$\langle \phi_k, \phi_m \rangle = \delta_{k,m} \quad (4)$$

where the inner product is defined by

$$\langle f, g \rangle = \sum_{n=0}^{\infty} f(n)g^*(n). \quad (5)$$

Recurrence relation:

$$\begin{aligned} n\phi_k(\lambda, n) &= \frac{\lambda(k+1)}{1-|\lambda|^2} \phi_{k+1}(\lambda, n) + \frac{|\lambda|^2 + k(1+|\lambda|^2)}{1-|\lambda|^2} \phi_k(\lambda, n) \\ &\quad + \frac{\lambda^*k}{1-|\lambda|^2} \phi_{k-1}(\lambda, n). \end{aligned} \quad (6)$$

As a last property, the following identity holds:

$$\begin{aligned} (n+1)\phi_k(\lambda, n+1) &= \frac{(k+1)\lambda^2}{1-|\lambda|^2} \phi_{k+1}(\lambda, n) + \\ &\quad \frac{(2k+1)\lambda}{1-|\lambda|^2} \phi_k(\lambda, n) + \frac{k}{1-|\lambda|^2} \phi_{k-1}(\lambda, n). \end{aligned} \quad (7)$$

The proof is given in the appendix. The Laguerre filter is defined by

$$F(z) = \sum_{k=0}^K \alpha_k(\lambda) \frac{\sqrt{1-|\lambda|^2}}{1-z^{-1}\lambda} \left(\frac{-\lambda^* + z^{-1}}{1-z^{-1}\lambda} \right)^{k-1}. \quad (8)$$

An implementation of the Laguerre filter is depicted in Fig. 2.

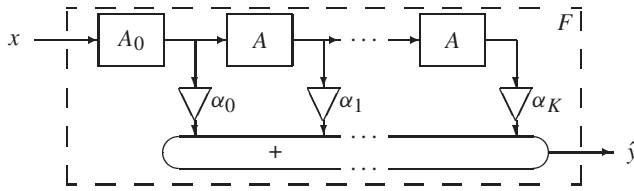


Fig. 2. The Laguerre filter consisting of pre-filter A_0 , a line of allpass sections A and weights α_k .

The optimal Laguerre coefficients are given by the normal equations

$$Q\alpha = P, \quad (9)$$

where $\alpha = [\alpha_0, \alpha_1, \dots, \alpha_K]^T$, and the matrix Q and the vector P have entries given by

$$Q_{k,m} = \sum_n f_m(n)f_k^*(n), \quad (10)$$

$$P_k = \sum_n y(n)f_k^*(n), \quad (11)$$

with $f_k = x * \phi_k$. We note that the matrix Q is a non-negative definite Hermitian Toeplitz matrix. In view of the Toeplitz character of Q , the coefficients can be determined efficiently using the Levinson algorithm. Other well-known adaptive techniques for the coefficients α_k include the RLS and the LMS algorithms.

Given a system G with impulse response $g(n)$, the optimal Laguerre pole according to a compaction criterion is given by [8]

$$\hat{\lambda} = \beta^* \left(1 - \sqrt{1 - 1/|\beta|^2} \right), \quad (12)$$

with

$$\left. \begin{aligned} \beta &= \frac{m_0/2 + m_1}{\mu}, \\ m_0 &= \sum_{n=0}^{\infty} |g(n)|^2, \\ m_1 &= \sum_{n=0}^{\infty} n|g(n)|^2, \\ \mu &= \sum_{n=0}^{\infty} (n+1)g(n)g(n+1). \end{aligned} \right\} \quad (13)$$

Having solved the normal equations for a given λ , we could determine the impulse response f . This impulse response is then the best approximation we can make of the impulse response g of the unknown system. In order to find the optimal λ for the series expansion of g , we can calculate the optimal λ associated with f by evaluating m_0 , m_1 and μ . We will show that this can be done more directly, i.e., from the (finite set of) optimal coefficients $\alpha(\lambda)$ instead of from the (infinite long) impulse response f .

Before we give the results, we introduce some auxiliary measures

$$a_0 = \sum_{k=0}^{\infty} |\alpha_k(\lambda)|^2, \quad (14)$$

$$a_1(\lambda) = \sum_{k=0}^{\infty} k|\alpha_k(\lambda)|^2, \quad (15)$$

$$a_2(\lambda) = \sum_{k=0}^{\infty} (k+1)\alpha_k(\lambda)\alpha_{k+1}^*(\lambda). \quad (16)$$

With these shorthand notations and the definitions (13), we find

$$m_0 = a_0 \quad (17)$$

by using the orthonormality (4),

$$m_1(1-|\lambda|^2) = |\lambda|^2 a_0 + (1+|\lambda|^2)a_1 + 2\Re\{\lambda a_2\} \quad (18)$$

by using the recurrence relation (6) and, finally,

$$\mu(1-|\lambda|^2) = \lambda a_0 + 2\lambda a_1 + a_2^* + \lambda^2 a_2 \quad (19)$$

by using (7).

In summary, where the processing chain was formerly

$$\alpha_k \rightarrow f(n) \rightarrow \{m_0, m_1, \mu\} \rightarrow \beta \rightarrow \hat{\lambda},$$

it is now

$$\alpha_k \rightarrow \{a_0, a_1, a_2\} \rightarrow \{m_0, m_1, \mu\} \rightarrow \beta \rightarrow \hat{\lambda}$$

i.e., avoiding the infinite sequence f .

3. EXAMPLE

As an example, we consider the situation as depicted in Fig. 1. The input signal x consist of a complex white noise signal. The additive noise s was set to zero. The signal G is a second-order Butterworth filter with a time-varying centre frequency. The centre frequency was switched at specific instances in time as illustrated by the spectrogram of y in Fig. 3.

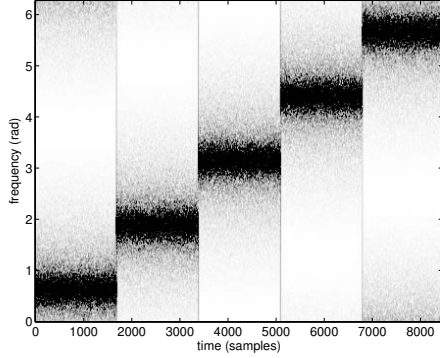


Fig. 3. Spectrogram of the signal y (using overlapping windows). The dark areas indicate the passband.

In the control box, a Hanning-windowed segment of 1024 samples (a frame) was used for calculation of the optimal filter coefficients. The hopsize between frames was taken as 256 samples, and the number of coefficients was 10 (i.e. $K = 9$). For each segment, we start with $\lambda = 0$ and three iterations to determine the optimal pole and the associated prediction coefficients.

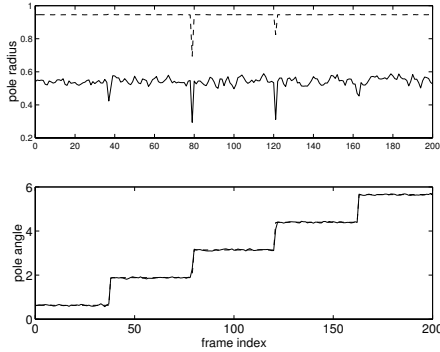


Fig. 4. Estimated pole after the first and third iteration (dashed and solid line, respectively) with initial pole set to 0.

In Fig. 4, the radius and angle of the pole as a function of frame and for iteration 1 and 3 are shown. Further iterations do not change the pole in any significant way. This means that we have a very fast convergence. We also note that the angle of the pole is already very accurate after the first iteration; the other iterations mainly affect the radius.

More important than the convergence is to consider if we profit from using an adaptive pole. The merit is illustrated by the gain that we get where the gain is defined as the ratio between the power

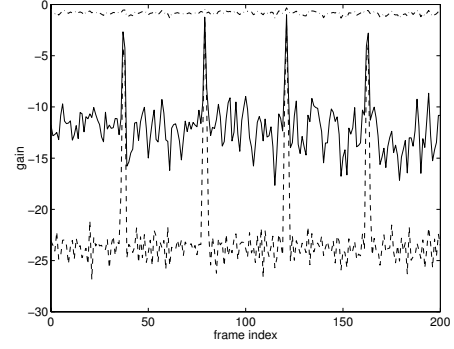


Fig. 5. Gain per frame for $\lambda = 0$ and after the first and third iteration (dash-dotted, solid and dashed line, respectively).

of e divided by that of y per segment. From the plot in Fig. 5, we infer that the gain for the conventional tapped-delay-line amounts to a 1 dB signal suppression, where for the pole after 3 iterations we have about 24 dB (except, obviously, at the transitions of G).

As a second experiment, we used the optimal pole of the previous frame as the initial pole for the current segment and determined the optimal pole and prediction coefficients by one iteration only. The results for the pole are shown in Fig. 6 where the pole after this single iteration is plotted as well as the pole from the previous experiment (i.e., with pole initialisation at 0 and 3 iterations). We observe that these poles are nearly identical except for the transition regions of G . As can be expected, the pole determined in this second experiment lags slightly behind that of the first experiment for a short period after each transition.

A similar effect can be observed when considering the gain. We have plotted the gain per frame together with the gain from iteration 3 in the previous experiment in Fig. 7. Since the pole is slightly lagging behind after a transition in G , so is the gain.

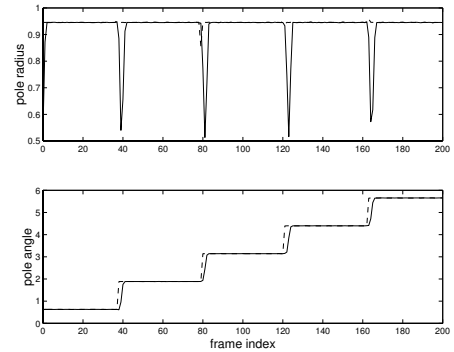


Fig. 6. Estimated pole after one iteration (solid line) when starting from the estimated pole of the previous frame compared the the result of iteration 3 in the previous example (dashed line).

The examples merely illustrate that a system using an adaptive pole can give a performance boost when compared to the traditional TDL adaptive filter and that computational costs for the pole determination are limited and certainly weigh up against the complexity (extra weights) needed to obtain a similar behaviour

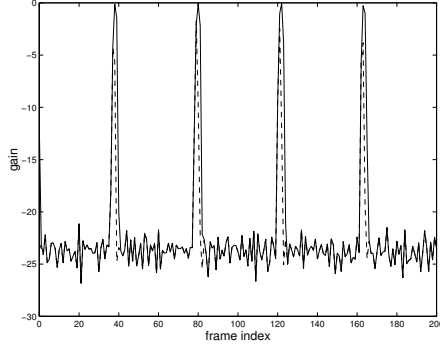


Fig. 7. Estimated gain after one iteration (solid line) when starting from the estimated pole of the previous frame compared the the result of iteration 3 in the previous example (dashed line).

with the TDL. We assume that these conclusions carry over to other more practical control boxes, e.g., involving RLS or LMS coefficient adaptation and an appropriately adapted procedure for pole tuning.

4. CONCLUSIONS AND DISCUSSION

We have derived expressions for a fast pole determination in an adaptive complex Laguerre filter. A simulation of an adaptive system incorporating the developed algorithm indicates good cancellation properties and fast adaptation.

In this paper, we addressed the complex Laguerre system. In practice, real systems and signals are of more importance. It is believed that, with some heuristic measures, the system can be adapted to this situation which would in fact be equal to a heuristic optimisation of the complex-conjugated pole pair in a Kautz system [14]. A simple heuristic rule proposed already in [9] is to apply the estimation of a complex Laguerre pole to the Hilbert transform of the impulse response and use this pole and its complex conjugate as the poles in a Kautz series. From our experiences, this already works satisfactory and eliminates the necessity of the searches as proposed in [15, 16].

Appendix

In this appendix, we give an outline how the relation (7) can be proved. First, we transform the relation to the z -domain. The left-hand side can then be written as

$$\begin{aligned} \mathcal{Z}\{(n+1)\phi_m(\lambda, n+1)\} &= -z^2 \frac{d\Phi_m(\lambda, z)}{dz} \\ &= -z^2 A^m(z) \frac{dA_0(z)}{dz} - z^2 m A_0(z) A^{m-1}(z) \frac{dA(z)}{dz}. \end{aligned} \quad (20)$$

The righthand side can be sorted similarly in two terms as

$$\begin{aligned} &\frac{1}{1-|\lambda|^2} [\lambda \Phi_m(\lambda, n) + \lambda^2 \Phi_{m+1}(\lambda, z)] + \\ &\frac{m}{1-|\lambda|^2} [\Phi_{m-1}(\lambda, z) + 2\lambda \Phi_m(\lambda, n) + \lambda^2 \Phi_{m+1}(\lambda, z)]. \end{aligned} \quad (21)$$

Substituting the definitions of A and A_0 in (20) and performing the differentiation with respect to z followed by some rearranging, proofs the identity of (20) and (21).

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