JOINT SEMIBLIND CHANNEL IDENTIFICATION IN PUNCTURED ARQ RETRANSMISSIONS

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ABSTRACT

We study the channel estimation of a bandwidth efficient automatic repeat request (ARQ) system in which re-transmissions are symbol or bit-wise punctured. Unlike simple ARQ in which an erroneous packet is re-transmitted, many hybrid ARQ systems aim to conserve bandwidth by only retransmitting a punctured or re-coded data packet. In this paper, we formulate a joint semiblind channel estimation algorithm for the punctured re-transmission in a hybrid ARQ network. We show that the joint semiblind estimation is quite simple and naturally combines training in the first transmission with retransmitted data statistics. We show that bandwidth savings resulting from puncturing the retransmission only lead to marginal degradation to the accuracy of channel estimation.

1. INTRODUCTION

Hybrid ARQ is an effective approach to ensure successful packet reception while conserving limited bandwidth resources [1]. After the receiver checks the CRC and determines that the packet contains errors despite forward error correction (FEC) decoding, a retransmission is requested. Instead of retransmitting the entire packet (frame), hybrid ARQ requests the transmitter to either retransmit part of the original data packet or to use a different code for forward error correction (FEC)[1]. For distortive channels, channel estimation in transitional ARQ is often achieved separately for each (re)transmission. Our fundamental idea is that by using joint channel estimation techniques, bandwidth may be conserved with little performance loss in terms of channel estimation accuracy.

To save bandwidth, it would be helpful if subsequent retransmissions can reduce the amount of training data, unlike in traditional ARQ systems. There are two important questions to ask regarding an improved hybrid ARQ receiver:

1) With training only in the first transmission, can a semiblind joint estimation approach be devised to perform comparably to traditionally trained channel estimation?
2) What effect will bit/symbol-wise puncturing have on the channel estimation performance?

In this work, our goal is to answer these questions.

It should be noted that blind channel estimation has been well formulated in [4, 7] for standard ARQ retransmissions. Collecting multiple retransmission over different channels at the receiver, a single input multiple output (SIMO) framework can directly utilize blind channel estimation algorithms based on second order statistics [2][3]. When puncturing is involved in an efficient hybrid ARQ system, only selective data are in common for multiple transmissions. We have already shown that under puncturing channel estimation condition can in fact be loosened and performance may even be improved [5].

In this work, our goal is to take advantage of the available training in many systems during the initial transmission. Specifically, we exploit both the training and the second order statistical information jointly to derive a semiblind algorithm with bandwidth efficiency and better performance.

The organization of this manuscript is as follows. In section II, the problem formulation and complete signal model of punctured hybrid ARQ is described. The next section briefly discusses training based estimation. Next, statistical approach to subspace channel estimation is described in Section IV, leading to the joint semiblind channel estimator in Section V. Finally, simulation results are given in section VI.

2. SIGNAL MODEL

Our focus of this work is on the joint semiblind channel estimation of unknown channels under hybrid ARQ when subsequent retransmission is punctured symbol-wise for higher efficiency. For simplicity and without loss of generality, we focus mainly on 2 transmissions over which the wireless channel may or may not change.

More specifically, let $s[k]$ be the information symbols originating from the transmitter. The ordered symbol sequence is first transmitted through a distortive channel with response $\{h_n\}$. When frame error remains after detection, a retransmission of the punctured symbol sequence, $s^p[k]$, sees a distortive discrete channel with response $\{h_n^p\}$. Here, h_n and h_n^p are the overall channel responses during the two transmissions that may be different. The sampled channel outputs of the two transmissions follow the convolution relationships

$$x_k = \sum_{n=-\infty}^{\infty} s_n h_{k-n} + n_k \tag{1}$$

$$x_k^p = \sum_{n=-\infty}^{\infty} s_n^p h_{k-n}^p + n_k^p. \tag{2}$$

In particular, for a half rate punctured hybrid ARQ,

$$s^p[k] = s[2k]. \tag{3}$$

We use $\mathbf{h}$ and $\mathbf{h}^p$ to represent vectors of the sampled channel parameters for the first and second transmissions respectively.

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By padding zeros to the shorter one, we assume that both channel vectors are of length, $L+1$.

$\mathbf{n}_k$ is a random vector that represents additive noise independent of $\mathbf{s}_k$. Although the noise may be colored Gaussian [3], here for brevity, we will consider only the simpler case of additive white Gaussian noise.

We rewrite equations (1)(2) in matrix-vector notation.

$$\mathbf{x}_k = \left[\begin{array}{cccc} x_k & x_{k-1} & \cdots & x_{k-m_1+1} \end{array}\right]^T, \quad (4)$$

$$\mathbf{s}_k = \left[\begin{array}{cccc} s_k & s_{k-1} & \cdots & s_{k-L-m_1+1} \end{array}\right]^T, \quad (5)$$

$$\mathbf{n}_k = \left[\begin{array}{cccc} n_k & n_{k-1} & \cdots & n_{k-m_1+1} \end{array}\right]^T \quad (6)$$

$$\mathbf{H} = \left[\begin{array}{ccccccc} h_0 & h_1 & \cdots & h_L & 0 & \cdots & 0 \\ 0 & h_0 & h_1 & \cdots & h_L & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & h_0 & h_1 & \cdots & h_L \end{array}\right]_{m_1 \times (L+m_1)} . \quad (7)$$

$\mathbf{H}$ is a Toeplitz matrix that represents the T-spaced channel convolution matrix. The following equation is the matrix-vector equivalent of (1):

$$\mathbf{x}_k = \mathbf{H}\mathbf{s}_k + \mathbf{n}_k. \quad (8)$$

Likewise, for a punctured re-transmission, we have

$$\mathbf{x}_k^p = \left[\begin{array}{cccc} x_k^p & x_{k-1}^p & \cdots & x_{k-m_2+1}^p \end{array}\right]^T, \quad (9)$$

$$\mathbf{s}_k^p = \left[\begin{array}{cccc} s_k^p & s_{k-1}^p & \cdots & s_{k-L-m_2+1}^p \end{array}\right]^T, \quad (10)$$

$$\mathbf{n}_k^p = \left[\begin{array}{cccc} n_k^p & n_{k-1}^p & \cdots & n_{k-m_2+1}^p \end{array}\right]^T \quad (11)$$

The input-output relationship of this punctured retransmission results in a new channel matrix of the same structure but with parameters possibly different from the first transmission, denoted by $\bar{\mathbf{H}}$. The relationship is thus

$$\mathbf{x}_k^p = \bar{\mathbf{H}}_{m_2 \times (m_2+L)} \mathbf{s}_k^p + \mathbf{n}_k^p. \quad (12)$$

The relationship, (12), can be rewritten such that the input-output relationship of the punctured retransmission has the same input signal vector as the first transmission. This results in a new convolutional transfer matrix, $\mathbf{H}^p$, which is directly related to $\bar{\mathbf{H}}$. To construct this new "punctured" channel matrix, we first form a puncturing matrix, $\mathbf{J}^{(\ell,n)}$, with dimension $(m_2+L) \times (m_1+L)$, which is a function of the puncturing rate $R = \frac{\ell}{n}$. In the following equation let $\mathbf{0}$ be the zero matrix with dimension $\mathbf{0}_{(m_2+L)\times(n-\ell)}$. We have

$$\mathbf{J}^{(\ell,n)} = \left[\begin{array}{c} \mathbf{e}_1, \ldots, \mathbf{e}_\ell, \mathbf{0}, \mathbf{e}_{\ell+1}, \ldots, \mathbf{e}_{2\ell}, \mathbf{0}, \ldots, \mathbf{e}_{m_2+L} \end{array}\right] \quad (13)$$

where $\mathbf{e}_n$ is the $n-$standard (identity) vector.

Now we have $\mathbf{s}_k^p = \mathbf{J}^{(\ell,n)}\mathbf{s}_k$. As a result, we can define

$$\mathbf{H}^p = \bar{\mathbf{H}}\mathbf{J}^{(\ell,\mathbf{n})}, \quad (14)$$

with which we can rewrite (12) as

$$\mathbf{x}_k^p = \mathbf{H}^p\mathbf{s}_k + \mathbf{n}_k^p. \quad (15)$$

Here $\mathbf{H}^p$ is no longer Toeplitz and has dimension $m_2 \times (L+m_1)$, where m_2 is chosen to satisfy identifiability conditions, and m_1 is determined such that for a given m_2 and rate $R = \frac{\ell}{n}$, the number of columns in $\mathbf{H}$ and $\mathbf{H}^p$ are the same. Let $C^{(R,m_2)}$ be the number of columns in $\mathbf{H}^p$ with m_2 rows and a puncturing rate, R. It then follows that

$$m_1 = C^{(R,m_2)} - L$$

$$m_1 = m_2 + \lfloor \frac{L+m_2-1}{\ell} \rfloor (n-\ell).$$

Finally, we combine the input-output relationships for both transmissions and form a composite channel transfer matrix.

$$\tilde{\mathbf{x}}_k = \left[\begin{array}{ccc|ccc} x_k & \cdots & x_{k-m_1+1} & x_k^p & \cdots & x_{k-m_2+1}^p \end{array}\right]^T,$$

$$\tilde{\mathbf{n}}_k = \left[\begin{array}{ccc|ccc} n_k & \cdots & n_{k-m_1+1} & n_k^p & \cdots & n_{k-m_2+1}^p \end{array}\right]^T$$

$$\tilde{\mathbf{h}} = \left[\begin{array}{c|c} \mathbf{h} & \mathbf{h}^p \end{array}\right]^T \quad (16)$$

$$\tilde{\mathbf{H}} = \left[\begin{array}{c} \mathbf{H} \\ \mathbf{H}^p \end{array}\right] \quad (17)$$

As a result, we have an equation in the desired form

$$\tilde{\mathbf{x}}_k = \tilde{\mathbf{H}}\mathbf{s}_k + \tilde{\mathbf{n}}_k \quad (18)$$

which allows us to define the statistical information to be used in our semiblind setup to estimate the channel $\tilde{\mathbf{h}}$.

3. CHANNEL ESTIMATION WITH TRAINING

Good performance and ease of implementation lead to the use of some training data for channel estimation. In the first transmission, we assume that a short training sequence of length greater than $2L+1$ is transmitted in the leading segment of the data frame prior to data payload,

Given $L+N$ training data points, N samples of channel output can be used for channel estimation. Specifically, we can collect the N samples in $\mathbf{x}$ and obtain

$$\mathbf{x} = \mathbf{S}\mathbf{h} + \mathbf{n} \quad (19)$$

in which $\mathbf{S}$ is a Toeplitz training data matrix of dimension $N \times (L+1)$ and $\mathbf{n}$ represents a vector of additive noise. A least square channel estimate $\hat{\mathbf{h}}$ can be found by

$$\hat{\mathbf{h}} = \min_{\mathbf{h}} \|\mathbf{x} - \mathbf{S}\mathbf{h}\|_2^2 \quad (20)$$

4. H-ARQ BLIND SUBSPACE CONSTRAINT

Following [5], we can conveniently generalize the subspace method of [3]. The basic approach is to first generate second order statistics of the channel output. The autocovariance matrix of $\tilde{\mathbf{x}}_k$ is decomposed into noise and signal subspaces. After subspace separation, the required full rank condition of the channel matrix $\tilde{\mathbf{H}}$ implies orthogonality between the resulting noise subspace and the channel matrix. This nullspace separated via subspace analysis will be used to estimate the unknown channel.

More specifically, let

$$\mathbf{R}_s = E(\mathbf{s}_k\mathbf{s}_k^H) = \sigma_s^2\mathbf{I}_{(L+m_1)} \quad (21)$$

$$\tilde{\mathbf{R}}_n = E(\tilde{\mathbf{n}}_k\tilde{\mathbf{n}}_k^H) = \sigma_n^2\mathbf{I}_{(m_1+m_2))} \quad (22)$$

$$\tilde{\mathbf{R}}_x = E(\tilde{\mathbf{x}}_k\tilde{\mathbf{x}}_k^h) = \sigma_s^2\tilde{\mathbf{H}}\tilde{\mathbf{H}}^H + \sigma_n^2\mathbf{I}_{(m_1+m_2)}. \quad (23)$$

Without loss of generality, we will assume that $\sigma_s^2 = 1$.

Given a full column rank matrix $\tilde{\mathbf{H}}$, $\tilde{\mathbf{H}}\tilde{\mathbf{H}}^{\mathbf{H}}$ is a Hermitian matrix that is positive semi-definite with rank $L+m_1$. As a result, its eigen-decomposition leads to

$$\mathbf{U}^H\tilde{\mathbf{H}}\tilde{\mathbf{H}}^H U = \mathbf{\Lambda} = diag(\lambda_1, \lambda_2, \cdots, \lambda_{L+m_1}, \underbrace{0, \cdots, 0}_{m_2-L}) \quad (24)$$

It is clear and well known that the first $L+m_1$ basis vectors of $\mathbf{U}$ span $\tilde{\mathbf{H}}$ and are therefore referred to as the signal subspace. The remaining $m_2 - L$ basis vectors represent noise subspace. As a result,

$$\mathbf{U}^H\tilde{\mathbf{R}}_x\mathbf{U} = \mathbf{\Lambda} + \sigma_n^2\mathbf{I}_{(m_1+m_2)} \quad (25)$$

Thus, if the eigenvalues of $\tilde{\mathbf{R}}_x$ are arranged in descending order, its first $L+m_1$ eigen-vectors still represent the complete signal subspace, denoted by $\mathbf{U}_s$. The remaining m_2-L eigenvectors still represent the nullspace of $\tilde{\mathbf{H}}\tilde{\mathbf{H}}^H$, and are denoted by $\mathbf{U}_n$. $\mathbf{U}_n$ is orthogonal to $\tilde{\mathbf{H}}$

$$\mathbf{U}_n^H\tilde{\mathbf{H}} = 0. \quad (26)$$

The above equation allows channel estimation under the structural constraint of $\tilde{\mathbf{H}}$ [3, 5]. following the general subspace algorithm outline [3], one will find that (26) can be equivalently written as

$$\hat{\tilde{\mathbf{h}}} = \min_{\|\tilde{\mathbf{h}}\|=1} \tilde{\mathbf{h}}^H\bar{\mathcal{U}}\tilde{\mathbf{h}}. \quad (27)$$

The matrix $\bar{\mathcal{U}}$ is a hermitian positive definite matrix formed by the vectors of $\mathbf{U}_n$ with dimension $2(L+1)\times 2(L+1)$.

It is interesting to note that the channel may still be identified even if channels for the 2 transmission are not coprime. For a detailed presentation of the relaxed identifiability conditions please see [5, 8].

5. H-ARQ SEMIBLIND SUBSPACE ALGORITHM

In practical scenarios, blind algorithms do not perform as well as training based solutions. Often the first frame of data transmission does contain a training segment. Thus, we would like to utilize both training and the subspace separation shown above. We now develop a simple semiblind channel estimation as a viable alternative to sending more training in subsequent retransmissions. Although training is only sent over the first transmission, our semiblind algorithm can estimate the channel during both transmissions.

Recall from (20) that $\mathbf{h}$ is part of $\tilde{\mathbf{h}}$. To formulate our semiblind algorithm, we simply combine the two optimization costs (20) and (27) to form a new joint cost surface. Although more complex weighting and constrained optimization may be more effective, we will only describe a simple combination here. In fact, simulations show that strictly adding the cost functions without weighting provides the very performance over random channels.

In summary, our semiblind minimization problem is the following

$$\hat{\tilde{\mathbf{h}}} = \min_{\tilde{\mathbf{h}}}(\|\mathbf{x}-\mathbf{S}\mathbf{h}\|_2^2 + \tilde{\mathbf{h}}^H\bar{\mathcal{U}}\tilde{\mathbf{h}}). \quad (28)$$

Note that

$$\|\mathbf{x}-\mathbf{S}\mathbf{h}\|_2^2 = \mathbf{h}^H\mathbf{S}^H\mathbf{S}\mathbf{h} - 2\mathbf{h}^H\mathbf{S}^H\mathbf{x} + \mathbf{x}^H\mathbf{x}. \quad (29)$$

Substituting (29) into (28) and setting the gradient with respect to $\tilde{\mathbf{h}}$ (not $\mathbf{h}$) of equation (28) to zero, we get the following expression

$$\begin{bmatrix} \mathbf{S}^H\mathbf{S} & 0 \\ 0 & 0 \end{bmatrix}\tilde{\mathbf{h}} - \begin{bmatrix} \mathbf{S}^H\mathbf{x} \\ 0 \end{bmatrix} + \bar{\mathcal{U}}\tilde{\mathbf{h}} = 0. \quad (30)$$

Finally, solving for the optimal semiblind channel estimate, we have

$$\hat{\tilde{\mathbf{h}}}_{opt} = \left(\begin{bmatrix} \mathbf{S}^H\mathbf{S} & 0 \\ 0 & 0 \end{bmatrix} + \bar{\mathcal{U}}\right)^{-1}\begin{bmatrix} \mathbf{S}^H\mathbf{x} \\ 0 \end{bmatrix}. \quad (31)$$

This constitutes the joint semiblind estimation of two transmission channels in a bandwidth efficient hybrid ARQ network.

6. SIMULATION

In this section, we summarize the performance of our proposed punctured subspace method (PSSM) in both the semiblind and totally blind scenarios. In addition, we compare its results to that of the standard subspace approach in a simple ARQ case, and to results completely based on training. We show these results in four different experiments.

Note that we assumed a 16 symbol training sequence known as the Constant Amplitude Zero AutoCorrelation sequence or CAZAC [6] in the first transmission. For each frame the channel is assumed to be static. Channel length is assumed to be known at the receiver. The first data payload is assumed to have 128 symbols. Half-rate puncturing is taken for the second re-transmission. QPSK modulated symbols are transmitted through two, 10 tap, T-spaced complex channels. We set $m_2 = L+1$, $m_1 = 2m_2 + L - 1$, and $2m = m_1 + m_2$ (i.e. the channel output covariance matrices have the same dimension).

We consider two different simulations, the first one examines the performance over the same set of 2 channels

$$\mathbf{h} = \begin{bmatrix} .297-.270j \\ -.422+.160j \\ .159-.173j \\ -.106+.037j \\ .292-.307j \\ .003+.174j \\ .184-.107j \\ -.172+.311j \\ -.273+.082j \\ -.063+.317j \end{bmatrix} \quad \mathbf{h}^p = \begin{bmatrix} -.004+.249j \\ .439-.209j \\ .35+.372j \\ .159+.075j \\ .349-.142j \\ .176+.223j \\ -.174+.051j \\ -.231-.061j \\ -.174+.214j \\ -.038+.133j \end{bmatrix}.$$

for every Monte Carlo iteration. The second example examines the performance over a collection of 20 randomly generated circular complex Gaussian channels.

The normalized mean square error (NMSE) is used to measure the performance of channel estimation. For every simulation we assume the receiver knows the channel length. In practice, the receiver must estimate the channel order using a moving average model order estimator, such as the minimum description length algorithm (MDL) [9].

The resulting NMSE in Figure 1 shows that totally blind PSSM and SSM have very similar performances. One can see that their performance is significantly worse than the training based method. This is what motivates the use of a semiblind approach. The semiblind approach is far more reliable than the totally blind method and thereby provides a more attractive alternative to all training estimation.

In the second simulation, 20 random complex Gaussian ISI channels are used. Figure 2 suggests that blind PSSM is slightly more effective than SSM. This can be explained because some random channels may be nearly coprime, leading to ill-conditioning for $\tilde{\mathbf{H}}$. On the other hand, co-prime-ness is not required for the channels in PSSM. Hence, the results are more dependable as various random channels are tested. What is more notable, however,

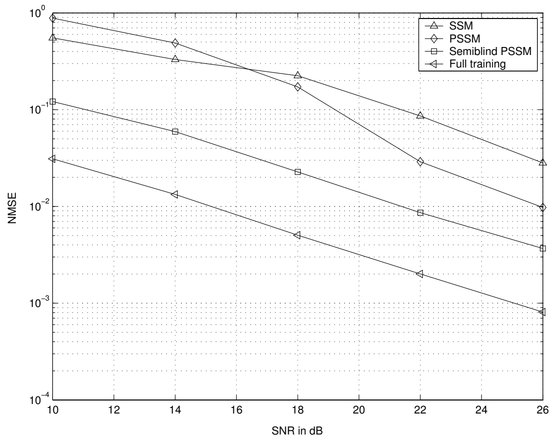

Fig. 1. Simulation using a single channel and 128 symbols

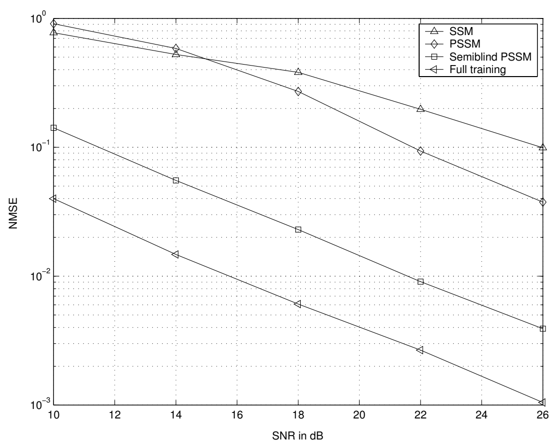

Fig. 2. Simulation using 20 random channels and 128 symbols

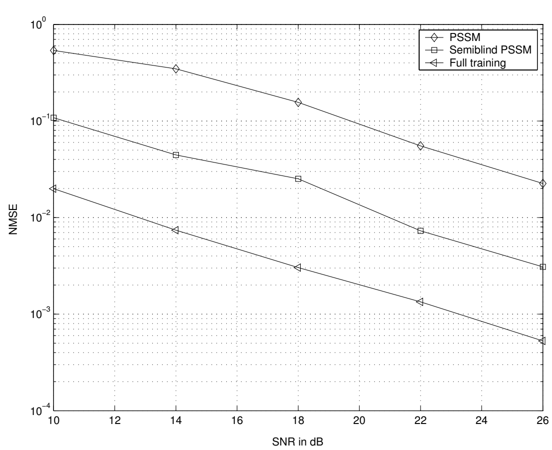

Fig. 3. retransmission channel estimation performance over random channels

is that the semiblind channel estimation performs very well even though training is absent for the estimation of the second channel. Finally, Figure 3 compares the channel estimation performance *only* for the second channel during retransmission. As there is no training for this channel, all methods are compared fairly. It is clear from the figure that, although training is only sent during the first transmission, the semiblind result still significantly outperforms the totally blind estimate.

7. CONCLUSION

Puncturing data before retransmission is an effective way to reduce bandwidth usage in ARQ. This bandwidth saving may be compromised partially if training must be present in retransmitted data packets. In this work, we present a joint semiblind channel estimation approach that does not require training in ARQ retransmissions. It provides much better performance than purely blind algorithms without obvious increase of complexity. In applications, many hybrid ARQ schemes send a fraction of the data, for instance the parity check bits, during the second transmission. Our framework for joint channel estimation using subspace techniques fits conveniently within that application. Semiblind PSSM algorithm is shown to be an accurate, bandwidth efficient approach that can be easily integrated into many existing ARQ schemes.

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