SWITCHING ARIMA MODEL BASED FORECASTING FOR TRAFFIC FLOW

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ABSTRACT

Switching dynamic linear models are commonly used methods to describe change in an evolving time series, where switching ARIMA model is a special case. Short-term forecasting of traffic flows is an essential part of Intelligent Traffic Systems (ITS). In this paper, we apply switching ARIMA model to traffic flow series. We have observed that the conventional switching model is inappropriate to describe the pattern changing. Thus the variable of duration is introduced and we use the sigmoid function to describe the influence of duration to the transition probability of the patterns. Based on the switching ARIMA model, the forecasting algorithm is presented. We apply the proposed model to the real data obtained from UTC/SCOOT systems in Traffic Management Bureau of Beijing. The experiments show that our proposed model is applicable and effective.

1. INTRODUCTION

One way of modeling change in an evolving time series is by assuming that the dynamics of some underlying model changes discontinuously at certain undetermined points in time [1] [2]. Switching model introduces a hidden variable (also called hidden state) to point out which model the series obeys. The hidden state is also modeled as a stochastic process. The model parameters of the time series are related to the hidden state. Using S_t and Z_t to denote the hidden state and the value of the time series at time t, respectively, the switching model can be described by the following two probability distributions.

State transition probability:

$$p(S_{t+1}|S_t, \cdots, S_0) \tag{1}$$

The probability of Z_t given its previous values and the current hidden state:

$$p(Z_t|Z_{t-1}, \cdots, Z_0, S_t) \tag{2}$$

Given the state S_t, the time series $\{Z_t\}$ is usually assumed to be dynamic linear model, a general form that includes ARIMA and classical regression models as special cases.

Short-term forecasting of traffic flows is an essential part of Intelligent Traffic Systems (ITS). A considerable amount of effort has been expended on this problem and some models are proposed, such as random walk, historical average, informed historical average [3], artificial neural networks [4], Kalman filter theory [5], nonparametric regression (K-nearest neighbor, non-linear regression) [6] and ARIMA based methods (Seasonal ARIMA [3] and KARIMA [7].

Autoregressive integrated moving average (ARIMA) process is a common used model for time series analysis. There have been great attempts to use ARIMA model for short-term traffic flow forecasting. ARIMA model assumes the traffic flow series can be made stationary by differencing. In most of the studies, the investigated models were ARIMA(p, $1, q$). Since a first difference will not yield a stationary transformation for traffic flow series [3], it would not have been surprising if these studies had reported inferior forecasting performance for the ARIMA models. To overcome the non-stationarity of the traffic flow series, many variations were proposed. Billy M. Williams [3] noticed the periodicity of the traffic flow series and assumed the traffic flow series can be stationary after normal and seasonal differencing, so he introduced seasonal ARIMA to model the traffic flow series. Another remedy to the basic ARIMA, named KARIMA, was proposed by Der Voot et al [7]. The "K" in the model name represents the method chosen for the cluster layer, namely a type of neural network known as a Kohonen self-organizing map. In this model, Kohonen map was firstly applied to cluster the traffic flow data. Then, a separate ARIMA(p,0,q) model was fitted to each cluster.

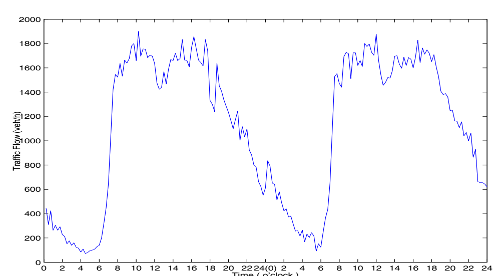

Fig. 1. Traffic flow series with two days

In this paper, a novel model, named switching ARIMA, is proposed to enhance the basic ARIMA model. For the data set obtained from UTC/SCOOT systems in Beijing, see Figure(1), we notice that the traffic flow series has several patterns, such as ascending pattern, descending pattern, peak pattern and bottom pattern. It is reasonable to construct a separate model for each of the patterns. In this paper we fit a separate ARIMA model to each pattern. We also notice that patterns change with time. Switching model can describe the changing of pattern very well. So switching ARIMA model is introduced to describe the traffic flow series. Based on the switching ARIMA model, we then present the forecasting algorithm for the traffic flow. We apply the proposed model to the real data obtained from UTC/SCOOT systems in Traffic Management Bureau of Beijing. The experiments show that our proposed model is applicable and effective.

The remainder of this paper is organized as follows. In section 2, we use switching ARIMA model to describe the traffic flow series. To make the switching model suitable to the traffic flow series, we introduce the variable of duration. In section 3, we give the forecasting algorithm based on switching ARIMA. In section 4, a case study has verified the effectiveness and applicability of the proposed model with real traffic data obtained from UTC/SCOOT System in Beijing. Finally, conclusions are drawn and comments for further research work are given.

2. SWITCHING ARIMA MODEL FOR TRAFFIC FLOW SERIES

Just as we have mentioned in the section of introduction, the traffic flow series has some patterns and a single ARIMA model is not sufficient to describe the traffic flow series. Here the patterns are the hidden states defined in switching model. We fit a separate ARIMA model to each pattern of the traffic flow and apply the transition of hidden states to describe pattern changing. Usually, the hidden state is assumed to be a time-invariant discrete one-order Markov process. However, this assumption is inappropriate to the traffic flow series. For a time-invariant discrete one-order Markov process, when the current state is given, the next state is independent to its previous states and the duration of the current state will not influence the transition probability from the current state to the next state. However, we notice that the patterns of traffic flow will last for some duration. When the duration is short, the state is less likely to change to other states. And when the duration is longer, the state is more likely to change to other states. We then introduce the variable of duration and use sigmoid function to model its influence to the transition probability to overcome this weakness of the conventional switching model.

Denote the traffic pattern at time t as S_t, which belongs to the set of M discrete symbols $\{1,\ 2,\ \ldots,\ M\}$. In this paper M is set to be 4. The 4 states correspond to bottom pattern, ascending pattern, peak pattern and descending pattern, respectively. It has been noticed that with time evolving patterns can only change from bottom pattern to ascending pattern, from ascending pattern to peak pattern, from peak pattern to descending pattern, or from descending pattern to bottom pattern. Symbol l_{S_t} is used to represent the duration of state S_t. We assume the state transition probability has the following form,

$$p(S_{t+1} = j | S_t = i \text{ at duration } l_i) = a_{ij}\left(l_i\right) \tag{3}$$

where $a_{ij}\left(l_i\right)$is the matrix $A\left(l\right)$'s element at row i and column j,

$$A(l) = \begin{bmatrix} 1-g_1(l_1),\ g_1(l_1),\ 0, 0 \\ 0, 1-g_2(l_2),\ g_2(l_2),\ 0 \\ 0, 0,\ 1-g_3(l_3),\ g_3(l_3) \\ g_4(l_4),\ 0, 0, 1-g_4(l_4) \end{bmatrix}$$

where $g_i(l_i) = sigmoid\left(\frac{l_i-\mu_i}{\sigma_i}\right)$, for i=1, ..., 4.

$sigmoid(x)$ is a sigmoid function [9], as shown in Figure(2). The parameters μ_i and σ_i control mean and standard variance of S_t's duration. This sigmoid function describes the characteristic of the state transition probability for the traffic flow series.

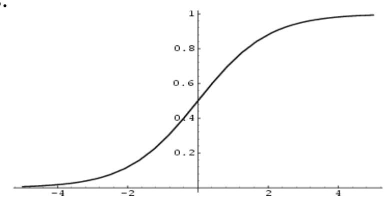

Fig. 2. The curve of the sigmoid function

Let $\{X_t\}$ denote the traffic flow series and $\{Y_t\}$ is the transformed series after differencing the original time series d times. $\{Y_t\}$ can be easily reverted to $\{X_t\}$, so in the following part we primarily confine our concern on $\{Y_t\}$. To sum up, the traffic flow series can be described as follows:

1. Equation (3) for the state process
2. Given the current state S_t, $\{Y_t\}$ can be modeled as

$$Y_t = \sum_{n=1}^{p} \Phi_n\left(S_t\right) Y_{t-n} + \sum_{m=1}^{q} \Theta_m\left(S_t\right) e_{t-m} + e_t \tag{4}$$

where the symbol S_t in the parentheses means that the value of the parameters Φ_n and Θ_m is related to the current state.

3. FORCASTING ALGORITHM BASED ON SWITCHING ARIMA

For classical ARIMA, given the current and previous values of the transformed traffic flow series $\{Y_t\}$, the optimal forecasting of Y_{t+1} is,

$$\hat{Y}_{t+1} = \sum_{n=1}^{p} \Phi_n Y_{t+1-n} + \sum_{m=1}^{q} \Theta_m e_{t+1-m} \tag{5}$$

For switching ARIMA, if the hidden state S_{t+1} is known beforehand, the forecasting will be exactly in the same way as the classical ARIMA. We write the forecasting of Y_{t+1} with the known S_{t+1} as

$$\hat{Y}_{t+1|S_{t+1}} = \sum_{n=1}^{p} \Phi_n\left(S_{t+1}\right) Y_{t+1-n} + \sum_{m=1}^{q} \Theta_m\left(S_{t+1}\right) e_{t+1-m}$$

Unfortunately, in general, the hidden State S_{t+1} isn't known beforehand. However, given the current and previous values of the time series, the probability of the hidden state can be calculated, so the forecasting with switching ARIMA can be written as,

$$\hat{Y}_{t+1} = \sum_{S_{t+1}=1}^{M} \hat{Y}_{t+1|S_{t+1}} p\left(S_{t+1}|Y_{t:0}, e_{t:0}\right) \tag{6}$$

where:

$$\begin{aligned} & p\left(S_{t+1}|Y_{t:0}, e_{t:0}\right) \\ & = \sum_{S_t=1}^{M} \sum_{l_{S_t}=1}^{L} p\left(S_{t+1}|S_t, l_{S_t}\right)\left(l_{S_t}|Y_{t:0}, e_{t:0}\right) p\left(S_t|Y_{t:0}, e_{t:0}\right) \end{aligned} \tag{7}$$

$$p(S_t|Y_{t:0}, e_{t:0}) = \frac{p(Y_t|Y_{t-1:0}, e_{t-1:0}, S_t) p(S_t|Y_{t-1:0}, e_{t-1:0})}{\sum_{S_t=1}^{M} p(Y_t|Y_{t-1:0}, e_{t-1:0}, S_t) p(S_t|Y_{t-1:0}, e_{t-1:0})} \tag{8}$$

$$\begin{aligned} & p\left(l_{S_t} = d|Y_{t:0}, e_{t:0}\right) \\ & = \begin{cases} p\left(l_{S_{t-1}} = d-1|Y_{t-1:0}, e_{t-1:0}\right) & \text{if} d \geq 2 \\ \sum_{\substack{S_{t-1}=1 \\ S_{t-1} \neq S_t}}^{M} \sum_{l_{S_{t-1}}=1}^{L} p\left(S_t|S_{t-1}, l_{S_{t-1}}\right) p\left(l_{S_{t-1}}|Y_{t-1:0}, e_{t-1:0}\right) & \\ \times\, p\left(S_{t-1}|Y_{t-1:0}, e_{t-1:0}\right) & \text{if} d = 1 \end{cases} \end{aligned} \tag{9}$$

After the new observation Y_{t+1} arrives, the error of forecasting can be calculated as,

$$e_{t+1} = Y_{t+1} - \hat{Y}_{t+1} \tag{10}$$

For the convenience of computing, it is usually assumed that maximum duration of states is L. For the completeness of the forecasting algorithm, the initial values of Y_0, e_0, $p\left(S_0|Y_0, e_0\right)$ and $p\left(l_{S_0}|Y_0, e_0\right)$ should be given before the starting of the forecasting algorithm. To sum up, we present the algorithm procedure as in Table(1).

At time t=0,
Step 0: *Initialization*
Initialize $Y_0, e_0, p\left(S_0|Y_0, e_0\right)$ and $p\left(l_{S_0}|Y_0, e_0\right)$
Set t=1
At time $t \geq 1$,
Step 1: *Calculate the posterior probability of hidden states*
Calculate$p\left(l_{S_t} = i|Y_t, ..., Y_0, e_t, ...e_0\right)$ according to equation (9)
Calculate $p\left(S_t|Y_t, ..., Y_0, e_t, ...e_0\right)$ according to equation (8)
Calculate $p\left(S_{t+1}|Y_t, ..., Y_0, e_t, ...e_0\right)$ according to equation (7)
Step 2: *Forecasting*
Calculate$\hat{Y}_{t+1}$ according to equation (6)
Calculate e_{t+1} according to equation (10)
Set $t \leftarrow t+1$ and go to **Step 1**

Table 1. The procedure of forecasting algorithm based on switching ARIMA

4. CASE STUDY

4.1. Data Description

The data used in this paper were obtained from the UTC/SC OOT system in Traffic Management Bureau of Beijing. The data are from Mar. 1 to Mar. 31, 2002. Some experiments have been done with the data from different sites and the experiment results show good performance of our method. Here, we just randomly select one of them to show the results. The raw data at Yuetanbei Street is selected. The discrete time series interval for the data is 15 minutes and the series length for each day is 96. Due to the malfunction of the detector or transmitter, 181 values are missing and we just remove the days including the missing values. Then, 26 days' data remain. The 26 data sets are shown in Figure(3). The former 20 days' data are used to train the model and residual data of 6 days are employed to test the effectiveness of the proposed model.

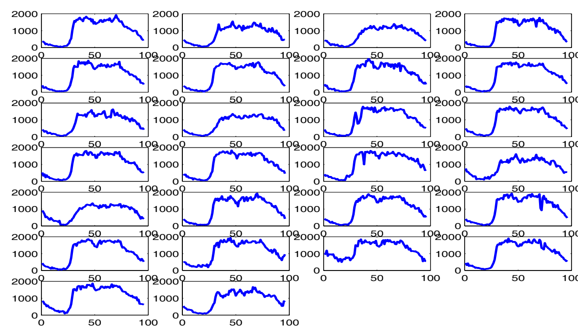

Fig. 3. Traffic flow vs. time plot for all 26 data sets

4.2. Experiment Results

To forecast the traffic flow, we should first learn the model parameters. The parameters include two parts, the parameters in switching model and the parameters in ARIMA model. For the parameters in the switching model, we should specify μ_i and σ_i. From the meaning of μ_i andσ_i, we can directly set their values as shown in Table(2). For the parameters in ARIMA models, we use the methods described in (6) to estimate the parameters of each pattern's ARIMA model.

	$i=1$	$i=2$	$i=3$	$i=4$
μ_i	10	6	40	40
σ_i	4	4	4	4

Table 2. The parameters in switching model

After the parameters are learned from the training set, we apply the forecasting algorithm as described in section 3 to forecast the traffic flow in the testing set. Two of the six days of predicted traffic flow vs. real traffic flow is shown in Figure(4). The error distribution of forecasts is shown in Figure(5). From these two figures, we can see that our proposed algorithm can forecast the traffic flow pretty well and most of the errors concentrate on the range between -5% and 5%.

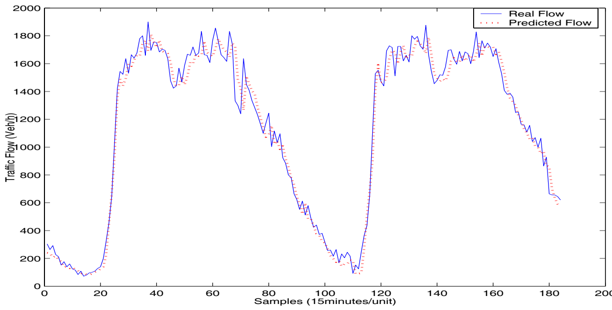

Fig. 4. Predicted traffic flow vs. real traffic flow

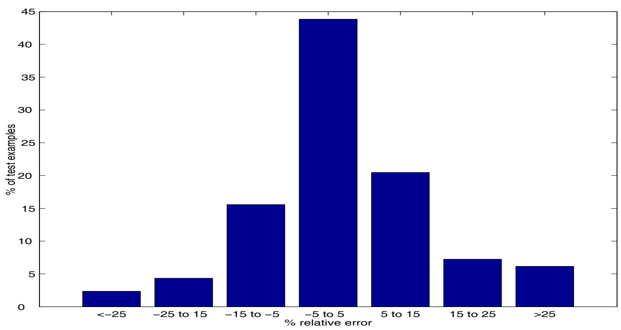

Fig. 5. Error distribution of forecasts

4.3. Comparisons with other models

Three other models are used to compare the performance of our model. For a full coverage of these three models the reader is referred to [3].

The results are compared in terms of two measures of performance, the mean root square error (MRSE) and the mean absolute relative error (MARE). The results of the comparison are given in Table(3). The results show that the proposed model outperforms the other three models in terms of mean root square error (MRSE) and the mean absolute relative error (MARE).

	MRSE	MARE
Random walk	118.9673	11.56%
Historical average	211.3237	18.42%
Informed historical average	108.2020	11.19%
Our Proposed Model	**105.2093**	**9.95%**

Table 3. Comparison of performance of four models

5. CONCLUSION

In this paper, we noticed the traffic flow has several patterns. The patterns have different characteristics. We use a separate ARIMA model to describe each of the patterns. We also noticed the patterns change with time. Thus we proposed the switching ARIMA model to describe the traffic flow series. Since the conventional switching model is inappropriate to describe the pattern changing of the traffic flow, we introduce the variable of duration and use the sigmoid function to describe duration's influence to the transition probability of the patterns. Based on our proposed switching ARIMA model, the forecasting algorithm for the traffic flow series is presented. A case study from Beijing has shown the applicability and the effectiveness of our proposed model.

However, there are still several topics to be further researched. Firstly, in this paper, we set parameters of the switching model manually. In practice, it will be a toilsome task. So, how to estimate these parameters automatically? Secondly, equation (3) is used to describe the switching model. Is it the best way? Thirdly, we use ARIMA to describe each of the patterns in traffic flow. What will it be if we use other models? Fourthly, we only utilize the data at a single site to forecast. Combining the other sites will enhance the performance of the proposed model. Lastly, the study of the relationship between our proposed model and other existing models will also be an interesting issue.

6. REFERENCES

[1] Shumway R. H. and Sroffer D. S., Dynamic linear models with switching, *Journal of the American Statistical Association*, Vol. 86, pp. 763-769, September 1991.

[2] Pavlovic, V., Rehg, J.M., Tat-Jen Cham, and Murphy, K.P., A Dynamic Bayesian Network Approach to Figure Tracking Using Learned Dynamic Models, Proceedings of the Seventh IEEE International Conference on Computer Vision, Volume: 1 , 20-27 Sept. 1999

[3] William, B.M, Modeling and Forecasting Vehicular Traffic Flow as a Seasonal Stochastic Time Series Process. *Doctoral dissertation*. Department of Civil Engineering, University of Virginia, Charlottesville, 1999

[4] Nadhir Messai, Philippe Thomas, Dimitri Lefebvre and Abdellah EI Moudni, A Neural Network Approach for Freeway Traffic Flow Prediction. *Proceedings of the 2002 IEEE International Conference on Control Applications*, September 18-20,2002 Glasgow, Scotland, U.K.

[5] Okutani, I. And Y.I. Stephanides, Dynamic Prediction of Traffic Volume through Kalman Filtering Theory, *Transportation Research Part B*, Vol. 18B, N0. 1, 1-11 1984

[6] Davis, G.A. and Nihan, N.L, Nonparametric Regression and Short-term Freeway Traffic Forecasting. *ASCE Journal of Transportation Engineering*, Vol.177, No 2, 178-188. 1991

[7] Der Voor, M., Dougherty, M. and Watson, S., Combining Kohonen maps with ARIMA time series models to forecast traffic flow, *Transportation Research –Part C*, 4, No. 5, 307-318.

[8] Box, G.E.P. and Jenkins, G.M., *Time Series Analysis: Forecasting and Control*, Revised Edition, Holden-Day, San Francisco.

[9] Seggern, D., CRC Standard Curves and Surfaces. Boca Raton, FL: CRC Press, p. 124, 1993