RING BASED FIR-IIR BEST DELAY LS INVERSE FILTERS

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ABSTRACT

In this paper, we investigate the best delay LS inverse filter design problem. We propose FIR-IIR LS inverse filters to improve the LS error performance for the same complexity. Furthermore we present a new approach for the design of FIR-IIR LS inverse filters by using a special selection procedure for the IIR part. We derived the closed form LS error expressions and compared with the practical results in order to effectively show the performance improvement in this case. In general 4-5 dB improvement is achieved compared to the best delay LS inverse filter design approach.

1. INTRODUCTION

The design of inverse filter is an important problem in telecommunications and signal processing. Inverse filtering is closely related to the deconvolution and equalization problems. The most convenient error criteria for the design of inverse filters is the LS error criteria. LS and weighted LS design methods [1]-[4] have been widely used. In [5] a best delay design strategy is introduced and best delay FIR-IIR LS inverse filters are used in the design of Wiener filters. IIR part in the FIR-IIR inverse filter allows exact inversion of the minimum-phase part while the FIR part is used for the maximum-phase component of the channel response. This approach has been shown to improve the LS error for the same complexity [5]. However there are some drawbacks of this approach as well. When the channel zeroes are close to z=0, the use of the IIR part becomes redundant and a single FIR inverse filter performs better for the same computational complexity.

In this paper, we propose a ring based approach for the selection of the IIR part in order to overcome the problems in [5]. In this case, IIR part of the FIR-IIR filter is constructed by considering the channel zeros inside a ring in unit circle. This approach increases the effectiveness of the IIR component and a significant improvement in LS error performance can be achieved.

Closed form LS error expressions are derived for the inverse filter when the channel order is small. These expressions are used to validate the proposed approach. In addition, several experiments are done in order to compare the ring based best delay FIR-IIR LS inverse filters with the best delay FIR LS inverse filters. It turns out that a significant improvement can be obtained when the channel has zeros close to the unit circle. We also considered the pre- and post-inverse filtering approaches when there is channel noise. It seems that the proposed approach also performs better for low SNR especially for the pre-filtering case.

2. BEST DELAY FIR LS INVERSE FILTER

In this part, we will summarize the design of best delay FIR LS inverse filter. The delay of the LS inverse filter seriously affects the LS error. Therefore it is important to find the optimum delay in the inverse filter design. Let $h(n)$ be the channel filter, and $h_{inv}^{FIR}(n)$ be FIR LS inverse filter. The LS error can be written as,

$$E_{LSE}^{k} = \sum_{n} \left| \delta(n-k) - \frac{1}{a} h(n) * h_{inv}^{FIR}(n) \right|^2 \tag{1}$$

where a is a constant scale factor and k is an arbitrary delay. The convolution operation above can also be written in matrix form as,

$$\mathbf{H}\mathbf{h}_{\mathbf{inv}}^{\mathbf{FIR}} = a\mathbf{d} \tag{2}$$

where $\mathbf{H}$ is the Toeplitz convolution matrix and $\mathbf{d}$ is the desired vector, which contains 1 at k^{th} location and all the other elements are zero.

$$\mathbf{d} = \begin{bmatrix} 0 & \cdots & 0 & 1 & 0 & \cdots & 0 \end{bmatrix}^T \tag{3}$$

We will find the best delay and LS inverse filter by using the SVD (Singular Value Decomposition) of the convolution matrix, namely,

$$\mathbf{H}=\mathbf{U}\begin{bmatrix} \Sigma & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}\mathbf{V}^{\mathbf{H}} \tag{4}$$

If we use the same error expression as in [6], the LS error can be written as,

$$E_{LSE} = \mathbf{d}^{\mathbf{H}}\left(\mathbf{d} - \mathbf{H}\mathbf{h}_{\mathbf{inv}}^{\mathbf{FIR}}\right) = \mathbf{d}^{\mathbf{H}}\mathbf{U}\left(\mathbf{U}^{\mathbf{H}}\mathbf{d} - \mathbf{U}^{\mathbf{H}}\mathbf{H}\mathbf{V}\mathbf{V}^{\mathbf{H}}\mathbf{h}_{\mathbf{inv}}^{\mathbf{FIR}}\right) \tag{5}$$

Above expression is due to the counterpart of the orthogonality principle and orthogonality of the left and right singular matrices $\mathbf{U}$ and $\mathbf{V}$. This expression is especially useful for the overdetermined case. Then, LS error for a delay of k samples can be written as,

$$E_{LSE}^{k} = \sum_{i=N}^{N+L-1} \left| \overline{u}_{k,i} \right|^2 \tag{6}$$

Above equation is nothing but the squared sum of the k^{th} row of left singular matrix $\mathbf{U}$. Best delay for a given channel $\mathbf{h}$ is found as the index of the row which gives the minimum LS error,

$$k_{opt} = \arg\min E_{LSE}^{k} \tag{7}$$

Once the best delay is found, desired vector $\mathbf{d}$ is determined and the best delay FIR inverse filter is found as,

$$\mathbf{h}_{\mathbf{inv}}^{\mathbf{FIR}} = \sum_{i=1}^{N} \frac{1}{\sigma_i^2} \mathbf{v}_i \mathbf{v}_i^H \mathbf{H}^H \mathbf{d} \tag{8}$$

3. FIR-IIR LS INVERSE FILTER

In general, any channel response can be decomposed into minimum and maximum-phase components as given below:

$$H(z) = H_{\min}(z) H_{\max}(z) \quad (9)$$

We assume that $H_{max}(z)$ includes the zeros on the unit circle as well. In the conventional FIR LS inverse filter design, the minimum-phase component is also inverted by an FIR filter. However we can use FIR-IIR filter in order to have a better inverse filter. FIR-IIR idea is based on the fact that $H_{min}^{-1}(z)$ is always a casual and stable IIR filter. Therefore the inverse filter for the FIR-IIR case is composed of an IIR part corresponding to $H_{min}^{-1}(z)$ and an FIR part for the maximum phase part,

$$H_{inv}^{FIIR}(z) = \frac{1}{H_{\min}(z)} H_{\max}^{FIR}(z) \quad (10)$$

where $H_{max}^{FIR}(z)$ is the best delay FIR LS inverse filter of the maximum-phase component.

Since IIR part is a perfect inverse filter, it produces no LS error. Therefore the LS error of FIR-IIR LS inverse filter is produced only by the FIR part. So the LS error performance comparison should be done between the FIR LS inverse filter and the FIR part of the FIR-IIR LS inverse filter. The expression after channel inversion is given as,

$$H_{inv}^{FIIR}(z) H(z) = H_{\max}(z) H_{\max}^{FIR}(z) \quad (11)$$

For a reliable comparison, the complexity of both FIR LS inverse filter and FIR-IIR LS inverse filter should be the same. Therefore the length of the $H_{max}^{FIR}(z)$ filter is less than the length of the FIR inverse filter $H_{inv}^{FIR}(z)$. The overall performance of the FIR-IIR filter depends on the location of channel zeros. Let "a" be one of the channel zero. Then the inverse operation to cancel the effect of this zero can be written as,

$$\frac{1}{1 - az^{-1}} = 1 + az^{-1} + a^2 z^{-2} + a^3 z^{-3} + \cdots + a^N z^{-N} + \cdots$$

When $|a| << 1$, $1/(1-az^{-1})$ can be effectively approximated with an FIR filter. The length of the FIR part of FIR-IIR filter will be decreased by one when a first order IIR part is used to compensate the pole of the inverse filter. This in turn increases the overall LS error in inverse filtering. So when the channel zeros are close to z=0, IIR part becomes redundant and FIR inverse filter has a better performance. If on the other hand, the channel zeros are close to the unit circle, IIR part of the FIR-IIR filter can effectively model the inverse filter while this is not possible for a purely FIR inverse filter. Therefore when the FIR-IIR filter is designed as in equation (10), there are certain cases where the performance of this filter becomes inferior to the FIR inverse filter. In order to solve this problem, we propose a special selection procedure for the IIR part of the FIR-IIR filter. This is based on a ring inside the unit circle.

3.1. Ring Based FIR-IIR LS Inverse Filter

In this case, we will select the IIR part such that it will be responsible only for the channel zeros inside a ring in the unit circle. Therefore IIR part of the $H_{inv}^{FIR}(z)$ will be a causal and stable filter in this case as well. The problem is the identification of the suitable radiuses for the ring region. This also corresponds to the identification of the suitable orders for the FIR and IIR parts of the FIR-IIR filter. Since the computational complexity of the FIR-IIR filter is equal to the computational complexity of the FIR inverse filter, it is sufficient to find the order of the IIR or the FIR part only.

Channel zeros significantly affect the LS error and therefore the zero locations should be taken into account when we choose the ring region. In Figure 1, we show the effect of the channel zero on the LS error for the inverse filter. In this case, the channel order is one and the FIR inverse filter length is three. It seems that the main LS error contributions come from the channel zeros which are close to the unit circle. For the best error performance, the poles of the FIR-IIR LS inverse filter should be used to cancel the zeros whose error contributions are sufficiently large. Therefore causal and stable IIR part should be defined based on a ring inside the unit circle. As it can be seen from Figure 1 this ring can be defined between the inner radius R_i and outer radius R_o such that,

$$R_{\mathbf{i}} \le R_{\mathbf{o}} \le R_{\mathbf{u}} = 1$$

This region is illustrated in Figure 2. Therefore IIR part of the FIR-IIR filter will be obtained from the channel zeros which fall into the ring. We can express the channel filter as,

$$H(z) = H_{A1}(z) H_{A2}(z)$$

where $H_{A1}(z)$ is composed of the channel zeros in the ring and $H_{A2}(z)$ is composed of the channel zeros outside the ring. Then the inverse filter can be obtained as,

$$H_{inv}^{FIIR}(z) = \frac{1}{H_{A1}(z)} H_{A2}^{FIR}(z) \quad (12)$$

In order to find suitable radiuses R_o and R_i , we obtained the closed form expressions for the LS error.

The FIR LS inverse filter impulse response can be found by using (2) and assuming that a is equal to 1 as,

$$\mathbf{h}_{\mathbf{inv}}^{\mathbf{FIR}} = \mathbf{H}^{\dagger}\mathbf{d} \quad (13)$$

where, **d** is the desired response, and

$$\mathbf{H}^{\dagger} = \left(\mathbf{H}^{\mathbf{H}}\mathbf{H}\right)^{-1} \mathbf{H}^{\mathbf{H}} \quad (14)$$

is the pseudoinverse [6] of **H.** But the inverse filter $\mathbf{h}_{\mathbf{inv}}^{\mathrm{FIR}}$ is not the perfect filter that produces desired response **d** when cascaded with the channel filter. The actual output is:

$$\mathbf{d}_{\mathbf{actual}} = \mathbf{H}\mathbf{h}_{\mathbf{inv}}^{\mathbf{FIR}} = \mathbf{H}\mathbf{H}^{\dagger}\mathbf{d} \quad (15)$$

There would be some error between the desired response and the actual output. This error term can be written as,

$$\mathbf{e} = \mathbf{d} - \mathbf{d}_{\mathbf{actual}} = \left(\mathbf{I} - \mathbf{H}\mathbf{H}^{\dagger}\right)\mathbf{d} \quad (16)$$

by using this error term LS error can be found as,

$$\begin{aligned} LSE &= \mathbf{e}^{\mathbf{H}}\mathbf{e} \\ &= \mathbf{d}^{\mathbf{H}} \left(\mathbf{I} - \mathbf{H}\mathbf{H}^{\dagger}\right)^{H} \left(\mathbf{I} - \mathbf{H}\mathbf{H}^{\dagger}\right)\mathbf{d} \\ &= \mathbf{d}^{\mathbf{H}} \left[\mathbf{I} - \left(\mathbf{H}^{\dagger}\right)^{H} \mathbf{H}^{H}\right]\left(\mathbf{I} - \mathbf{H}\mathbf{H}^{\dagger}\right)\mathbf{d} \\ &= \mathbf{d}^{\mathbf{H}} \left(\mathbf{I} - \mathbf{H}\mathbf{H}^{\dagger}\right)\left(\mathbf{I} - \mathbf{H}\mathbf{H}^{\dagger}\right)\mathbf{d} \\ &= \mathbf{d}^{\mathbf{H}} \left(\mathbf{I} - \mathbf{H}\mathbf{H}^{\dagger}\right)\mathbf{d} \end{aligned}$$

$$LSE = \mathbf{d}^{\mathbf{H}}\mathbf{B}\mathbf{d} \quad (17)$$

Let the optimum delay for FIR LS inverse filter be "k". So the desired response can be written as given in (3). Then LSE can be simplified as,

$$LSE = \mathbf{d}^{\mathbf{H}} \begin{bmatrix} b_{k1} & \ldots & b_{k(k-1)} & b_{kk} & b_{k(k+1)} & \ldots & b_{k(N+L)} \end{bmatrix}^T$$

where b_{kj} elements are the terms in the k^{th} column of B, N is the length of the FIR LS inverse filter and L is order of channel filter. Then

$$LSE = b_{kk} = \mathbf{B}(k,k) \tag{18}$$

By using this result, we derived the LS error for the FIR inverse filter when the channel order is one. It is given as,

$$\begin{aligned} LSE &= \frac{|a|^{2N}}{1+|a|^2+|a|^4+\cdots+|a|^{2N}} \qquad for\,|a|<1 \\ LSE &= \frac{1}{1+|a|^2+|a|^4+\cdots+|a|^{2N}} \qquad for\,|a|>1 \end{aligned} \tag{19}$$

where a is the zero of the channel filter. We also derived the LSE formulas for second order channel filter. These expressions will not be presented here due to space limitations.

4. PERFORMANCE EVALUATIONS

4.1. No Noise Case

We will use the results in the previous section and find the most convenient ring for the IIR filter selection. We will consider a channel with order two and the FIR best delay LS inverse filter length will be chosen as three for simplicity. One of the zeros of the channel is assumed to be inside the unit circle and the other is at the outside of the unit circle. For the above simple case, we can model channel filter components $H_{A1}(z)$ and $H_{A2}(z)$ as follows,

$$H_{A1}(z) = 1 + az^{-1} \qquad |a| < 1$$

$$H_{A2}(z) = 1 + bz^{-1} \qquad |b| \geq 1$$

For this case, we chose the outer radius of the ring as one and $R_o=R_u$. Then the value of |a|, which is equal to R_i, is found as a function of |b| and the LSE difference between the FIR LS inverse filter and the FIR-IIR LS inverse filter. The results are graphically illustrated in Figure 3. In this figure, the area in which inner roots are shown as –1 represents that for a given outer root b and a LSE difference, there is no solution for an inner root a where the FIR-IIR LS inverse filter has better LSE than the FIR LS inverse filter. For such cases, inner radius should be equal to the outer radius. Therefore there will be no IIR part for the FIR-IIR LS inverse filter and it will be same as the FIR LS inverse filter. For a given outer root, inner radius increases when the LSE difference increases. Also when the outer root magnitude is decreased, inner radius increases for a constant LSE difference. It is also evident that there is a large region where the LSE performance of the FIR-IIR filter is better than the LSE performance of the FIR inverse filter (LSE>0).

For more general channels, the effect of the radiuses of the ring on the normalized LS error is illustrated in Figure 4. In this figure LS error difference between the FIR-IIR LS inverse filter and FIR LS inverse filter is evaluated for 100 normal distributed random real channels. The inner radius is chosen to be smaller than the outer radius. As it can be seen from the figure the optimum ring area can be defined between the unit circle and the inner radius which is approximately 0.6. For this case, approximately 4-5 dB performance improvements can be achieved.

4.2. Noisy Case

We also considered the case where there is noise in the inverse filtering operations. The two alternatives for inverse filtering are treated, namely the pre- and post-filtering as shown in Figure 5. In the pre-filtering case, data is pre-filtered before transmission. Inverse filtering is done after the channel processing for the post-filtering. Figure 6 shows the performance comparison between the FIR-IIR and FIR inverse filters for the pre-filtering approach. Positive values of LSE indicate the cases where the FIR-IIR filter has better performance. In this case, the difference between the inner and outer radius is chosen as 0.5 and the LSE difference is shown for different SNR and outer radius selections. It seems that the best choice for the outer radius is close to one. Similar experiments are done for the post-filtering case as well. The results are shown in Figure 7. It seems that the SNR increase leads to better performance for the FIR-IIR filter for both cases. However the performance for pre-filtering is better than the post-filtering. This is reasonable since there is some noise enhancement in case of post-filtering.

5. CONCLUSION

In this paper, we present a new approach for the design of LS inverse filters. We propose FIR-IIR based approach together with a special selection for the IIR part. IIR part of the FIR-IIR filter is selected from the channel zeros which fall into a ring region inside the unit circle. We derived closed form LS error expressions and compared the performance of the FIR-IIR filter with the FIR best delay inverse filter for different channel, inner and outer radius values. It is shown that one of the best choice for the outer radius of the ring is one. A good choice for the inner radius is 0.6. We had considerable performance improvement over FIR inverse filters. In addition, FIR-IIR inverse filters have a large region where they have better performance than their FIR counterparts.

5. REFERENCES

[1] S. Sunder, "An Efficient Weighted Least-Squares Design of Linear-Phase Nonrecursive Filters", IEEE Transactions On Circuit and Systems-II: Analog and Digital Signal Processing, Vol 42. No 5. May 1995.

[2] Wu-Sheng Lu, and Tian-Bo Deng, "An Improved Weighted Least-Squares Design for Variable Fractional Delay FIR Filters", IEEE Transactions On Circuit and Systems-II: Analog and Digital Signal Processing Vol. 46, No. 8, August 1999.

[3] P. Stoica and T. Soderstrom, "A Method for the Identification of Linear Systems Using the Generalized Least Squares Principle", IEEE Transaction On Automatic Control., Vol. AC-22, No. 4, August 1977.

[4] Eli Fogel, "A Fundamental Approach to the Convergence Analysis of Least Squares Algorithms", IEEE Transaction On Automatic Control., Vol AC-26, No. 3, June 1981.

[5] T. Engin Tuncer, "Causal and stable FIR-IIR Wiener filters", IEEE Workshop on Statistical Sig. Proc., SSP-03, St. Louis, Sep. 28., 2003.

[6] S. Haykin, "Adaptive Filter Theory", Prentice Hall, 1996.

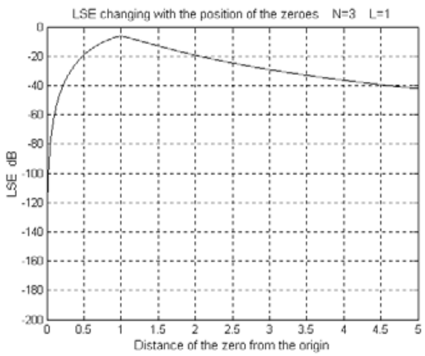

Figure 1 LSE changing with the position of zeroes for N=3, L=1

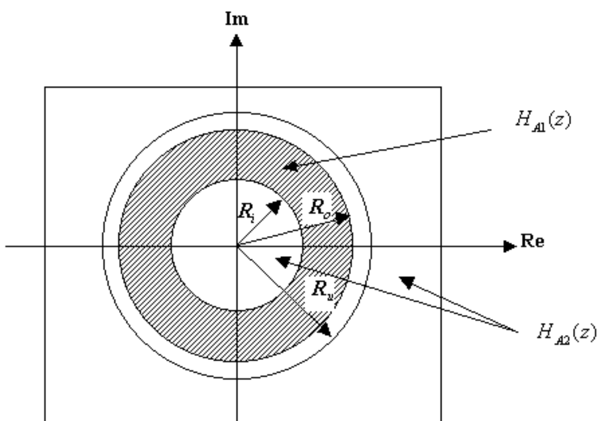

Figure 2. Ring based separation of the z-plane for the IIR part.

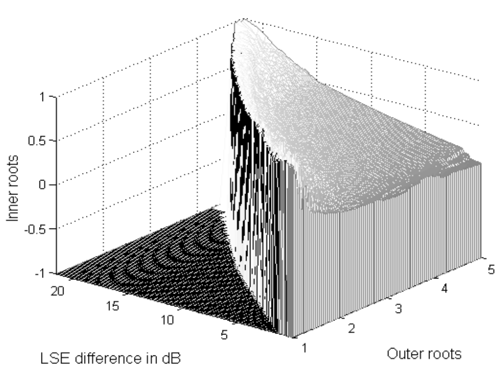

Figure 3 Theoretical results for the inner radius selection for N=3, L=1.

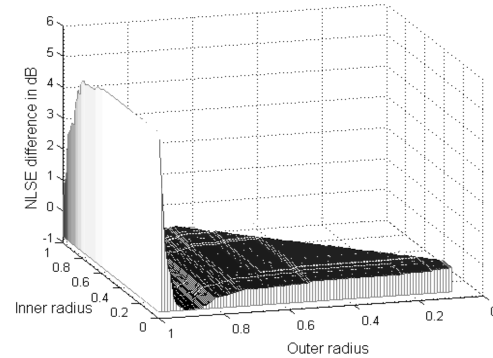

Figure 4 Inner , outer radius and LSE difference for N=32, L=15.

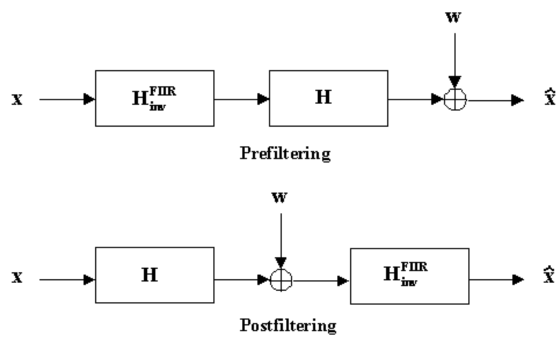

Figure 5 Two structures that inverse filter can be used

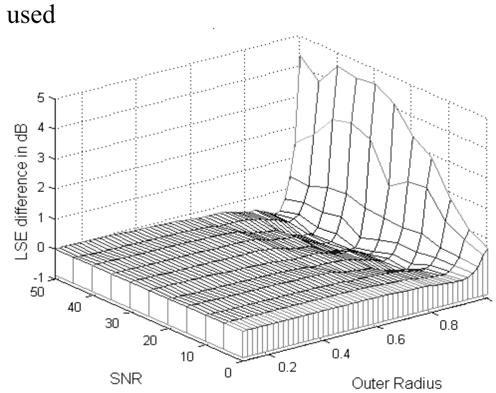

Figure 6 Performance comparison for the pre-filtering for N=32, L=15.

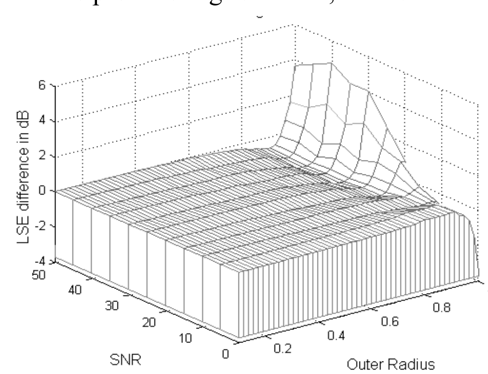

Figure 7 Performance comparison for the post-filtering for N=32, L=15.