WAVELET PACKETS-BASED DIRECTION-OF-ARRIVAL ESTIMATION

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ABSTRACT

The wavelet packets-based MUSIC (WP-MUSIC) is proposed in this paper to improve the performance of classical MUSIC in scenarios of closely spaced DOA and low signal-to-noise ratio (SNR). The WP-MUSIC is to decompose the fullband signal into several subbands by wavelet packets, and then apply MUSIC to each subband. The computation savings of WP-MUSIC are proven superior to MUSIC. Some simulation results, proving the validity and improved performance of the proposed approach, are presented.

1. INTRODUCTION

The high-resolution methods of direction-of-arrival (DOA) estimation have been a topic of great importance in recent years for their wide-spread applications in radar, sonar, and mobile communication. Especially the multiple signal classification (MUSIC) method [1] captures many attentions for its super resolution capability and independence of array geometrical structure. However in the scenarios of low signal-to-noise ratio (SNR), small number of snapshots, or closely spaced signal directions, the performance of many methods degrades rapidly. Also, the computation loads of most eigenstructure-based methods increase exponentially with the growth of array size. Although some preprocessing methods like Toeplitz approximation method (TAM) [2] and spatial smoothing method (SSM) [3] are suggested to improve the estimation when the sources are correlated and coherent, the inherent disadvantages of fullband analysis of received signal can not be alleviated.

Subband decomposition is a technique to split the spectrum of original signal with bandpass filters and subsampled to have a time-frequency decomposition. Rao and Pealman proved in [4] that with subband decomposition, some superiorities can be obtained comparing with the direct estimation on the fullband signal. Classical wavelet transform (WT) [5] is a two-subband decomposition method, which presents the resolution both in frequency domain and time domain. Different mother wavelets are designed to deal with different scenarios. Wavelet packet (WP) is a much detailed WT by decomposing the *approximations* and *details* as well. Both WT and WP have their successful applications in harmonic retrieval [6], [7]. B Wang *et al.* [8] suggested a spatial discrete wavelet transform preprocessing method (SDWTP) for direction-of-arrival estimation by analyzing the spatial-temporal duality , but SDWTP method failed to decompose the spatial spectrum of the *details*.

Inspired by the work of [6] and [8], a novel WP-based method for DOA estimation (WP-MUSIC) is proposed in this paper by combining subband decomposition with MUSIC. Wavelet packet is chosen to decompose the subbands for its capability of *details*-decomposition, which provides a solution to the problem in [8]. The proposed method is performed by wavelet packets decomposition of the signal, using a minimum description length (MDL) criterion [9] as the guidance of each decomposition . A best bases method (BBM) [10] is employed to choose the best bases of the WP tree . MUSIC method is applied to all the best bases, yielding all the DOAs. The proposed method provides both savings of computation and improvement of resolution, which is a good candidate of foremost preprocessing method for many DOA estimation algorithms and even for TAM and SSM to refine the performance, especially in scenarios of low SNR and closely spaced spatial frequency.

2. PROBLEM FORMULATION

2.1. Spatial-Temporal Equivalence

Consider a uniform linear array (ULA) with M isotropic sensors spaced by the distance d and D ($D < M$) narrowband incoherent plane waves centered at frequency ω_0 , impinging from directions $\{\theta_1, \theta_2, ..., \theta_D\}$. Using complex representation, the received signal at the ith sensor can be expressed as

$$x_i(t) = \sum_{k=1}^{D} s_k(t) e^{-j\omega_0 (i-1)\sin\theta_k d/c} + n_i(t) \qquad (1)$$

where $s_k(t)$ is the complex envelop signal of the kth wavefront, c is the propagation speed of the wavefronts, and $n_i(t)$ is the additive noise at the ith sensor.

This work is supported by Key Program of Science and Technology from the Ministry of Education of China, under Grant no. 02085

In matrix form, we have

$$\mathbf{x}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{n}(t) \quad (2)$$

where $\mathbf{x}(t)$, $\mathbf{s}(t)$, and $\mathbf{n}(t)$ are respectively the $M \times 1$ vector of received signal, $D \times 1$ vector of D wavefronts, and $M \times 1$ vector of additive noise. $\mathbf{A}$ is the matrix of array manifold

$$\mathbf{A}(t) = [\mathbf{a}(\theta_1), \mathbf{a}(\theta_2), ..., \mathbf{a}(\theta_D)] \quad (3)$$

where $\mathbf{a}(\theta_k) = [1, e^{-j\hat{\omega}_k}, ..., e^{-j(M-1)\hat{\omega}_k}]^T$ is the steering vector of the array corresponding to the direction θ_k, where $\hat{\omega}_k = \omega_0 \sin\theta_k d/c$ denotes the spatial frequency of kth wavefront, superscript T denotes transpose.

It is assumed that the signals and noises are stationary and ergodic zero mean complex valued random processes, the noises are assumed to be uncorrelated between sensors, uncorrelated with the signals, and have identical variance σ^2 in each sensor. With the assumptions above the $M \times M$ spatial covariance matrix of the received signals is denoted by

$$\mathcal{R}_{\mathbf{xx}} = E[\mathbf{x}\mathbf{x}^H] = \mathbf{A}\mathcal{R}_{\mathbf{ss}}\mathbf{A}^H + \sigma^2\mathbf{I} \quad (4)$$

where $\mathcal{R}_{\mathbf{ss}} = E[\mathbf{s}\mathbf{s}^H]$ is the $D \times D$ signal covariance matrix, superscript H denotes Hermitian transpose (complex conjugate transpose), and $\mathbf{I}$ denotes the $M \times M$ identity matrix.

The temporal frequency ω is defined by the ratio of angular change and its time duration, which can be extended to a generalized parameter frequency if given the ratio of a unit parameter and its unit time duration. As we have defined in this paper, $\hat{\omega}$ is the spatial frequency, which follows the definition of its counterpart.

The response difference between two sensors to the kth wavefront is the phase difference caused by the spatial transmission delay, termed as $\hat{\omega} \cdot 1_t$, in which 1_t denotes the unit spaced distance and it plays the role of unit sampling duration of time. Suppose an azimuth-only scenario and the array sensors is placed positively along the x-axis with the first sensor locating at the coordinate origin. Thus x-axis can be seen as the spatial distance. With these assumptions, the nth noiseless snapshot of the array to the kth wavefront can be expressed as

$$\mathbf{x}(n) = [s_k(n), s_k(n)e^{-j\hat{\omega}_k}, ..., s_k(n)e^{-j(M-1)\hat{\omega}_k}]^T \quad (5)$$

It can easily prove that $\mathbf{x}(n)$ is the sampling sequence of a signal with the amplitude $s_k(n)$ and frequency $\hat{\omega}_k$. In the scenario of N snapshots, the equivalence between temporal and spatial signal model is that the realizations and samplings in temporal scenario correspond respectively to the snapshots and sensors in spatial one.

2.2. Subband Decomposition Superiorities

Subband decomposition is proven in [4] to have its superiorities upon fullband signal, which are concluded in the following aspects.

1. The aggregate of the minimum prediction error of the subbands is less than that of the fullband signal.
2. For Gaussian source, the composite entropies of the subbands are closer to the the entropy rate of the source than that of the fullband.
3. The spectral flatness measure (SFM) of the Gaussian source is less than that of the subbands, which means the SFM of the subbands is much flatter.

Besides that, subband decomposition for spatial frequency estimation in this paper has two advantages: not only are the various modes isolated in separate subbands, but the SNR and spatial frequency spacing are doubled in each two-subbands' splitting. It is well known that the Cramer-Rao bound (CRB) of the estimation goes better if there is less number of modes in each subband.

2.3. Proposed WP-MUSIC Method

For computation's sake, a pseudo-2D method is suggested in this paper. Pseudo-2D means to decompose each snapshot of the ungrouped signal matrix by wavelet packets while keeping the number of snapshots unchanged. After all the snapshots are decomposed all the *approximations* and *details* are regrouped into two matrices for next-level decomposition. Pseudo-2D method is performed with the supervision of minimum description length (MDL) criterion and the decomposed wavelet packets tree is also pruned by the best base method (BBM). The following is a summary of WP-MUSIC method based on the signal model (2).

Step 1. Take snapshots of signal model (2) and form an $M \times N$ matrix $\mathbf{X} = [\mathbf{x}(1), \mathbf{x}(2), ..., \mathbf{x}(N)]$, where $\mathbf{x}(i), i = 1, 2, ...N$ is defined in (2) .

Step 2. Perform pseudo-2D method on $\mathbf{X}$ and yield the *approximation* $\mathbf{X}_1$ and *detail* $\mathbf{X}_2$.

Step 3. Apply MDL criterion on the mother node $\mathbf{X}$ and its children nodes $\mathbf{X}_1$ and $\mathbf{X}_2$ respectively, if there are to modes lost, accept two nodes and goto Step 2. for next decomposition. Otherwise stop the decomposition at node $\mathbf{X}$.

Step 4. Prune the decomposed tree by BBM.

Step 5. Apply MUSIC method to all the leaves to obtain the estimates of the corresponding subbands.

It is also worth noting that the WP-based DOA estimation method is not confined to the classical algorithm. The proposed method focuses on the algorithm itself. The goal is to increase the estimation of resolution in low SNR and small number of snapshots , and to reject the noise by more detailed decomposition. To generalize the WP-based DOA

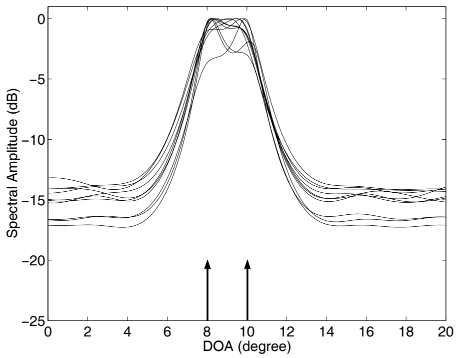

Fig. 1. MUSIC spatial spectrum for two signals with equal power (SNR = -5 dB) from 8^o and 10^o by ten batches.

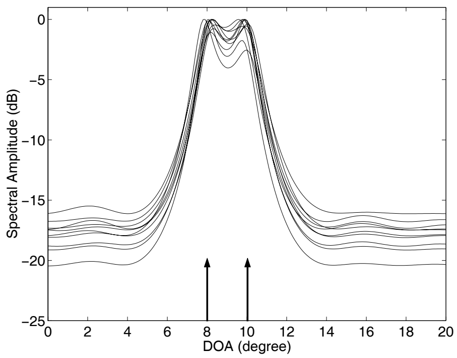

Fig. 2. WP-MUSIC spatial spectrum for two signals with equal power (SNR = -5 dB) from 8^o and 10^o by ten batches.

estimation methods, other high resolution algorithms, such as ML, ESPRIT, and MEM, can be used.

The computation loads of classical MUSIC method are due to two main parts, the decomposition of matrix ($\mathcal{O}(M^3)$) and the searching for the peaks ($\mathcal{O}(D^3)$), where M and D are the numbers of sensors and plane waves respectively. The computational advantage of WP-MUSIC is that it reduces the number of sensors to its half, and only adds a small amount of computation loads of wavelet packet transform. For the one-level D_1 wavelet packets decomposition, the added computation loads are only $\mathcal{O}(M)$. Even for the l-level, the added computation loads are about $\mathcal{O}(M) + 2\mathcal{O}(2^{-1}M) + ... + 2^{l-1}\mathcal{O}(2^{-l+1}M)$. The computation superiority of WP-MUSIC is obvious with easy validations.

3. SIMULATION RESULTS

In this section, we present some simulations to justify the performance of proposed WP-MUSIC algorithm and compare it with classical MUSIC algorithm. In both examples, simulations is carried out according to signal model (2) for a half wavelength spaced ($d = \lambda/2$) ULA with 32 sensors

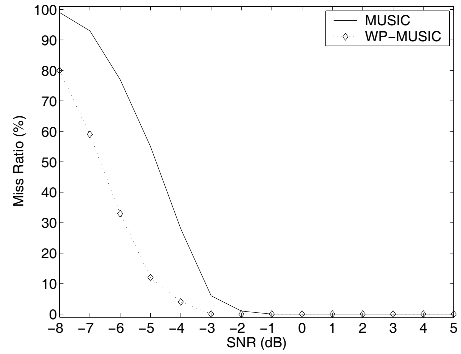

Fig. 3. Miss Ratio of MUSIC and WP-MUSIC vs. SNR for two signals from 8^o and 10^o.

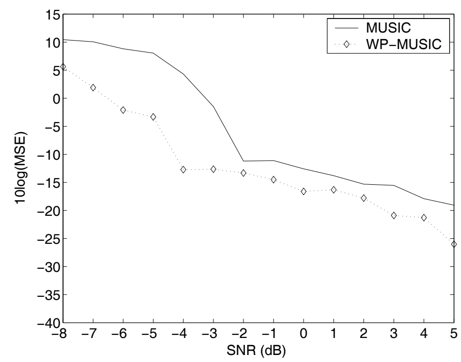

Fig. 4. 10log(MSE) of MUSIC and WP-MUSIC vs. SNR for two signals from 8^o and 10^o.

($M = 32$), and the number of snapshots taken from the array is 100 ($N = 100$). The mother wavelet used for WP-MUSIC method is Daubechies' D_1 wavelet (the same as Haar wavelet) for computations' saving [11].

Example 3.1: Low SNR case

In the first example, the low SNR case (SNR = -5 dB) is considered. Two plane waves at closely spaced directions 8^o and 10^o with equal power are impinging on the array. Suppose a one-level wavelet packets decomposition and two subbands yielded. By applying ten batches on the MUSIC and WP-MUSIC method, we obtain the spatial spectrums shown in Figs. 1 and 2, respectively. Five ones miss presenting two dominant peaks corresponding to the DOAs of the two waves in ten batches with the classical MUSIC method, while all ten batches resolve two DOAs with WP-MUSIC method. The capability of closed DOA resolving in low SNR and small number of snapshots owes to the spatial frequency spacing and SNR amplification. Figs. 3 and 4, presenting the performance of WP-MUSIC (the diamond dashed line) upon classical MUSIC (the solid line) in the sense of miss ratio and MSE versus SNR, are depicted by 100 Monte Carlo simulations. The superiorities of resolu-

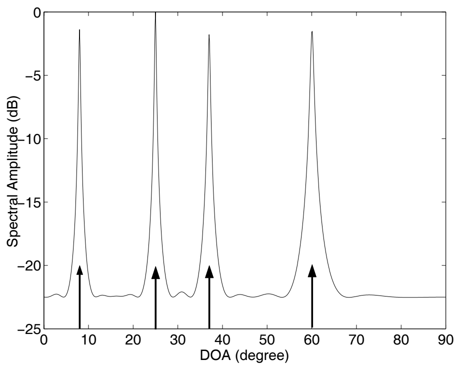

Fig. 5. MUSIC spatial spectrum for four signals with equal power (SNR = 0 dB) at directions 8^o, 25^o, 37^o, and 60^o by a two-level decomposition.

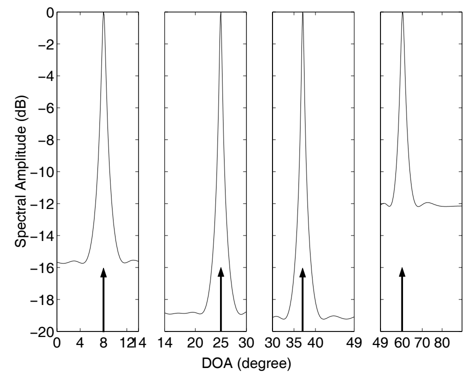

Fig. 6. WP-MUSIC spatial spectrum for four signals with equal power (SNR = 0 dB) at directions 8^o, 25^o, 37^o, and 60^o by a two-level decomposition.

tion enhancing and noise rejecting of the supposed method are obvious.

Example 3.2: Multiple waves case

A multiple waves case is chosen in the second example. Four plane waves at directions 8^o, 25^o, 37^o, and 60^o with equal power and SNR = 0 dB imping on the array. Fig. 5 depicts the spatial spectrum of the classical MUSIC method applied on the source wave, in which four dominant peaks corresponding to the DOAs are presented. Attracted by the MSE's decrease of all the estimations, a two-level wavelet packets decomposition is performed on the source waves and four subbands are yielded on the four leaf-nodes. In this example, four subbands correspond to the best bases of the wavelet packets tree. Fig. 6 shows the spatial spectrum of our proposed WP-MUSIC method applied on each selected best subband wave. Figs. 5 and 6 prove the validity of spatial frequency decomposition by properly selecting the level and the best bases.

4. CONCLUSION

The proposed WP-MUSIC method in this paper relies on the spatial-temporal equivalence of the signal. The novelties of WP-MUSIC are both savings in computation and improvement in resolution, especially in the scenarios of low SNR and closely spaced spatial frequency. Moreover, WP-MUSIC decomposes subbands much detailed to make full use of the subband priorities. Computer simulations demonstrate the validity of the proposed approach.

5. REFERENCES

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