A BOOTSTRAP SCHEME FOR TIME-FREQUENCY AUTO-TERM SELECTION IN ANTENNA ARRAYS

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ABSTRACT

A method for detection of source signal auto-term regions in the time-frequency plane, based on spatial time-frequency distribution matrices is presented. As opposed to previous methods, a multiple hypothesis test is used in order to strongly control the family wise error rate when testing multiple locations on the time-frequency plane simultaneously. A bootstrap based method for estimating the distribution of the test statistic is also proposed, and the performance in terms of operating characteristics is compared to that of using an asymptotic distribution.

1. INTRODUCTION

Non-stationary signals such as frequency modulated (FM) and polynomial phase signals (PPS) arise in a number of fields including sonar, radar and telecommunications. Recently, the application of time-frequency (TF) analysis to sensor array processing for non-stationary signals has received significant attention in the literature. The use of spatial time-frequency distribution (STFD) matrices in particular has emerged as a natural means for exploiting both the spatial diversity and TF localization properties of non-stationary sources impinging on a sensor array.

Methods for blind source separation [1], direction-of arrival estimation [2, 3] and signal synthesis [4] have been proposed based on STFD processing. It has been shown that the relationship between the STFD of the sensor data and the time-frequency distributions (TFDs) of the sources is identical to that of the sensor data covariance matrix and the sources' correlation matrix. This key property permits direction finding and blind source separation to be performed *using the sources' TF localization properties*. In a 'blind' scenario, no *a priori* knowledge of the source TF localization can be assumed and must therefore be estimated [5, 6, 7].

This paper presents a multiple hypothesis testing (MHT) framework for detecting the locations in the TF plane at which source signals exhibit a significant power concentration. The MHT approach allows testing of multiple points on the TF plane simultaneously while controlling the overall probability of a false detection. A bootstrap based scheme for estimating the null distribution of the test statistics is also proposed. This potentially allows the detector to be used in non-Gaussian, temporally correlated noise environments. Performance of the detector using both the asymptotic and bootstrap estimates of the null distributions is presented in terms of the operating characteristics, for a number of signal-to-noise (SNR) values.

2. SIGNAL MODEL AND STFD MATRICES

We consider an m-element sensor array observing an instantaneous linear mixture of signals emitted from $d < m$ narrowband far-field sources. The vector $\boldsymbol{x}(n) \in \mathbb{C}^{m\times 1}$ represents a snapshot of the baseband array output at sampling instant n, which may be corrupted by an additive noise process $\boldsymbol{v}(n)$. The baseband array output model is

$$\boldsymbol{x}(n) = \boldsymbol{A}\boldsymbol{s}(n) + \boldsymbol{v}(n), \quad n = 1, \ldots, N \tag{1}$$

where $\boldsymbol{A} \in \mathbb{C}^{m\times d}$ is termed the mixing matrix and $\boldsymbol{s}(n) \in \mathbb{C}^{d\times 1}$ is a (deterministic) vector of the source signals. $\boldsymbol{A}$ is assumed to be of full column rank. We also assume that the sources have different localisation properties in the TF plane and are 'uncorrelated' such that

$$\boldsymbol{R_s} \stackrel{\text{def}}{=} \lim_{T\to\infty} \frac{1}{T} \sum_{n=0}^{T-1} \boldsymbol{s}(n)\boldsymbol{s}^{\mathrm{H}}(n) = \boldsymbol{I}. \tag{2}$$

The different receive power of the source signals is assumed in the matrix $\boldsymbol{A}$, so that $\boldsymbol{R_s}$ may be identity without loss of generality. The noise process is assumed to be wide-sense stationary and independently, identically distributed at each sensor, with variance σ_v^2. In the sequel a 'whitened' signal vector $\boldsymbol{z}(n) \in \mathbb{C}^{d\times 1}$ shall be considered

$$\boldsymbol{z}(n) = \boldsymbol{W}\boldsymbol{x}(n) = \boldsymbol{U}\boldsymbol{s}(n) + \boldsymbol{W}\boldsymbol{v}(n) \tag{3}$$

where $\boldsymbol{W}$ is termed the whitening matrix which is chosen so that $\boldsymbol{U} = \boldsymbol{W}\boldsymbol{A}$ is unitary. $\boldsymbol{W}$ may be estimated using and eigen-decomposition of the sample correlation matrix, as detailed in [8]. Having a unitary transformation of the sources allows a test statistic for auto-terms to be computed, which is free of cross-source terms.

The STFD matrix for a vector $\boldsymbol{x}(n)$ is defined according to

$$[\boldsymbol{D}_{\boldsymbol{xx}}(n,\omega;\varphi)]_{ij} = D_{x_i x_j}(n,\omega;\varphi) \quad (4)$$

where $D_{x_i x_j}(n,\omega;\varphi)$ is assumed to be a bilinear TFD of Cohen's class, for which the kernel function is φ. In the following we shall assume the use of the pseudo Wigner Ville Distribution (PWVD) with odd window length L and omit φ from the notation.

3. HYPOTHESIS TESTING

The goal of the signal detection problem, is to determine at which points of a set, $\mathcal{S}$, of TF locations

$$\mathcal{S} = \{\boldsymbol{\zeta}_i = (n_i, \omega_i) : \quad i = 1, \ldots, p\}, \quad (5)$$

the auto-source TFDs have significant peaks. Following the definition of a STFD matrix given in (4), we note that the auto-source distributions lie on the diagonal entries of the source STFD matrix. The signal detection problem can therefore be presented under a MHT framework, with the ith null and alternative hypothesis for each $\boldsymbol{\zeta}_i$, given respectively by

$$\begin{array}{ll} \mathsf{H}_i: & \mathsf{Tr}\,[\boldsymbol{D}_{\boldsymbol{ss}}(\boldsymbol{\zeta}_i)] = 0 \\ \mathsf{K}_i: & \mathsf{Tr}\,[\boldsymbol{D}_{\boldsymbol{ss}}(\boldsymbol{\zeta}_i)] > 0. \end{array} \quad (6)$$

From a single set of observation data, the set $\mathcal{S}$ may contain some points for which the null H_i is true and points for which it is false. If the subset of points for which H_i is true is given by $\{\boldsymbol{\zeta}_i : i \in \Omega_{\mathsf{H}}\}$, then we wish to control the level of the test according to a null hypothesis

$$\mathsf{H} = \bigcap_{i \in \Omega_H} \{\mathsf{H}_i\}. \quad (7)$$

Since H is a subset of the global null hypothesis $\bigcap_i \{\mathsf{H}_i\}$ we require a test procedure which strongly controls the family wise error (FWE) rate according to

$$\mathsf{Pr}\,[\text{Reject } \mathsf{H}|\mathsf{H}] = \mathsf{Pr}\,[\text{Reject any element of } \mathsf{H}|\mathsf{H}] \leq \gamma,$$

where γ is the set level of significance for the test. A number of MHT procedures exist which strongly control the FWE, such as those of Holm, Hochberg and others [9]. We make use of Holm's MHT procedure which is a sequentially rejective Bonferroni algorithm.

For testing (6) we consider the quantity $\mathsf{Tr}\,[\boldsymbol{D}_{\boldsymbol{zz}}(\boldsymbol{\zeta}_i)]$ which has mean and variance given respectively by

$$\mu = \|\boldsymbol{W}\|^2 \sigma_v^2 + \mathsf{Tr}\,[\boldsymbol{D}_{\boldsymbol{ss}}(\boldsymbol{\zeta}_i)] \quad (8)$$

$$\sigma^2 \approx L\sigma_v^2 \left(2\|\boldsymbol{W}\|^2 + \left\|\boldsymbol{W}\boldsymbol{W}^{\mathrm{H}}\right\|^2 \sigma_v^2\right). \quad (9)$$

1. Randomly draw a set of data from a single sensor (a row of the matrix $\boldsymbol{X}$), with replacement.
2. Repeat the random selection m times to obtain a resample of the array data matrix, $\boldsymbol{X}^*$.
3. Compute the eigen-value decomposition of the sample correlation matrix

   $$\hat{\boldsymbol{R}}^* = N^{-1}\boldsymbol{X}^*(\boldsymbol{X}^*)^{\mathrm{H}}$$

   and form estimates $\hat{\boldsymbol{W}}$ and $\hat{\sigma}_v^2$ of the whitening transform and noise variance respectively. [a]
4. Substitute $\hat{\boldsymbol{W}}$ and $\hat{\sigma}_v^2$ into Equations (8) and (9) to obtain $\hat{\mu}$ and $\hat{\sigma}^2$. Compute the test statistic T_i^* from Equation (10).
5. Repeat steps 1 to 4 B times to obtain the bootstrap test statistics $T_i^*(b)$, $b = 1, \ldots, B$.

[a] Estimation of $\boldsymbol{W}$ and σ_v^2 is discussed in [8].

Table 1. Bootstrap procedure for resampling non-stationary array data.

The test statistic used in (6) at the point $\boldsymbol{\zeta}_i$ is given by

$$T_i = \left(\mathsf{Tr}\left[\hat{\boldsymbol{W}}\boldsymbol{D}_{\boldsymbol{xx}}(\boldsymbol{\zeta}_i)\hat{\boldsymbol{W}}^{\mathrm{H}}\right] - \hat{\mu}\right)/\hat{\sigma} \quad (10)$$

where $\hat{\mu}$ and $\hat{\sigma}$ are estimates of (8) and (9) under the null, respectively. In the general case, the distribution of the test statistic is unknown. In the following section we propose a bootstrap based procedure for estimating the distribution of (10) under the null hypotheses.

4. BOOTSTRAP PROCEDURE

For each test statistic T_i defined in (10) we require the distribution under the null hypothesis in order to evaluate significance levels for the MHT. As opposed to using the asymptotic distribution described in the previous section, we estimate the null distribution using the bootstrap [10].

Due to the non-stationary nature of the source signals, we cannot directly resample columns from the array data matrix $\boldsymbol{X} = (\boldsymbol{x}(1), \ldots, \boldsymbol{x}(N))$. However, the noise is assumed to be independently identically distributed across the sensors. We may therefore resample rows of $\boldsymbol{X}$ to generate a bootstrap data set $\boldsymbol{X}^*$. Recalculating the test statistic from $\boldsymbol{X}^*$ according to (10) gives us T_i^* and repeating the procedure B times then gives $T_i^*(b), b = 1, \ldots, B$ for $i = 1, \ldots, p$. The procedure is summarised in Table 1.

One must show caution in the procedure of Table 1 that the bootstrap sample correlation matrix $\hat{\boldsymbol{R}}^*$ retains at least rank $d + 1$ for the estimation of the whitening transform. This problem is less likely to occur as the ratio m/d of sensors to sources becomes large. However, to ensure the rank condition is met one can add a test for the rank of $\hat{\boldsymbol{R}}^*$ to the procedure of Table 1, at step 3.

Since the null distribution of the test statistic is zero mean, it can be approximated using the bootstrap test statistics by forming $T_i^*(b) - B^{-1} \sum_{b=1}^{B} T_i^*(b)$, $b = 1, \ldots, B$. One can then use this distribution to determine significance levels or p-values to be used in the multiple test. In order to accurately estimate the tails of the null distribution one must have a large enough number of sensors to resample from, especially if one wishes to set a very low level of significance for the test. Experience has shown that 30 sensors are sufficient for significance levels of 0.05 and greater.

An advantage of the proposed resampling scheme is that we can estimate the null distribution in the presence of temporally correlated and non-Gaussian noise. The only requirement is that the noise is independently distributed with respect to the sensors. Whitening methods can be considered if this scheme is to be applied in a spatially coloured noise environment. An obvious disadvantage of the proposed scheme is increased computational burden since the test statistic must be computed B times instead of once.

5. SIMULATIONS

The asymptotic null distribution of the test statistic is standard Gaussian [11] and this is especially close to the true distribution when the sensor noise is Gaussian. Since the Gaussian sensor noise case is the only case where a detector based on the test statistic (10) has been proposed in the literature [6, 7, 11], we use this scenario to illustrate that the bootstrap detector can achieve similar and in some cases better performance. However, it should be remembered that unlike the previously proposed methods, the bootstrap detector is not restricted to this case of Gaussian sensor noise.

Here we present simulation results demonstrating the performance of the MHT procedure for point selection. In the following examples, we use a uniform linear array of $m = 32$ sensors and $N = 128$ snapshots of data. There are three linear FM (chirp) source signals present, having directions of arrival (DOA) with respect to the array broadside given by $(-3^o, 0^o, 3^o)$ respectively. The noise-free sum of the chirp source signals, $\mathsf{Tr}\,[\boldsymbol{D}_{\boldsymbol{ss}}(n, \omega)]$, is shown in Figure 1, where the PWVD is computed with a window length of 33. In the following we refer to the detector based on the bootstrap and the asymptotic distributions as $D1$ and $D2$ respectively.

To evaluate the performance of the point selection procedure, a set of 30 points from the TF plane is tested. We choose 15 points at which there are auto-source terms, and 15 locations which are dominated by noise. For this set of points, we apply Holm's sequentially rejective Bonferroni procedure, using both the asymptotic distribution and the bootstrap distribution of the test statistic. The number of bootstrap replications is $B = 200$. Shown in Figure 2 are the operating characteristic (OC) curves for SNRs of -9, -6 and -3 dB which plot the probability of detection versus the probability of false alarm.

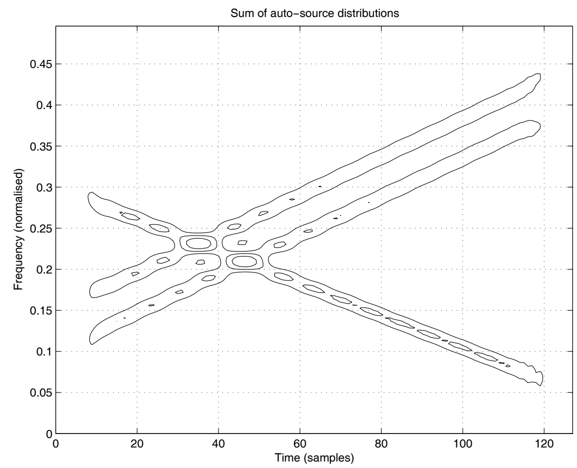

Fig. 1. Sum of the auto-source TFDs of the three chirp source signals.

It can be seen that both $D1$ and $D2$ have similar performance, and $D1$ slightly outperforms $D2$ for the middle SNR value. The probability of detection plotted is the overall probability of detection, i.e. the probability of accepting *all* of the true K_i's. The individual decisions for each K_i are actually accepted at a higher rate than that shown on the OC.

Shown in Figure 3 is the achieved FWE rate versus the nominal level of significance of the test. It can be seen that $D1$ maintains the FWE rate at less than or equal to γ as desired, while the bootstrap detector $D2$ deviates slightly for small values of γ. This is due to the difficulty in accurately estimating the tails of the null distribution when there are a limited number of sensors from which to resample. This problem is alleviated as the number of available sensors increases.

Both $D1$ and $D2$ exhibit a significantly lower FWE rate than the nominal level of significance for higher values of γ which indicates that the MHT procedure is conservative. However, for typically used values of γ (less than 0.1) this is not a problem. It is also possible to employ a more sophisticated MHT procedure [12], which may yield a less conservative test.

6. CONCLUSIONS

The work presented in this paper demonstrates two concepts. Firstly, multiple hypothesis testing procedures may be successfully applied to TF point selection techniques to control the FWE rate. This allows a global level of significance to be specified for testing a number of TF locations simultaneously. Secondly, the application of a bootstrap resampling scheme may be applied to the sensor array data, in order to estimate the null distribution of the test statistic used for point selection. This could potentially be applied to a wide range of noise models, since few assumptions on the noise distribution are made.

7. REFERENCES

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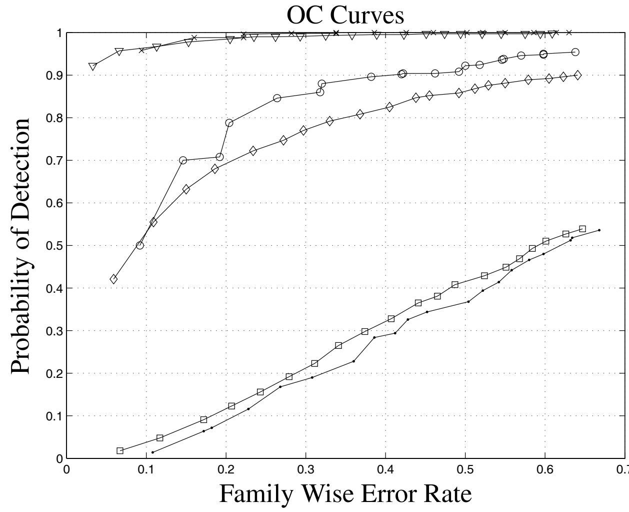

Fig. 2. OC curves. SNR = -3 dB ($*$ D1, $\vee$ D2), SNR = -6 dB ($\circ$ D1, $\diamond$ D2), SNR = -9 dB ($\bullet$ D1, $\square$ D2).

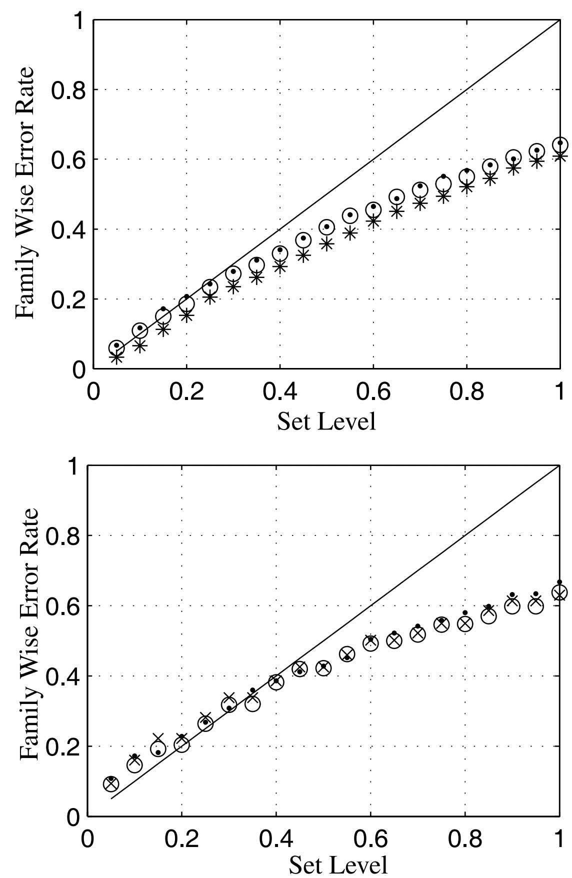

Fig. 3. FWE for (top) normal approximation and (bottom) bootstrap based detector. SNR = -3 dB ($*$), SNR = -6 dB ($\circ$), SNR = -9 dB($\bullet$).