CHANNEL MODELING FOR SPREAD SPECTRUM VIA EVOLUTIONARY TRANSFORM

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ABSTRACT

Given the importance of direct sequence spread spectrum (DSSS) communications, the modeling of its transmission channel is of great interest. Due to multipath and Doppler effects in the transmission channel, the transmitted signal is spreaded in both time and frequency. Transmission channels that spread the message in time and frequency are modeled as random, time-varying systems. It is shown in this paper that the estimation of the parameters of such models is possible by means of the spreading function which is related to the time-varying frequency response of the system and the associated evolutionary kernel of the DSSS signal. Applying the time-frequency or frequency-frequency discrete evolutionary transforms, we show how to compute the spreading function from the received DSSS signal. The procedure is efficiently implemented with the discrete evolutionary transform. Once the number of paths, delays, Doppler frequencies and gains characterizing the channel are found we use this information to obtain an estimate of the pseudo-noise and a decision parameter to determine the bit sent. Our procedure is illustrated with simulations of the process, and the corresponding bit-error rate.

1. INTRODUCTION

Direct sequence spread spectrum (DSSS) communications provides a very efficient use of the spectrum. It has advantages such as code division multiple access (CDMA), low probability of intercept, and robustness to intentional jamming or interference from other users [1]. This is achieved by spreading the message so that it ocuppies a bandwidth in excess of the minimum needed for transmission. Despreading at the receiver with a synchronized replica of the spreading sequence permits not only recovery of the message but reduction of interferences added in the transmission. Due to multipath and Doppler effects in the transmission channel, the transmitted signal is further spreaded in both time and frequency [1, 2]. The transmission channel is thus commonly modeled as a random, time-varying linear system [3, 4, 5]. For each bit, the channel can be characterized by a number of paths with delays, Doppler frequencies and attenuation factors. Signal fading caused by the channel limits the performance of DSSS. The RAKE receiver used in CDMA works well in the case of slow fading, when the channel characteristics vary slowly with time, and avoids the channel estimation but at the cost of performance.

The time-varying frequency response of the channel, also known as Zadeh's function [6], is connected with the spreading function which provides a characterization of the channel in terms of number of paths, time delays, Doppler frequencies and gains, all of which vary randomly. The existing connection between the Zadeh's function and the evolutionary spectral theory can thus be exploited to obtain a characterization of the channel, and to provide a way in the receiver to detect the bit that was sent. When transmitting the m^{th} data bit using DSSS, the received baseband signal is given by

$$r_m(n) = y(n) + i_m(n) \qquad 0 \le n \le (N-1). \tag{1}$$

The interference signal

$$i_m(n) = j_m(n) + \eta_m(n) \qquad 0 \le n \le (N-1),$$

is composed of interference from other users, $j_m(n)$, and white

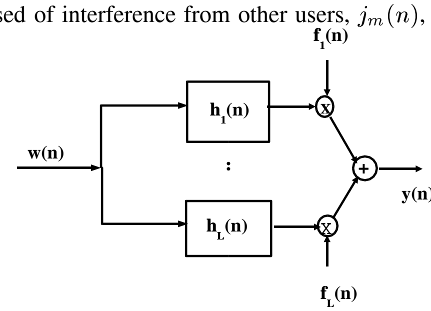

Fig. 1. Transmission channel with multi-path and Doppler effects

noise $\eta_m(n)$ from the transmission media. The signal $y(n)$ is the transmitted data bit $d_m = \pm 1$ multiplied by $p(n)$, the pseudo-noise sequence with N chips, and affected by multipath delays and frequency shifts due to Doppler effects. Characterizing the channel as a linear time-varying system with impulse response $g(n,k)$, we can express $y(n)$ as

$$y(n) = \sum_k g(n,k) w(n-k),$$

where $w(n) = dp(n)$.

The impulse response of the system can change either bit by bit –fast fading– or slowly corresponding to slow fading. Typically, the impulse response is considered separable:

$$g(n,k) = \sum_{\ell=0}^{L-1} h_\ell(n-k) f_\ell(n), \tag{2}$$

where $h_\ell(n)$ is the impulse response of the all-pass systems, corresponding to delays $\{N_\ell\}$, and $f_\ell(n) = \alpha_\ell e^{j\psi_\ell n}$ where $\{\psi_\ell\}$ are the Doppler frequencies with gains $\{\alpha_\ell\}$ shown in Fig.1. The $\{\alpha_\ell\}$ gains are inversely related to the delays, thus a path q with small delay will have an α_q close to unity, and a path s with a large delay will have a α_s close to zero.

The spreading function $S(\Omega, k)$ provides the spreading in the time and frequency produced by the channel [7, 5], and is connected with the time-varying impulse response $g(n, k)$, the Zadeh's frequency response $G(n, \omega)$ and the bifrequency function $B(\Omega, \omega)$ as indicated in Fig. 2. We will show that the discrete evolutionary transform, in the time-frequency or frequency-frequency domains [8], can be used to characterize the channel by means of the spreading function.

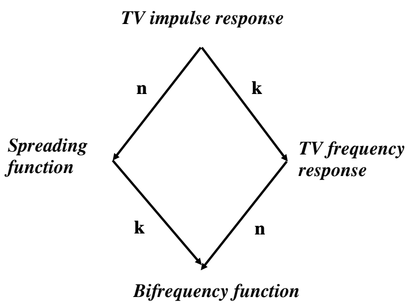

Fig. 2. Relation of different functions

2. CHANNEL MODELING AND THE SPREADING FUNCTION

Consider the LTV model for the channel for the transmission of one bit, which without loss of generality we assume to be $d = 1$ so that $w(n) = p(n)$. The signal $y(n)$, as indicated above, is then given by

$$y(n) = \sum_{\ell=0}^{L-1} \alpha_\ell p(n - N_\ell) e^{j\psi_\ell n}, \tag{3}$$

for the case of L paths. For each bit, we characterize the channel by the number of paths, the delays, the Doppler frequencies and the attenuation factors. These parameters change, at every bit or set of bits, at random within specified limits.

To obtain the spreading function consider first the computation of the Zadeh frequency response function of the LTV system. This can be obtained using the linearity of the system, so that if the pseudo-noise sequence $p(n)$ is represented as

$$p(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} P(\omega) e^{j\omega n} d\omega,$$

then we have that when we replace $p(n)$ in (3) we get

$$y(n) = \frac{1}{2\pi} \sum_{\ell=0}^{L-1} \alpha_\ell e^{j\psi_\ell n} \int_{-\pi}^{\pi} P(\omega) e^{j\omega(n - N_\ell)} d\omega, \tag{4}$$

and that the equivalent response to $p(n)$, as an infinite sum of exponentials, is

$$y(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} P(\omega) G(n, \omega) e^{j\omega n} d\omega, \tag{5}$$

where $G(n, \omega)$ is the frequency response of the LTV system. Thus, comparing the above two equations, the Zadeh function is given by

$$G(n, \omega) = \sum_{\ell=0}^{L-1} \alpha_\ell e^{j\psi_\ell n} e^{-j\omega N_\ell}, \tag{6}$$

which can be easily verified to be the Fourier transform of the separable impulse response $g(n, k)$.

Now, the bifrequency function $B(\Omega, \omega)$ is found by computing the Fourier transform of $G(n, \omega)$ with respect to the n variable:

$$B(\Omega, \omega) = 2\pi \sum_{\ell=0}^{L-1} \alpha_\ell e^{-j\omega N_\ell} \delta(\Omega - \psi_\ell). \tag{7}$$

$-\pi \leq \Omega, \omega \leq \pi$. Finding the inverse Fourier transform of $B(\Omega, \omega)$ with respect to ω we have that the spreading function is given by

$$S(\Omega, k) = 2\pi \sum_{\ell=0}^{L-1} \alpha_\ell \delta(\Omega - \psi_\ell) \delta(k - N_\ell), \tag{8}$$

which displays peaks located at the delays and the corresponding Doppler frequencies, and with $2\pi\alpha_\ell$ as their amplitudes. If we extract this information from the received signal, we should then be able to figure out what the sent bit d was. We will now consider the computation of the spreading function by means of the evolutionary transformation of the received signal.

2.1. The Discrete Evolutionary Transformation

Considering $y(n)$ a non-stationary signal of length N, it can be represented in terms of a time-varying kernel $Y(n, \omega_k)$ or its corresponding bifrequency kernel $Y(\Omega_s, \omega_k)$. The time-frequency discrete evolutionary transform (DET) and it inverse are given by

$$Y(n, \omega_k) = \sum_m y(m) V_k(m, n) e^{-j\omega_k m}, \tag{9}$$

$$y(n) = \sum_k Y(n, \omega_k) e^{j\omega_k n},$$

where $V_k(m, n)$ is a window that can be obtained from the Gabor or the Malvar representation of $y(n)$ [8], and $\omega_k = 2\pi k/N$, $0 \leq k \leq N-1$. Computing the discrete Fourier transform with respect to n of $Y(n, w_k)$ we obtain the frequency-frequency DET and its inverse as

$$Y(\Omega_s, \omega_k) = \sum_m y(m) V_k(m, \Omega_s) e^{-j\omega_k m},$$

$$y(n) = \sum_k \sum_s Y(\Omega_s, \omega_k) e^{j(\omega_k + \Omega_s)n}.$$

The Zadeh function and the corresponding spreading function are obtained from the evolutionary kernel of $y(n)$ as follows. According to equation (4), the discrete representation of $y(n)$ (obtained by representing $p(n)$ by its discrete Fourier transform) is

$$\begin{aligned} y(n) &= \frac{1}{N} \sum_{\ell=0}^{L-1} \sum_{k=0}^{N-1} \alpha_\ell e^{j\psi_\ell n} P(k) e^{j\omega_k(n - N_\ell)} \qquad (10) \\ &= \sum_{k=0}^{N-1} Y(n, \omega_k) e^{j\omega_k n}, \end{aligned}$$

where $Y(n, \omega_k) = \sum_\ell Y_\ell(n, \omega_k)$ is the time-frequency evolutionary kernel of $y(n)$, and

$$Y_\ell(n, \omega_k) = \frac{1}{N} \alpha_\ell P(k) e^{j(\psi_\ell n - \omega_k N_\ell)}.$$

Using this we have that according to equation (6) the Zadeh function can be expressed in terms of the evolutionary kernel as

$$\begin{aligned} G(n,\omega_k) &= \frac{N}{P(k)}\sum_{\ell=0}^{L-1} Y_\ell(n,\omega_k) \\ &= \frac{NY(n,\omega_k)}{P(k)}. \end{aligned}$$

The discrete Fourier transform of $G(n,\omega_k)$ with respect to n gives the bifrequency function

$$B(\Omega_s,\omega_k) = \frac{NY(\Omega_s,\omega_k)}{P(k)},$$

using the frequency-frequency evolutionary kernel $Y(\Omega_s,\omega_k)$. Finally, the inverse discrete Fourier transform of the above bifrequency function gives us the spreading function $S(\Omega_s, m)$.

To compute the discrete evolutionary kernel from the received signal, we need to consider what would be an appropriate function $V_k(m,n)$. First, suppose there is no noise in the received signal, so that $r(n) = y(n)$. To find the time-frequency evolutionary kernel $Y(n,\omega_k)$ we replace $y(n)$, equation (10), to get

$$\sum_{m=0}^{N-1} y(m)V_k(m,n)e^{-j\omega_k m} = \frac{1}{N}\sum_{s,\ell}\alpha_\ell P(s)e^{-j\omega_s N_\ell} \times \sum_{m=0}^{N-1} V_k(m,n)e^{j(\psi_\ell+\omega_s-\omega_k)m}.$$

The Gabor and Malvar windows do not work well in this case. We consider $V_k^p(m,n) = e^{j\psi_p(n-m)}$ which depends on the Doppler frequency ψ_p. This function will give us the correct representation of $Y(n,\omega_k)$ only when $\psi_p = \psi_\ell$, in fact, the last sum in the above representation using $V_k^\ell(m,n)$ gives

$$e^{j\psi_\ell n}\sum_{m=N_\ell}^{N-1} e^{j\frac{2\pi}{N}(s-k)m} = e^{j\psi_\ell n}(N-N_\ell)\Delta(s-k),$$

where the function $\Delta(s-k)$ is a very good approximation of a delta function $\delta(s-k)$. Using it, we have finally

$$\frac{N-N_\ell}{N}\sum_{\ell=0}^{L-1}\alpha_\ell P(k)e^{j(\psi_\ell n-\omega_k N_\ell)} \approx Y(n,\omega_k)(N-N_\ell),$$

which is the expected result multiplied by $N-N_\ell$. If the frequency in $V(.)$ does not coincide with one of the Doppler frequencies the result is completely different from the expected result.

Given that the Doppler frequencies are not known, to implement the computation of $Y(n,\omega_k)$, and then the spreading function, we consider $V_k^p(m,n) = e^{j\omega_p(n-m)}$, $0 \le \omega_p \le \pi$. When ω_p coincides with one of the Doppler frequencies the spreading function displays a large peak at the corresponding delay and Doppler frequency. For those frequencies ω_p not equal to the Doppler frequencies the spreading function displays a random sequence of peaks spread over all possible delays. It is possible to determine a threshold that permit us to obtain the most significant peaks of the spreading function corresponding to possible delays and Doppler frequencies.

Finally, the attenuation values can be estimated by considering the spreading function corresponding to the corresponding Doppler frequency. For instance, when there is a unique Doppler frequency ψ_q, the spreading function as computed using the evolutionary transform will be of the form

$$S_q(\Omega_s,k) = (N-N_q)\alpha_q\Delta(\Omega-\omega_q)\Delta(k-N_q) \qquad (11)$$

so that finding its peak and dividing by the factor $N-N_q$ provides α_q. For the noisy case, the above computations will be affected by the noise but in general they can be done in a similar way.

2.2. Bit detection using the channel characterization

The spreading function provides a way to characterize the changes in the channel, bit by bit, and can be used to determine what the value of the sent bit was. Since we know the pseudo-noise sequence $p(n)$, we would like to obtain the signal component that has the smallest delay and the largest gain α to determine the value of d. That is, the signal

$$\begin{aligned} r_0(n) &= r(n) - \sum_{\ell=1}^{L-1}\hat{\alpha}_\ell p(n-\hat{N}_\ell)e^{j\hat{\psi}_\ell} \\ &= \hat{\alpha}_O p(n-\hat{N}_0)e^{j\hat{\psi}_0 n} + \mu(n) \end{aligned}$$

where $\mu(n)$ contains the original noise $\eta(n)$ as well as the errors caused by the estimates of the parameters of the channel. Demodulating $r_0(n)$ by multiplying it by $e^{j\hat{\psi}_0 n}$, shifting it $\hat{N}_0$ samples and dividing it by $\hat{\alpha}_0$ we obtain a noisy estimate of the pseudo-noise sequence. Multiplying it by $p(n)$ and adding it, we have a decision parameter to determine the value of d. If the decision parameter is positive then $d = 1$, otherwise $d = -1$.

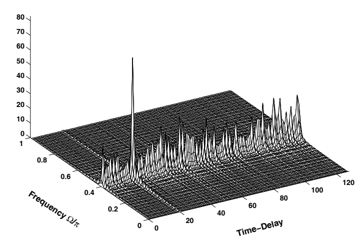

Fig. 3. Spreading function corresponding to a delay of 20 and a Doppler frequency of 0.4π in a multi-path channel

3. SIMULATIONS

To illustrate our procedure, we let the time-varying channel have a random number of paths, from 1 to 5, for each bit. For each path, we assigned at random the delays (from 0 to 127), the Doppler frequencies (from 0 to π); the gains were linearly related to the delays. The results shown in Figs. 3 to 6 correspond to a simulation with an additive media noise with $SNR = 3$ dB. Figure 3 clearly shows the peak of the spreading function corresponding to one of the Doppler frequencies and its corresponding delay and gain, while figure 4 displays the thresholded spreading function for a channel with 4 multi-paths. The recovered signal $r_0(n)$ is compared with the actual pseudo-noise sequence in Fig. 5, which

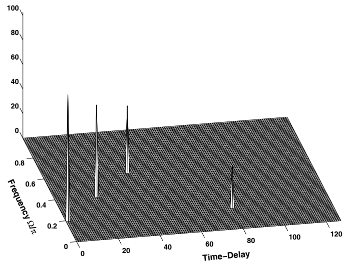

Fig. 4. Thresholded spreading function for a multipath channel with four paths

can be seen as very close, and the correlated signal in Fig. 6, corresponding to a bit with $d = 1$, shows a positive mean indicating $d = 1$.

In order to determine the goodness of the process in detecting the correct bit sent, we then did a Monte-Carlo simulation, where the model changes from bit to bit randomly as explained above – attempting to simulate fast fading– and for each bit we performed 500 trials with different media noise, with the same SNR. The chosen SNRs ranged from -9 dB to 0 dB. The results are very encouraging and are displayed in Fig. 7.

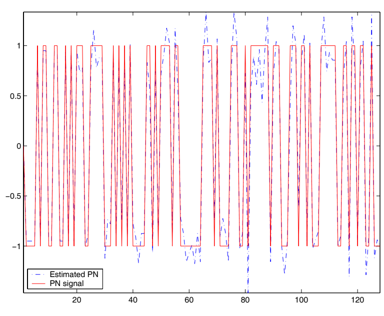

Fig. 5. Recovered $r_0(n)$ (dotted line) and the actual pseudo-noise sequence (solid line)

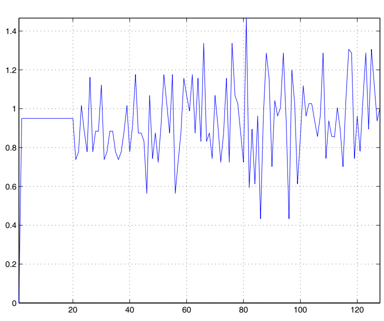

Fig. 6. Correlated signal at the receiver

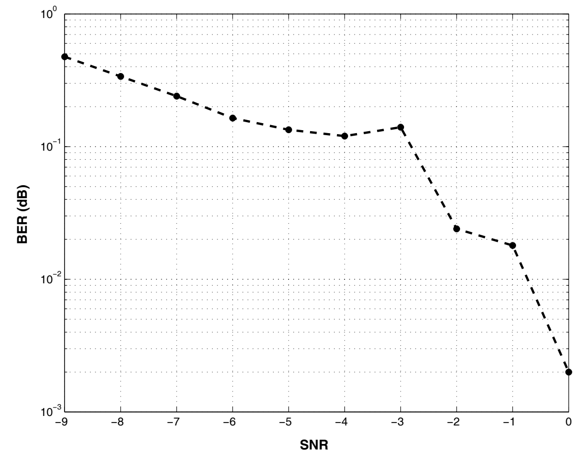

Fig. 7. Bit error rate (BER) vs SNR

4. CONCLUSIONS

In this paper we propose a way to characterize the transmission channel of DSSS, considering fast fading. The model varies from bit to bit, and for each bit is characterized by the spreading function that provides the number of paths, the delays, Doppler frequencies and attenuations which are randomly varying. We showed that the spreading function can be obtained using the discrete evolutionary transform. As a preliminary result it was illustrated the detection of the sent bit using the channel characterization and the pseudo-noise sequence. Further work is needed to improve the computational load of the algorithm and the receiver.

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