NONPARAMETRIC REGULARIZED TIME DELAY ESTIMATION

Oscar Barrero M.[1] *and Bart L.R. De Moor*[2]

[1]oscar.barrero@esat.kuleuven.ac.be, [2]bart.demoor@esat.kuleuven.ac.be
Katholieke Universiteit Leuven, ESAT/SISTA-SCD department
Kasteelpark Arenberg 10 - 3001 Heverlee (Leuven) - Belgium
Telephone +32-16-321709, fax +32-16-321970, webpage: www.esat.kuleuven.ac.be/∼sista-scd

ABSTRACT

In this paper a novel non-parametric estimator, which we call the Regularized Time Delay Estimator (RTDE), is introduced for the time delay estimation problem in MISO (Multiple Input-Single Output) linear systems. This estimator decouples the input-output signals in the frequency domain using a regularized Wiener-Hopf filter. An RLS (Regularized Least Squares) problem is formulated based on the coherence spectrum in order to find the optimal filter that can decouple the signals. Then, the corresponding delayed impulse responses of the system are computed. As a result, the time delay between the several input-output signals can be estimated.

1. INTRODUCTION

As a consequence of industrial automation, many process industry companies like to exploit the enormous and valuable data they have recorded from Data Acquisition Systems to build models of their processes. Although today several methods exist to build models based on data, there are still many problems to be solved, one of which is time delay estimation. In this paper, the problem of time delay estimation for MISO (Multiple Inputs - Single Outputs) systems based on noisy measured data is treated.

Basically, there are two methodologies to solve the problem. The first one is model-based or parametric estimation, where some assumptions have to be made about the system dynamics, and the second one is model-independent or nonparametric estimation, where the estimation is done based on the analysis of the data. In this work, we will focus on the nonparametric case.

In the literature, some classical methods for time delay estimation based on the technique of cross-correlation are found (see [1], [2], [3]). Although, they have been shown to work pretty well, they were basically designed to deal with SISO (Single Input - Single Output) systems. Hence, some extra considerations like input signal decoupling have to be done in order to extend these methods to the MISO (Multiple Inputs - Multiple Outputs) case. As a result, a new time delay estimator for MISO systems based on generalized cross-correlation is introduced in this paper, which will be called Regularized Time Delay Estimator (RTDE). This estimator decouples the input-output signals in the frequency domain using Wiener-Hopf filter which is regularized by the coherence spectrum (section 2.2). Therefore, an RLS (Regularized Least Squares [4]) problem is formulated based on the coherence spectrum such that this filter is able to decouple the signals in an optimal way. Consequently, the corresponding delayed impulse responses and time delays of the system are computed.

2. SOME DEFINITIONS

2.1. Cross-Correlation function

The Cross-Correlation function between two signals $u(k)$ and $y(k)$ in the discrete time domain is defined as

$$R_{uy}(\tau) = E[u(k+\tau)y(k)], \;\; 0 < k < N-1, \; -T < \tau < T, \tag{1}$$

where $E[\cdot]$ denotes the mathematical expectation.

Applying the DFT (Discrete Fourier Transform) to (1), gives

$$G_{uy}(\omega_n) = Y(\omega_n)U^*(\omega_n), \tag{2}$$

for $\omega_n = \frac{2\pi n}{NT}, n = 0, 1, 2, \ldots, N-1$ and sampling time T. Where '$*$' denotes conjugate, $Y(\omega_n)$ and $U(\omega_n)$ are the DFT of $y(k)$ and $u(k)$, respectively, and $G_{uy}(\omega_n)$ is the *Cross-Power Spectrum*.

2.2. Coherence Spectrum

The coherence spectrum between two signals $u(k)$ and $y(k)$ is equal to the cross-power spectrum $G_{uy}(\omega_n)$ divided by

Dr. Bart De Moor is a full professor at the Katholieke Universiteit Leuven, Belgium. This research was supported by Research Council KUL, IWT: PhD Grants, Soft4s (softsensors), Belgian Federal Government

the square root of the product of the two auto power spectra. Specifically, the complex coherence is defined by

$$\gamma_{uy}(\omega_n) = \frac{G_{uy}(\omega_n)}{\sqrt{G_{uu}(\omega_n)G_{yy}(\omega_n)}}. \tag{3}$$

The coherence is a normalized cross-spectral density function; in particular, the normalization constrains so that the Magnitude-Squared Coherence (MSC) defined by

$$C_{uy}(\omega_n) \triangleq |\gamma_{uy}(\omega_n)|^2 \tag{4}$$

lies in the range $0 \leq C_{uy}(\omega_n) \leq 1$, for all frequencies ω_n.

One interesting interpretation of the MSC is that it is a measure of the relative linearity of two processes (for more details, see [2]).

In this paper the coherence spectrum will be used as a measure of confidence of the cross-power spectrum in each frequency ω_n, this in order to improve the time delay estimation, as shown below in figure (1). In this figure we can see that for low frequencies the coherence spectrum is higher, this is due to the fact that the input and output sequences contain more energy in this frequency range. On the other hand, for higher frequencies the coherence spectrum tends to zero, this is due to the fact that most of the energy in this frequency range comes from the disturbance signal. As a result, we can say that the coherence spectrum can be used to detect in which frequency band the estimation of the cross-power spectrum between the two signals is reliable or not.

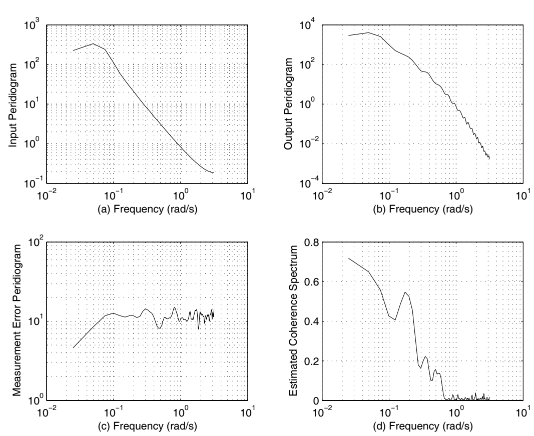

Fig. 1. Example of estimated coherence spectrum. (a) Peridiogram of the input signal; (b) Peridiogram of the output signal; (c) Peridiogram of the measurement noise signal; (d) Estimated coherence spectrum.

3. PROBLEM FORMULATION

Let us assume that $\boldsymbol{U} \in \mathbb{R}^{N\times p}$, $\boldsymbol{E} \in \mathbb{R}^{N\times 1}$, and $\boldsymbol{Y} \in \mathbb{R}^{N\times 1}$ are measured data sequences consisting of N equidistant time points of an LTI MISO system with p inputs and one output as shown in figure (2), such that

$$y(k) = f(u_1(k), u_2(k), \ldots, u_p(k)) + e(k), \; 0 \leq k \leq N-1. \tag{5}$$

The problem considered here is to estimate the time delays τ between the i_{th} input $u_i(k)$ and the output $y(k)$, taking into account that $y(k)$ is also affected by the other inputs and the noise signal $e(k)$. Some classical methods like *PHAT,*

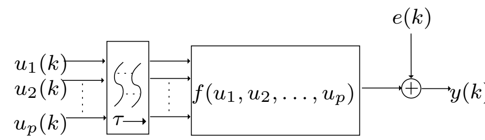

Fig. 2. LTI MISO system with p known inputs $u(k)$ with unknown delays, one known output $y(k)$, and the unknown noise signal $e(k)$ which represents the input-output noise measurement, where $f(u_1, u_2, \ldots, u_p)$ is an unknown linear function (i.e., linear dynamic system) of $u(k)$.

ROTH, SCOT, CRA, Bispectrum (see [1], [5] for a survey), compute the time delay between two signals, typical input and output, for SISO (Single Input-Single Output) systems by estimating the impulse response based on the Generalized Cross-Correlation function [1], as can be seen in figure (3). Thus, depending on the particular form of the filtering function $\Psi_g(\omega_n)$, one can have a different performance of the cross-correlation function.

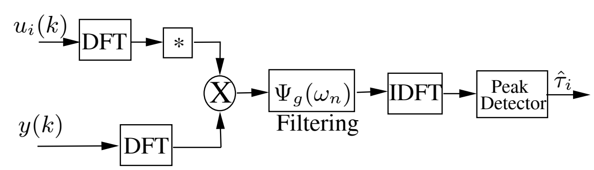

Fig. 3. Time delay estimation for SISO systems based on the generalized cross-correlation. Where $u_i(k)$ and $y(k)$ are the input and output signal respectively, '*' denotes conjugate, $\Psi_g(\omega_n)$ is a filtering function that depends on the method to be used to make the time delay estimation such as CRA, PATH, ROTH, SCOT or ML. Finally $\hat{\tau}_i$ is the time delay estimation.

4. REGULARIZED TIME DELAY ESTIMATION

In this section we will extend the generalized cross correlation function idea to the MISO systems case, assuming the system is linear-time-invariant (LTI), and taking into account that the input and output signals are coupled. Therefore, given the system shown in figure (4), where $U_1(\omega_n), \ldots, U_p(\omega_n)$, $Y(\omega_n)$, and $E(\omega_n)$ are respectively input, output, and noise variables in the discrete Fourier domain, and a set

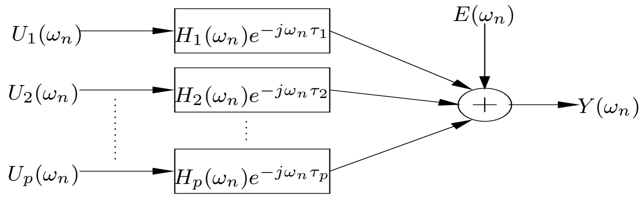

Fig. 4. LTI MISO delayed system in Laplace domain. with $U(\omega_n)$, $Y(\omega_n)$, and $E(\omega_n)$ the input, output and measurement noise variables in the Laplace domain respectively. $H_i(\omega_n)e^{-j\omega_n\tau_i}$ the delayed transfer function from the i_{th} input to the output.

of unknown transfer functions $H_i(\omega_n)$ from each input i to the output, thus, $Y(\omega_n)$ can be written as

$$\begin{aligned} Y(\omega_n) &= H_1(\omega_n)e^{-j\omega_n\tau_1}(\omega_n)U_1(\omega_n) + \dots \\ &\quad + H_p(\omega_n)e^{-j\omega_n\tau_p}U_p(\omega_n) + E(\omega_n). \end{aligned} \tag{6}$$

Consequently, the delayed impulse responses $H_1(\omega_n)e^{-j\omega_n\tau_1}$, $\dots, H_p(\omega_n)e^{-j\omega_n\tau_p}$ can be computed from the cross correlation function between input and output signals as follows,

$$\underbrace{\begin{bmatrix} \hat{G}_{U_1Y}(\omega_n) \\ \vdots \\ \hat{G}_{U_pY}(\omega_n) \end{bmatrix}}_{\hat{\boldsymbol{G}}_{p\times 1}} = \underbrace{\begin{bmatrix} \hat{U}_1(\omega_n)U_1^*(\omega_n) & \dots & \hat{U}_p(\omega_n)\hat{U}_1^*(\omega_n) \\ \vdots & \ddots & \vdots \\ \hat{U}_1(\omega_n)\hat{U}_p^*(\omega_n) & \dots & \hat{U}_p(\omega_n)\hat{U}_p^*(\omega_n) \end{bmatrix}}_{\hat{\boldsymbol{A}}_{p\times p}} \cdot \underbrace{\begin{bmatrix} \hat{H}_1(\omega_n)e^{-j\omega_n\tau_1} \\ \vdots \\ \hat{H}_p(\omega_n)e^{-j\omega_n\tau_p} \end{bmatrix}}_{\boldsymbol{H}_{p\times 1}} + \underbrace{\begin{bmatrix} V_1(\omega_n) \\ \vdots \\ V_p(\omega_n) \end{bmatrix}}_{\boldsymbol{V}_{p\times 1}} \tag{7}$$

where $\hat{\boldsymbol{A}} \in \mathbb{C}^{p\times p}$ is positive definite, and contains the estimated power and cross-power spectrum $U(\omega_n)U^*(\omega_n) \in \mathbb{R}$ and $U(\omega_n)U_j^*(\omega_n) \in \mathbb{C}$ of the input variables, respectively. $\hat{\boldsymbol{G}} \in \mathbb{C}^{p\times 1}$ contains the estimated cross-power spectrum between the input i and the output, $\boldsymbol{H} \in \mathbb{C}^{p\times 1}$ contains the delayed transfers functions $H_i(\omega_n)$ between the i_{th} input $u_i(k)$ and the output $y(k)$ at the frequency ω_j, and $\boldsymbol{V}$ is the unknown cross-power spectrum between the measurement error and $u_i(k)$. As a result, equation (7) can be written as [1]

$$\hat{\boldsymbol{G}} = \hat{\boldsymbol{A}}\boldsymbol{H} + \boldsymbol{V}. \tag{8}$$

[1]The estimation of the cross and power spectrum in equation (7) is made based on P. Welch's method. (Peridiograms) [6]. This estimation is due to the fact that we have a finite number of samples.

On the other hand, since the MSC $C_{u_iy}(\omega_n)$ gives information about how reliable the cross-power spectrum $G_{u_iy}(\omega_n)$ is at certain frequencies ω_j, this will be used as a weighted parameter in the solution of equation (8). Consequently, the problem can be formulated as an RLS (Regularized Least Squares) problem, as follows:

$$\min_{\boldsymbol{H}} \|\hat{\boldsymbol{A}}\boldsymbol{H} - \hat{\boldsymbol{G}}\|_2^2 + \lambda^2\|\boldsymbol{L}\boldsymbol{H}\|_2^2, \tag{9}$$

where $\boldsymbol{L} = \boldsymbol{I} - \boldsymbol{C}$, $\boldsymbol{I} \in \mathbb{R}^{p\times p}$ is the identity matrix, and $\boldsymbol{C} \in \mathbb{R}^{p\times p}$ is defined as

$$\boldsymbol{C} = diag([C_{u_1y}(\omega_j)\dots C_{u_py}(\omega_j)]), \tag{10}$$

a diagonal matrix containing the MSC at the frequency ω_j, and λ is a regularization parameter that must be chosen such that the solution of equation (9) $\boldsymbol{H}$, is a compromise between the minimization of $\|\hat{\boldsymbol{A}}\boldsymbol{H} - \hat{\boldsymbol{G}}\|_2$ and $\lambda\|\boldsymbol{L}\boldsymbol{H}\|_2$ (see [4], [7]). A large λ (equivalent to a large amount of regularization) favors small solution seminorm at the cost of a large residual norm, while a small λ (i.e., a small amount of regularization) has the opposite effect. Also can be shown that λ controls the sensitivity of the regularized solution $\boldsymbol{H}$ to perturbation in $\hat{\boldsymbol{A}}$ and $\hat{\boldsymbol{G}}$, and the perturbation bound is proportional to λ^{-1}. Thus, the regularization parameter λ is an important quantity which controls the properties of the regularized solution, and λ should therefore be chosen with care.

One very well-known method to compute λ is the *L-Curve* which is a plot -for all valid regularization parameters- of the (semi)norm $\|\boldsymbol{L}\boldsymbol{H}\|_2$ of the regularized solution versus the corresponding residual norm $\|\hat{\boldsymbol{A}}\boldsymbol{H} - \hat{\boldsymbol{G}}\|_2$. In this way, the *L-Curve* clearly displays the compromise between minimization of these two quantities, which is the heart of any regularization method. Then, after choosing λ in accordance to the *L-Curve* method, the minimization problem described in equation (9) is solved using the Tikhonov regularization approach (see [4] for details).

The Tikhonov's method computes the regularized solution $\boldsymbol{H}$ of equation (9) as the solution to the following least squares problem

$$min \left\| \begin{pmatrix} \hat{\boldsymbol{A}} \\ \lambda^{1/2}\boldsymbol{L} \end{pmatrix} \boldsymbol{H} - \begin{pmatrix} \hat{\boldsymbol{G}} \\ \boldsymbol{0} \end{pmatrix} \right\|_2^2, \tag{11}$$

Finally, the IDFT is applied to $\boldsymbol{H}$ in order to find the estimated delayed impulse responses in the time domain. Then, the time delays are obtained by looking at the maximum values of the estimated impulse responses.

5. NUMERICAL RESULTS

In this section we will compare RTDE with the classical nonparametric Maximum Likelihood (ML) estimator method, for a MISO system with four inputs, one output,and different values of SNR. To make the simulations four delayed linear systems were selected

$$G_1(s) = \frac{e^{-5s}}{(s+0.7)(s+0.5)} \quad G_2(s) = \frac{e^{-20s}}{(s+0.8)(s+1.3)}$$

$$G_3(s) = \frac{e^{-9s}}{(s+0.3)(s+1.8)} \quad G_4(s) = \frac{e^{-13s}}{(s+0.5)(s+1.5)}$$

with their respective time delays:

$$\tau_1 = 5 \; sec, \; \tau_2 = 20 \; sec, \; \tau_3 = 9 \; sec, \; \tau_4 = 13 \; sec$$

Initially four different step piece-wise input signals of length 4850 were generated, assuming a sampling time of 1 second. Then, in order to investigate the performance of RTDE for different conditions of noise: without measurement noise, measurement uncorrelated white noise, measurement uncorrelated and correlated coloured noise, were added to the original input and output signals. The computations and simulations were made in Matlab 6.5.

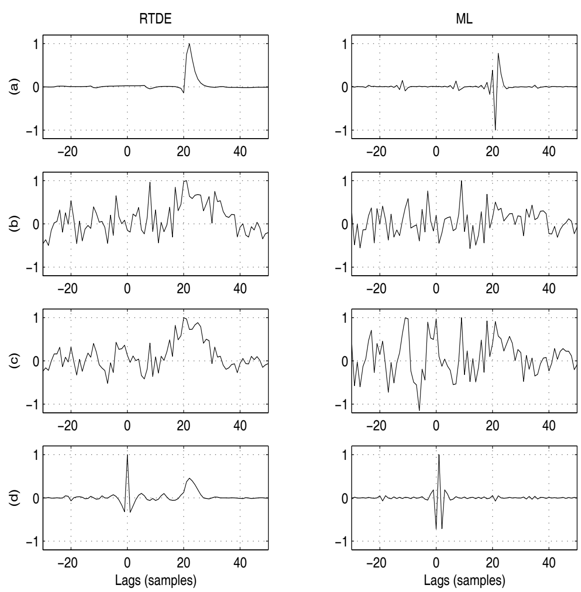

Fig. 5. Comparison between the maximum likelihood (ML) time delay estimator [1] and RTDE estimators for the delayed impulse response $H_2(\omega_n)e^{-j\omega_n\tau_2}$, with $\tau_2 = 20$ sec. (a) without measurement noise. (b) with measurement uncorrelated white noise. (c) with measurement uncorrelated coloured noise. (d) with correlated coloured noise.

In figure (5) we observe that RTDE performs at least as ML for the (a) and (c) cases, and much better for the (b) and (d) cases, showing that RTDE works better in the sense of robustness than ML under different measurement noise conditions. This, due to the fact that with the help of the coherence spectrum we can identify which frequency bandwidth of the cross-power spectrum is corrupted by noise. As a result, RTDE neglects this information resulting in a more robust estimation, as was shown in section (4).

6. CONCLUSIONS

In this paper we have introduced a new non-parametric approach, RTDE. This approach has shown to be more reliable than the classical ones, for the MISO systems case in very critical noise conditions. By using the information that the coherence spectrum gives about the reliability of the cross-power spectrum, the problem of time delay estimation can be formulated as a RLS problem. Due to the RLS formulation we can have a very robust estimator in the sense of better performance under problems of noise input correlation and noisy signals.

7. REFERENCES

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