ESTIMATING FREQUENCIES OF TWO DIMENSIONAL HARMONICS WITH HYPERCOMPLEX

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ABSTRACT

The complex signal of two dimensional harmonics is common. And the pairing steps were always needed when we estimated the frequency pairs of harmonics which was described by complex signals. We introduced hypercomplex signal in this paper, and used it to study two dimensional harmonics. First, we constructed hypercomplex signal using the original two dimensional harmonics and its Hilbert transform. Then, we presented our algorithm of estimating frequencies of hypercomplex signal through taking advantage of the properties of Hamilton's quaternion. Some simulations illustrated the prospect of using hypercomplex signal in estimating parameters of two dimensional harmonics without pairing steps.

1. INTRODUCTION

In many applications, such as radar, sonar and communication, parameters estimation of two dimensional harmonics was very important. In order to get high resolutions estimation, many subspace methods of one dimensional harmonics, such as MUSIC, MEMP, ESPRIT, have been developed to estimate parameters of two dimensional harmonics[1][2][3].

When additive noise was colored Gaussian noise M.ibrahim[4] and R.R.Gharieb[5] analyzed this problem with high-order cumulants since high-order cumulants is insensitive to any Gaussian noise. In addition, Dou[6] presented special sixth-order moment to extract frequencies of two dimensional harmonics in correlated multiplicative and additive noise.

So far, pairing steps were needed directly or indirectly in all of the algorithms of parameters estimation of two dimensional harmonics. Recently, hypercomplex has been used in image processing more and more popular [7]-[12]. At the same time, Bulow, T. and Sommer, G [13] extended the common complex signal to hypercomplex signal for researching multi-dimensional signal.

Section II, we constructed the hypercomplex signal of two dimensional harmonics and presented some properties of Hamilton's quaternion. Section III, the corresponding algorithm was presented to illustrate the theory.

2. SIGNAL MODEL AND HAMILTON'S QUATERNION

2.1. Signal model

Considering two dimensional harmonics model as:

$$x(m,n)=\sum_{l=1}^{L} a_l \cos(\omega_{1l} m+\omega_{2l} n) \tag{1}$$

where, L represents the number of signal, $(\omega_{1l},\omega_{2l})$ denotes the l th frequency pairs and a_l represents amplitude.

Let

$$\begin{aligned} f(p,q) &= x(m+p,n-q)+x(m+p,n+q) \\ &+ x(m-p,n-q)+x(m-p,n+q) \\ &= 4\sum_{l=1}^{L} a_l \cos(\omega_{1l} m+\omega_{2l} n)\cos(\omega_{1l} p)\cos(\omega_{2l} q) \\ &= \sum_{l=1}^{L} b_l \cos(\omega_{1l} p)\cos(\omega_{2l} q) \end{aligned} \tag{2}$$

where, $b_l = 4a_l \cos(\omega_{1l} m+\omega_{2l} n)$. We assumed that $b_l \neq 0$. Then, equations (1) and (2) have the same frequency pairs.

The partial and total Hilbert transforms [15] of (2) were written as:

$$f_H^P(p,q)=\sum_{l=1}^{L} b_l \sin(\omega_{1l} p)\cos(\omega_{2l} q) \tag{3}$$

$$f_H^Q(p,q)=\sum_{l=1}^{L} b_l \cos(\omega_{1l} p)\sin(\omega_{2l} q) \tag{4}$$

$$f_H^T(p,q)=\sum_{l=1}^{L} b_l \sin(\omega_{1l}p)\sin(\omega_{2l}q) \tag{5}$$

where, $f_H^P(p,q)$ and $f_H^Q(p,q)$ were partial Hilbert transforms along P and Q directions respectively. $f_H^T(p,q)$ was total Hilbert transform.

Define hypercomplex model:

$$\begin{aligned} F(p,q) &= f(p,q)+if_H^P(p,q) \\ &+ jf_H^Q(p,q)+kf_H^T(p,q) \end{aligned} \tag{6}$$

If (6) was seen as Hamilton's quaternion we could rewrite (6) as:

$$\begin{aligned} F(p,q) &= f(p,q)+if_H^P(p,q) \\ &+ jf_H^Q(p,q)+kf_H^T(p,q) \\ &= \sum_{l=1}^{L} b_l \exp(i\omega_{1l}p)\exp(j\omega_{2l}q) \end{aligned} \tag{7}$$

2.2. Properties of Hamilton's quaternion

The multiplication rules of Hamilton's quaternion were

$$ii=jj=kk=-1\ ,\ \ ij=-ji=k$$

and the conjugate $\bar{e}$ of Hamilton's quaternion $e=a+ib+jc+kd$ was defined as: $\bar{e}=a-ib-jc-kd$

Meanwhile, we could get (8)-(12), which were similar to the famous Euler relations

$$\begin{aligned} \cos(\omega_{1l}p)\cos(\omega_{2l}q) &= [\exp(i\omega_{1l}p)\exp(j\omega_{2l}q) \\ &+ \exp(-i\omega_{1l}p)\exp(j\omega_{2l}q) \\ &+ \exp(i\omega_{1l}p)\exp(-j\omega_{2l}q) \\ &+ \exp(-i\omega_{2l}q)\exp(-j\omega_{1l}p)]/4 \end{aligned} \tag{8}$$

$$\begin{aligned} \sin(\omega_{1l}p)\sin(\omega_{2l}q) &= [\exp(i\omega_{1l}p)\exp(j\omega_{2l}q) \\ &- \exp(-i\omega_{2l}q)\exp(j\omega_{1l}p) \\ &- \exp(i\omega_{1l}p)\exp(-j\omega_{2l}q) \\ &+ \exp(-i\omega_{2l}q)\exp(-j\omega_{1l}p)]/4k \end{aligned} \tag{9}$$

$$\begin{aligned} \sin(\omega_{1l}p)\cos(\omega_{2l}q) &= [\exp(i\omega_{1l}p)\exp(j\omega_{2l}q) \\ &- \exp(-i\omega_{2l}q)\exp(j\omega_{1l}p) \\ &+ \exp(i\omega_{1l}p)\exp(-j\omega_{2l}q) \\ &- \exp(-i\omega_{2l}q)\exp(-j\omega_{1l}p)]/4i \end{aligned} \tag{10}$$

$$\begin{aligned} \cos(\omega_{1l}p)\sin(\omega_{2l}q) &= [\exp(i\omega_{1l}p)\exp(j\omega_{2l}q) \\ &+ \exp(-i\omega_{2l}q)\exp(j\omega_{1l}p) \\ &- \exp(i\omega_{1l}p)\exp(-j\omega_{2l}q) \\ &- \exp(-i\omega_{2l}q)\exp(-j\omega_{1l}p)]/4j \end{aligned} \tag{11}$$

$$\begin{aligned} \exp(i\omega_{1l}p)\exp(j\omega_{2l}q) &= \cos(\omega_{1l}p)\cos(\omega_{2l}q) \\ &+ i\sin(\omega_{1l}p)\cos(\omega_{2l}q) \\ &+ j\cos(\omega_{1l}p)\sin(\omega_{2l}q) \\ &+ k\sin(\omega_{1l}p)\sin(\omega_{2l}q) \end{aligned} \tag{12}$$

When $x\in(-\pi/2,\pi/2)$, $y\in(-\pi/2,\pi/2)$, $x^{'}\in(-\pi/2,\pi/2)$ and $y^{'}\in(-\pi/2,\pi/2)$, it could be proved that if and only if $x=x^{'}$ and $y=y^{'}$, equation (13) existed

$$\exp(ix)\exp(jy)=\exp(ix^{'})\exp(jy^{'}) \tag{13}$$

From (8)-(13), it could be illustrated that it was possible to get the parameters of real two dimensional harmonics through studying the corresponding Hamilton quaternion.

3. ALGORITHM OF ESTIMATING FREQUENCY PAIRS

Assume that

$$y(m,n)=\sum_{l=1}^{L} a_l \exp(i\omega_{1l}m)\exp(j\omega_{2l}n) \tag{14}$$

where, L represent the number of signal, $(\omega_{1l},\omega_{2l})$ denotes the l th frequency pairs . Frequencies were mutually unequal among ω_{1l}, so did to ω_{2l}

Let

$$\begin{aligned} Y_1 &= \begin{bmatrix} y(0,0) & y(0,1) & \cdots & y(0,N) \\ y(1,0) & y(1,1) & & y(1,N) \\ & & & \\ y(M,0) & y(M,1) & & y(M,N) \end{bmatrix} \\ &= \begin{bmatrix} 1 & 1 & \cdots & 1 \\ e^{i\omega_{11}} & e^{i\omega_{12}} & \cdots & e^{i\omega_{1L}} \\ \vdots & \vdots & \vdots & \vdots \\ e^{i\omega_{11}M} & e^{i\omega_{12}M} & \cdots & e^{i\omega_{1L}M} \end{bmatrix} \\ &\bullet \begin{bmatrix} a_1 & & & \\ & a_2 & & \\ & & \ddots & \\ & & & a_L \end{bmatrix} \begin{bmatrix} 1 & e^{j\omega_{21}} & \cdots & e^{j\omega_{21}N} \\ 1 & e^{j\omega_{22}} & \cdots & e^{j\omega_{22}N} \\ \vdots & \vdots & \cdots & \vdots \\ 1 & e^{j\omega_{2L}} & \cdots & e^{j\omega_{2L}N} \end{bmatrix} \end{aligned} \tag{15}$$

$$Y_2 = \begin{bmatrix} y(1,1) & y(1,2) & \cdots & y(1,N+1) \\ y(2,1) & y(2,2) & & y(2,N+1) \\ & & & \\ y(M+1,1) & y(M+1,2) & & y(M+1,N+1) \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & \cdots & 1 \\ e^{i\omega_{11}} & e^{i\omega_{12}} & \cdots & e^{i\omega_{1L}} \\ \vdots & \vdots & \vdots & \vdots \\ e^{i\omega_{11}M} & e^{i\omega_{12}M} & \cdots & e^{i\omega_{1L}M} \end{bmatrix}$$

$$\bullet \begin{bmatrix} e^{i\omega_{11}} a_1 e^{j\omega_{21}} & & & \\ & e^{i\omega_{12}} a_2 e^{j\omega_{22}} & & \\ & & \ddots & \\ & & & e^{i\omega_{1L}} a_L e^{j\omega_{2L}} \end{bmatrix}$$

$$\bullet \begin{bmatrix} 1 & e^{j\omega_{21}} & \cdots & e^{j\omega_{21}N} \\ 1 & e^{j\omega_{22}} & \cdots & e^{j\omega_{22}N} \\ \vdots & \vdots & \cdots & \vdots \\ 1 & e^{j\omega_{2L}} & \cdots & e^{j\omega_{2L}N} \end{bmatrix} \tag{16}$$

when $\lambda_L = e^{i\omega_{1l}}, \lambda_R = e^{j\omega_{2l}}$, $\lambda_L, \lambda_R \in (-\pi/2, \pi/2)$, we knew that $|Y_2 - \lambda_L Y_1 \lambda_R| \equiv 0$ since frequencies were mutually unequal among ω_{1l} and ω_{2l}.

Since Hamilton's quaternion doesn't agree with the law of commutation of multiplication we didn't use the common rules of general matrix to get the rank of determinant.

Fortunately, Cheng [14] presented that Hamilton's quaternion matrix may be expressed by complex matrix. And the rank of determinant of quaternion is equal to half of the rank of determinant of complex matrix which is used to express the quaternion matrix.

Assume that $a \in R, b \in C, e \in Q$, R represents real number field, C denotes complex field and Q represents Hamilton's quaternion field. The complex expression of $x = x_0 + ix_1 + jx_2 + kx_3 \in Q$ was defined as:

$$f(x) = \begin{bmatrix} x_0 + x_1\sqrt{-1} & x_2 + x_3\sqrt{-1} \\ -x_2 + x_3\sqrt{-1} & x_0 - x_1\sqrt{-1} \end{bmatrix} = \begin{bmatrix} \alpha & \beta \\ -\overline{\beta} & \overline{\alpha} \end{bmatrix} \in C^{2\times 2} \tag{17}$$

The complex expression of Hamilton's quaternion matrix $A = (a_{ij}) \in Q^{m\times n}$ was defined as:

$$f(A) = A^f = (f(a_{ij})) = \begin{bmatrix} \alpha_{ij} & \beta_{ij} \\ -\overline{\beta}_{ij} & \overline{\alpha}_{ij} \end{bmatrix} \in C^{2n\times 2n} \tag{18}$$

The relation of the rank Hamilton's quaternion matrix and complex matrix is

$$R(A) = \frac{1}{2} R(A^f) \tag{19}$$

So, we could write λ_L, λ_R as:

$$\lambda_L = \begin{bmatrix} \cos\omega_{1l} + \sin\omega_{1l}\sqrt{-1} & \\ & \cos\omega_{1l} - \sin\omega_{1l}\sqrt{-1} \end{bmatrix} \tag{20}$$

$$\lambda_R = \begin{bmatrix} \cos\omega_{2l} & \sin\omega_{2l}\sqrt{-1} \\ \sin\omega_{2l}\sqrt{-1} & \cos\omega_{2l} \end{bmatrix} \tag{21}$$

From $|Y_2 - \lambda_L Y_1 \lambda_R| \equiv 0$ we could estimate λ_L, λ_R using general rule of calculation of complex matrix.

4. SIMULATION

we generated (14) with $L = 2$, $a_1 = a_2 = 1$, $(\omega_{11}, \omega_{21}) = (1, 0.5)$ and $(\omega_{12}, \omega_{22}) = (-0.5, 1)$. Then, we took $M = N = 2$ and constructed (15) and (16). It could be observed from figure (1) and figure (2) that the position of peaks were around $(1, 0.5)$ and $(-0.5, 1)$.

5. CONCLUSION

This paper presented a new approach to estimate frequency pairs of two dimensional harmonics. Algorithm and simulation illustrated that Hamilton's quaternion was useful to avoid pairing steps in estimating frequencies of two dimensional harmonics. However, many problems needed to be further investigation. For example, how to get λ_L, λ_R directly through decomposition of Hamilton's quaternion matrix need further study.

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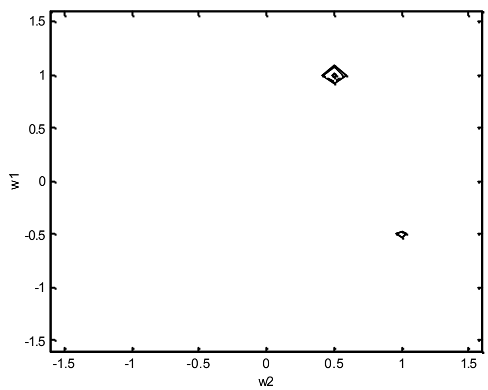

Fig. 1 Top view of extracting frequency pairs with Hamilton's quaternion

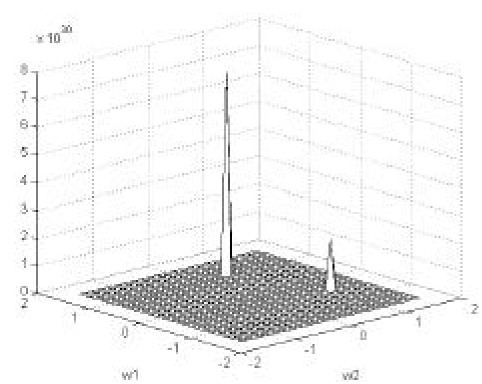

Fig. 2 three dimensional view of extracting frequency pairs with Hamilton's quaternion