LINEARLY PRECODED OFDM SYSTEM WITH ADAPTIVE MODULATION

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ABSTRACT

We consider the linear precoded OFDM approach proposed in [1], where a non-redundant precoding was applied to the symbol blocks before entering the OFDM system. The precoding, while maintained the transmit power, introduced a structure to the transmitted signal that allowed for blind channel estimation by a simple auto-correlation performed at the receiver. We propose an adaptive modulation based extension of the method of [1] in order to combat channel with deep fading. Bits are allocated on each subcarrier so that the overall transmit power is minimized under a fixed bit error rate (BER). The obtained bit allocation can also be viewed as minimizing BER for the precoded system, under a fixed overall transmit power constraint. The proposed approach provides large performance gains over the uniformly loaded one, especially under deep fading conditions for the same overall throughput and transmit power.

keywords-**OFDM, linear precoding, adaptive modulation, blind channel estimation.**

1. INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM) is a popular modulation technique for wireless network due to its modest requirement for equalization and its high spectral efficiency. It has been implemented in a number of applications, such as the Discrete Multitone (DMT) in ADSL for high speed data on the twisted pair local loop, in the digital audio and video broadcasting standards in Europe (DAB, DVB), and in the IEEE 802.11a wireless LAN standard.

Conventional OFDM is performed in the following steps. First, data are serial to parallel converted and segmented into blocks of length N. An N-point IDFT (Inverse Discrete Fourier Transform) is performed on each block, and a cyclic prefix (CP), consisting of the last N_{cp} IDFT samples, is appended in front of the resulting block. These augmented blocks are sent one after the other through the communication channel. At the receiver, the cyclic prefix is discarded and a DFT is performed on the remainder of each block. As long as the channel length is smaller than the length of the cyclic prefix, the channel affects less than N_{cp} symbols in the beginning of each received block, and as the cyclic prefix portion of the received block is discarded, interblock interference is eliminated. Finally, after demodulation, the symbol at the k-th carrier equals the transmitted one scaled by the complex k-th channel gain.

The gain of each subchannel is needed for coherent symbol detection. It can be estimated based on training symbols [10], at the expense of bandwidth, or via blind channel estimation methods that are more spectrally efficient [11],[15],[14, 16, 17]. One relatively simple blind technique was proposed in [1] based on linear precoding. The symbol blocks are premultiplied by a non-redundant non-unitary precoding matrix before they enter the OFDM systems. While maintaining the transmit power, the precoding introduces a structure to the transmitted signal that allows for blind channel estimation at the receiver. A weakness of the precoding approach, however, was lack of multipath diversity, which in the case where some carriers experience deep fading results in degradation of performance. Although the effect of the deep fading channels can be mitigated by diversity and coding [2, 3], such approaches involve high computation complexity. However, high complexity defies the purpose of OFDM, which is not supposed to need a complex equalizer.

In the context of [1], we here propose an adaptive modulation approach to achieve performance gains in the case of deep fading. The goal is, for a given overall transmit power and throughput, to allocate bits on each subcarrier so that we minimize the BER of the precoded OFDM system. The proposed bit allocation scheme is a modification of that of [4]. Since the precoding introduces intercarrier interference, the criterion in [4] is not directly applicable in our case. We derive the analytic expression for the signal-to-interference ratio (SINR) at the receiver, and based on it get the BER expression for the precoded system, then we can calculate the required transmit SNR under a given BER. The expression for SNR is used in the optimization criterion to get the optimum bit allocation. The bit allocation got here can minimize the overall transmit power under a given BER, or can also be viewed as minimizing the system BER under constraint overall throughput and transmit power. The bit allocation is determined at the receiver, and sent to the transmitter via a low-rate channel, to be used in subsequent transmissions.

The paper is organized as follows. Section II summarizes the blind channel estimation method of [1] and provides background on adaptive OFDM. Section III provides the BER analysis and corresponding bit allocation result. The implementation and performance of the proposed adaptive loaded OFDM system are presented in Section IV, while conclusions are given in Section V.

2. BLIND CHANNEL ESTIMATION AND ADAPTIVE MODULATION

2.1. Blind Channel Estimation based on Linear Precoding

Let $d_{i,n},\ n = 0, ..., N-1$ denote the information bearing symbols of the i-th OFDM block. We assume that the $d_{n,i}$'s are zero-mean i.i.d., temporally white with unit variance, $\sigma_n^2 = 1$, and spatially uncorrelated. The precoded symbols can be written as:

$$\mathbf{s}_{i,k} = \frac{1}{\sqrt{1+|A|^2}}(d_{i,k} + (-1)^k A d_{i,T}),\ k = 0, \ldots, N-1 \quad (1)$$

where A is a predefined imaginary number and T is chosen as the subchannel with maximum channel gain (see [1] for more details). This kind of precoding maintains power, while it does not introduce any offset within the transmitted block, nor over any carrier. In vector notation, it holds:

$$\mathbf{s}_i = \mathbf{Q}\mathbf{d}_i \tag{2}$$

where the precoding matrix $\mathbf{Q}$ only has nonzero elements on the diagonal and the T_{th} column.

Let $\mathbf{y}_i$ denote the received symbol block. The autocorrelation of the received blocks equals:

$$\mathbf{R} \quad \triangleq \quad E\{\mathbf{y}_i \mathbf{y}_i^H\} = \mathbf{H}(\mathbf{Q}\mathbf{Q}^H)\mathbf{H}^H \tag{3}$$

where the expectation is taken over successive symbol blocks; $\mathbf{H}$ is an $N \times N$ diagonal matrix whose k-th diagoanal element is the gain of the k-th carrier, i.e.; $\mathbf{Q}$ is the normalized precoding matrix.

Based on $\mathbf{R}$, the channel H_k within the complex constant H_T^*, can be obtained as:

$$\hat{H}_k \triangleq \begin{cases} \frac{1+|A|^2}{(-1)^k A + (-1)^{k+T}|A|^2} \hat{\mathbf{R}}_{kT}, & k \neq T, \\ \hat{\mathbf{R}}_{k,T}, & k = T. \end{cases} \tag{4}$$

where $\hat{\mathbf{R}}_{kT}$ denotes the (k, T) element of an estimate of $\mathbf{R}$.

2.2. Adaptive Modulation

Conventional OFDM system allocate power and bit uniformly among all subchannels. However, this will result in high overall bit error rate when some subchannels are in deep fade. On the other hand, adaptive modulation allocates bits and/or power of subchannels based on the channel amplitude response. To achieve the objective to maximize the average throughput, or minimize the overall probability of error, subchannels with large amplitude response are allocated more bits per OFDM symbol than those in a deep fading. The optimum solution to the bit and power allocation is well known as the *water filling approach* [13], which provides the optimum power distribution that maximizes the channel capacity for a non-ideal channel with additive Gaussian noise.

There are several published works [4, 5, 6, 7] that discuss bit and/or power allocation algorithms to perform optimum or near-optimum bit and/or power allocation among subchannels OFDM systems. One relatively simple algorithm is the successive bit allocation algorithm of [4]. By iterating through the number of bits to be allocated, in each iteration, one bit is allocated to the subchannel that requires the least amount of additional power to transmit the bit under a BER. The achieved bit allocation corresponds to the solution of the following minimization problem:

$$P_T^* = min_{c_k \in D} \sum_{k=1}^{K} \frac{1}{|H^2(k)|} f_k(c_k) \tag{5}$$

under the constraint: $\sum_{k=1}^{N} c_k = e_f \cdot N$, where $f_k(c_k)$ is the required receiver power in a subchannel for reliable (i.e. BER $= 10^{-4}$) reception of c_k information bits, when the subchannel gain is equal to unity. Since $f_k(c_k)$ is a convex and increasing function with $f_k(0) = 0$, the bit allocation scheme can achieve the best BER under a given overall transmit power.

3. BER ANALYSIS AND BIT ALLOCATION

Since the precoding introduces intercarrier interference, the criterion in [4] is not directly applicable in our case. We next derive the analytic expression for the signal-to-interference ratio at the receiver, and based on it define the transmitter SNR to be used in the optimization criterion.

Let us assume an MMSE equalizer at the receiver. The recovered symbols are:

$$\hat{\mathbf{d}}_i = \mathbf{G}\mathbf{H}\mathbf{s}_i + \mathbf{G}\mathbf{v}_i \tag{6}$$

where

$$\mathbf{G} = \mathbf{Q}^H \hat{\mathbf{H}}^H (\hat{\mathbf{H}}\mathbf{Q}\mathbf{Q}^H \hat{\mathbf{H}}^H + \sigma_v^2 \mathbf{I}_N)^{-1} \tag{7}$$

where σ_v^2 is the noise variance.

The signal-to-interference and noise ratio (SINR) at k-th subcarrier of the symbol block equals:

$$SINR_k = \frac{\sigma_k^2 |\boldsymbol{e}_k^H \boldsymbol{G}\boldsymbol{H}\boldsymbol{Q}\boldsymbol{e}_k|^2}{\sigma_v^2 \boldsymbol{e}_k^H \boldsymbol{G}\boldsymbol{G}^H \boldsymbol{e}_k + \sigma_m^2 \sum_{m \neq k} |\boldsymbol{e}_k^H \boldsymbol{G}\boldsymbol{H}\boldsymbol{Q}\boldsymbol{e}_m|^2} \tag{8}$$

Taking into account that $\sigma_k^2 = 1$ for all k's, and defining $\boldsymbol{g}_k = \boldsymbol{G}^H \boldsymbol{e}_k$, $\boldsymbol{q}_k = \hat{\boldsymbol{H}}\boldsymbol{Q}\boldsymbol{e}_k$ ($\boldsymbol{e}_k$ is column vector with its k-the element equal to one and all other elements equal to zero), we get:

$$SINR_k = \frac{\boldsymbol{g}_k{}^H \boldsymbol{q}_k \boldsymbol{q}_k^H \boldsymbol{g}_k}{\frac{1}{SNR_k} \boldsymbol{g}_k{}^H \boldsymbol{g}_k + \boldsymbol{g}_k{}^H (\boldsymbol{H}\boldsymbol{Q}\boldsymbol{Q}^H \boldsymbol{H}^H - \boldsymbol{q}_k \boldsymbol{q}_k^H) \boldsymbol{g}_k} \tag{9}$$

Substituting the above SINR into the BER expression [13] we get:

$$\bar{P}_e^k \approx \frac{4(1 - 1/\sqrt{M})}{\log_2 M} \left[\mathcal{Q}\left(\sqrt{\gamma \cdot SINR_k} \right) \right] \tag{10}$$

where, for M-QAM, $\gamma = \frac{3}{M-1}$, and $\mathcal{Q}(\cdot)$ denotes the Marcum $\mathcal{Q}$-function. So for any given BER, we can compute the corresponding $SINR_k$ from (10), and then compute the required transmitter SNR_k according to

$$SNR_k = \frac{\boldsymbol{g}_k{}^H \boldsymbol{g}_k}{\frac{\boldsymbol{g}_k{}^H \boldsymbol{q}_k \boldsymbol{q}_k^H \boldsymbol{g}_k}{SINR_k} - \boldsymbol{g}_k{}^H (\boldsymbol{H}\boldsymbol{Q}\boldsymbol{Q}^H \boldsymbol{H}^H - \boldsymbol{q}_k \boldsymbol{q}_k^H) \boldsymbol{g}_k} \tag{11}$$

In (5), the role of SNR_k is played by the term $P_k = \frac{f_k(c_k)}{|H(k)|^2}$. However, in our case, the transmitter SNR for carrier k is a function of the total channel estimate $\hat{\mathbf{H}}$ rather than $\hat{H}(k)$ only; the precoding matrix, $\mathbf{Q}$; the given BER; and the number of bits assigned to subcarrier k, i.e., c_k. For M-QAM signals, the parameter c_k equals $log_2(M)$.

The optimum c_k's are obtained as the solution to the following minimization problem:

$$SNR_T^* = min_{c_k \in D} \sum_{k=1}^{N} SNR_k(c_k, \hat{\mathbf{H}}, \mathbf{Q}, \mathbf{BER}) \tag{12}$$

under the constraint: $\sum_{k=1}^{N} c_k = e_f \cdot N$. To get the solution of (12), we use the successive bit allocation algorithm proposed in [4]. We iterate through the number of bits to be allocated, assigning in each iteration one bit to the subchannel that requires the least amount of incremental transmitter SNR to transmit that bit under a given BER requirement. By noting that $SNR_k(c_k, \hat{\mathbf{H}}, \mathbf{Q}, \mathbf{BER})$ is a convex and increasing function with $SNR_k(0, \hat{\mathbf{H}}, \mathbf{Q}, \mathbf{BER}) = \mathbf{0}$, the bit allocation vector c $(N \times 1)$

obtained here is also the one we need to minimize the system BER under a given overall transmit power and throughput. As shown in [8], the SNR penalty for quantizing the number of bits to 0,2,and 4 to achieve $e_f = 2$ is less than 1 dB, so it is good enough for us to approximate the best performance of adaptive modulation by setting $e_f = 2$, and allocating 0,1,2 or 4 bits to each subchannel according to (12).

The bit allocation scheme assigns BPSK, 4-QAM or 16-QAM modulation types on each subcarrier. A sample output of the bit allocation algorithm is shown in Fig.1. The top half of the figure shows the average amplitude response of the channel over one block time, while the bottom half shows the corresponding bit allocation. As we can observe, no bit is allocated to the subchannels in deep fading(i.e. around subchannel 30 and 50), while 4 bits are allocated to those subchannels with high amplitude response. A total of 128 information bits are transmitted in each OFDM symbol block, for an average of 2 bits/subchannel.

4. SYSTEM IMPLEMENTATION AND PERFORMANCE

4.1. System Model

The proposed adaptive loaded OFDM system is constructed in the following way.

First, we assume that the channel will remain the same over M OFDM symbol blocks, thus bit allocation is performed once every M blocks. The bit allocation vector is generated at the receiver according to (12) using the blindly estimated channel response $\hat{H}$ of the previous segment. At the first iteration, $\hat{H}$ is set equal to identity matrix. The bit allocation vector is fed back to the transmitter, to be used in subsequent transmissions.

At the transmitter, according to the fed back bit allocation vector, M blocks of information bearing bits, i.e., $b_m(i)(m = 1, ..., M; i = 1, ..., e_f \cdot N)$ are adaptively modulated on the subchannels of M OFDM symbol blocks. Then linear precoding is applied to each symbol block as described in (1). The precoded OFDM symbol blocks are transmitted through a conventional OFDM system one block after another. At the receiver, a minimum mean square error (MMSE) equalizer is used to recover the original signal as shown in (6), also based one segment (M OFDM symbol blocks).

4.2. Simulation Results

Simulations are carried out according to the IEEE 802.11a standard, which calls for frames of length $N = 64$, and cyclic prefix of length $N_{cp} = 8$. The channel was simulated according to modified Jakes' model [12]. Here we set channel in deep fading, the minimum normalized channel gain is less than 0.05 (the normalization is with respect to the maximum channel gain).

For slow varying channel, the maximum Doppler frequency was set to $f_d = 20$ Hz. At the 5.2 GHz carrier frequency and 54 Mbps data rate with 4 QAM modulation and 3/4 coding, $f_d = 20$Hz corresponds to user speed roughly 1.2 m/s. Three independent paths ($L = 3$) of the multipath channel were generated with different initial phases. The correlation matrix $\mathbf{R}$ is estimated by time averaging over M OFDM blocks, i.e.,

$$\hat{\mathbf{R}} = \frac{1}{M} \sum_{i=0}^{M-1} \mathbf{s}_i \mathbf{s}_i^H \tag{13}$$

The above estimate converges in the mean square sense to $\mathbf{R}$.

The normalized mean square error (NMSE) is computed according to:

$$NMSE = \frac{1}{J \times K} \sum_{j=1}^{J} \sum_{k=1}^{K} \frac{\sum_{i=0}^{N-1} |H_{kj}^i - \hat{H}_{kj}^i|^2}{\sum_{i=0}^{N-1} |H_{kj}^i|^2} \tag{14}$$

where K is the number of segments of one realization of channel(we assume the channel will remain the same in one segment); J is the number of independent input realizations; and $H_{kj}^i, \hat{H}_{kj}^i$ are the i_{th} subchannel and its estimation during the k_{th} segment of the i_{th} time realization, respectively. The scaling ambiguity could be resolved via the use of a pilot symbol or the finite alphabet approach proposed in [17]. The channel was estimated based on the blind approach of [1]. The obtained estimate was used to recover the transmitted symbols via an MMSE equalizer(6). Fig. 2 shows the BER comparison as a function of SNR for the (I) proposed adaptive modulation based on blind estimated channel information; (II) adaptive modulation with no precoding and using real channel information as proposed in [4]; (III) uniform $4-QAM$ proposed in [1]. It can clearly be seen that the proposed method outperforms the uniform loaded $4-QAM$ approach when channel in deep fading.

We also provide the BER comparison (3) between the proposed adaptive modulation method and the standard training method,i.e. for M OFDM symbol blocks, we will send two training OFDM symbols $(HR = (Y(:,1)./x(:,1) + Y(:,2)./x(:,2))/2)$. We compare the result for (I) proposed adaptive modulation based on blind channel estimation(M=50); (II) the training method with M=50,200 and 300. For the proposed method, the overhead is $1/64$ OFDM symbol per 50 symbol blocks; while for the training method with $M = 200$, the overhead is 2 OFDM symbols per 200 symbol blocks. We can see that the overhead of the training based adaptive modulation (M=200) is 16 times of that of our proposed method, while the performance is almost the same (only 1 dB difference).

5. CONCLUSIONS

We presented an blind adaptive modulation scheme for OFDM system over frequency selective channel. The proposed method results in good performance, especially for channel in deep fading, while is bandwidth efficient, maintains transmission power, and is computationally simple.

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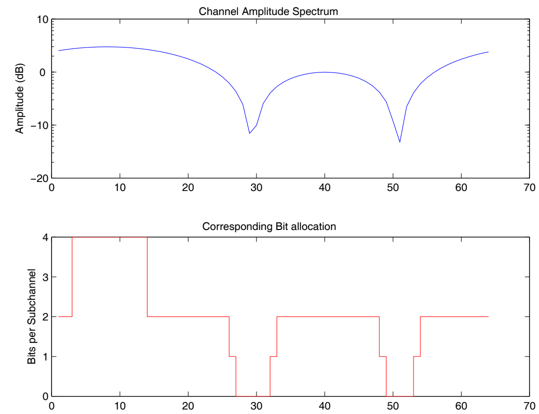

Fig. 1. Example of bit allocation

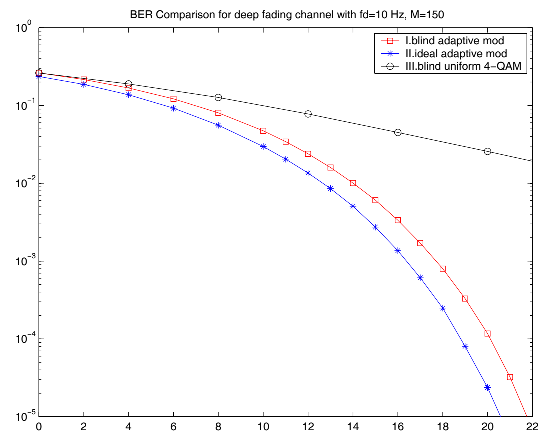

Fig. 2. BER verus SNR in dB (1)

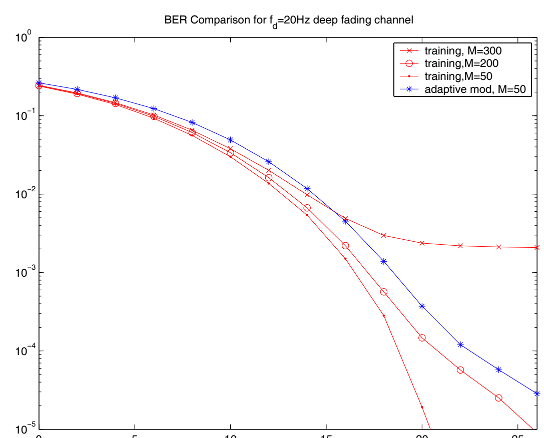

Fig. 3. BER verus SNR in dB (2)