GENERALIZED SYMBOL SYNCHRONIZATION USING VARIABLE IIR AND FIR FRACTIONAL-DELAY FILTERS WITH ARBITRARY OVERSAMPLING RATIOS

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ABSTRACT

In this paper, a generalized maximum likelihood symbol synchronization scheme is proposed. It enables use of variable fractional delay (interpolation) filters when the desired sampling rate conversion factor is non-integer or time varying. Furthermore, both IIR and FIR type variable filter structures can be used in efficient configuration. The paper includes both performance analysis and discussion of implementation complexity.

1. INTRODUCTION

The current trend is to use digital receivers where the sampling of the demodulated baseband signal is performed by a fixed sampling rate oscillator. This new design approach reduces the number of required analog components as most of the receiver functions are performed digitally. Using digital signal processing instead of analog signal processing allows increased flexibility, configurability, and integrability of the receiver. From these properties arises the software radio (SWR) concept [1] which is a natural progression of the digital radio receivers towards multimode, multistandard terminals where most of the functionalities are defined by software.

These SWR systems employ direct conversion receivers with asynchronous sampling such that the actual sampling instants are not synchronized with the incoming symbol stream. In order to evaluate the received symbols at optimum instants we must operate in synchronism with the symbol stream. Maximum Likelihood theory can be used to develop optimal timing recovery schemes and digital fractional-delay filters can be used for synchronization. The novel gathering structure offers an efficient and flexible realization of the required fractional-delay interpolators [2].

In multi-mode multi-service systems the symbol rates of the different services may vary. This means that the oversampling ratios of the respective streams will be different. Furthermore, some applications will find FIR FD synchronization filters more suitable than their IIR counterparts, and others vice versa. Thus it is desirable to develop a generalized symbol synchronization scheme that can be used with any carried service, irrespective of the specific oversampling ratio (OSR) and choice over FIR or IIR type.

There have been publications on synchronization algorithms for specific oversampling ratios and on sampling rate conversion by arbitrary factors using fractional delay filters [2]-[6]. However, there has not yet been a generalized algorithm that would clearly present a synchronization algorithm suitable for both FIR and IIR filters using arbitrary oversampling ratios. This is the void that this paper aims to fill by proposing a generalized maximum likelihood symbol synchronization scheme applicable for both FIR and IIR filters running at arbitrary oversampling ratios.

In Fig. 1 symbol synchronization is performed digitally for arbitrary oversampling ratios using an FD filter. The received signal $s_c(t)$ is first digitally sampled at a fixed sampling rate $F_S = 1/T_S$. We assume that the overall transfer function before synchronization meets the Nyquist criterion. Symbol synchronization is usually performed near a sampling rate twice the symbol rate. In order to achieve this, economical fixed sample-rate conversion techniques can be utilized, such as the cascaded integrator-comb (CIC) filters [5, 7]. Before symbol decision the residual timing offset is corrected using an FD interpolation filter and ML feed-forward timing estimation. It is assumed that there is only little jitter between expected transmit and receive symbol rates such that timing offset can be considered constant for each block of M symbols.

2. GENERALIZED SYMBOL SYNCHRONIZATION SCHEME FOR ARBITRARY OVERSAMPLING RATIOS

The input signal $s_{T_S}(n)$ has sample rate F_S. The output signal $s_T(d,m)$ sample rate is equal to the symbol rate $F = 1/T \leq F_S = 1/T_S$ [samples/second]. This means that the synchronization process actually involves the process of decimation [5] as the input sampling rate F_S is simultaneously converted into the lower output sampling rate F as shown in Fig. 1. Formally this can be expressed as

$$s_c(t) = \sum_n s_{T_S}(n) h_c(t - nT_S) \tag{1}$$

$$\begin{aligned} \Rightarrow\ s_T(d,m) &\equiv s_c(mT + dT) \\ &= \sum_n s_{T_S}(n) h_c(mT + dT - nT_S) \\ &= \sum_n s_{T_S}(n) h_c(k_m T_S - \mu_m T_S - nT_S) \end{aligned} \tag{2}$$

where $h_c(t)$ is a continuous-time interpolation filter and $s_T(d,m)$ the mth soft output symbol estimate whose timing has been corrected by the fractional delay $d = \tau/T$ relative to the symbol interval T.

*This work was supported by the Research Council of Norway through project number 157716/432.

The ideal continuous-time interpolation filter $h_c(t)$ can be *approximated by a digital interpolation filter* giving $\hat{s}_T(d,m) \approx s_T(d,m)$ as

$$\begin{aligned}\hat{s}_T(d,m) =& \sum_n s_{T_S}(n) h(\mu_m, k_m - n) \quad (3)\\ =& \sum_n s_{T_S}(k_m - n) h(\mu_m, n)\\ =& s_{T_S}(k_m) \otimes h(\mu_m, k_m) \quad (4)\\ \equiv& \hat{s}_{T_S}(\mu_m, k_m) \quad (5)\end{aligned}$$

where $\otimes$ denotes convolution,

$$\begin{aligned}mT + dT =& k_m T_S - \mu_m T_S \quad (6)\\ k_m =& \mathrm{round}\left\{(m+d)\frac{T}{T_S}\right\} \in \mathbb{Z} \quad (7)\\ \mu_m =& k_m - (m+d)\frac{T}{T_S} \in [-0.5, 0.5[, \quad (8)\end{aligned}$$

and $h(\mu_m, k_m) \approx h_c(k_m T_S - \mu_m T_S)$ is a continuous function of $\mu_m \in [-0.5, 0.5[$ such that it can be used to resample the received signal $s_{T_S}(k_m)$ at the required fractional delays $\{\mu_m\}$ relative to the sampling interval T_S.

2.1. Maximum Likelihood Feedforward Timing Estimation

The log likelihood function (LLF) for symbol timing estimation, assuming an additive Gaussian noise Nyquist channel, is given by [4, 8]

$$\Lambda(d) = \left| \sum_{m=1}^{M} \hat{a}_m^* \hat{s}_T(d,m) \right| \quad (9)$$

with * denoting complex conjugation. Here $\{\hat{a}_m\}$ are the correct (data aided, i.e., training mode) or estimated (decision directed) symbol values, $\hat{s}_T(d,m)$ the fractionally delayed symbol estimates, d a fractional delay (relative to T), and M the number of used past symbols. The delay d is assumed to remain constant within the block of M symbols. We will assume a training signal and thus the symbols in (9) are known, i.e., $\{\hat{a}_m\} \to \{a_m\}$.

The maximum likelihood (ML) feedforward fractional delay estimate $\hat{d}$ is defined as

$$\hat{d} = \frac{\hat{\tau}}{T} = \arg\max_d \{\Lambda(d)\} \in [-0.5, 0.5[\quad (10)$$

where $\hat{\tau}$ is the timing error estimate in seconds and T the symbol interval. Here, $\Lambda(d)$ is approximated by a polynomial

$$\Lambda(d) \approx A_P d^P + A_{P-1} d^{P-1} + \cdots + A_0 \quad (11)$$

of order P. The coefficients $\{A_0, A_1, \ldots, A_P\}$ can be obtained by solving the system of linear equations

$$\begin{bmatrix} 1 & d_0 & d_0^2 & \cdots & d_0^P \\ 1 & d_1 & d_1^2 & \cdots & d_1^P \\ \vdots & & & & \\ 1 & d_P & d_P^2 & \cdots & d_P^P \end{bmatrix} \begin{bmatrix} A_0 \\ A_1 \\ \vdots \\ A_P \end{bmatrix} = \begin{bmatrix} \Lambda(d_0) \\ \Lambda(d_1) \\ \vdots \\ \Lambda(d_P) \end{bmatrix} \quad (12)$$

provided we have an interpolation filter that can evaluate $\Lambda(d_i)$ for a chosen set of distinct delay values $\{d_p\}_{p=0}^{P}$. The maximum-point estimate is then obtained in closed form by solving for the peak point of the polynomial [3]. The structures presented in this paper can be used with any P. Furthermore, it is possible to divide the interval $d \in [-0.5, 0.5]$ into more than one section and use separate polynomials for each section.

With integer-ratio oversampling (i.e., $T/T_S \in \mathbb{Z}^+$) the set of delay values $\{d_p\}$, relative to the symbol interval T, will map into a respective set of delay values $\{\mu_{mp}\}$ according to (8) as

$$\mu_{mp} = k_m - (m+d_p)\frac{T}{T_S} \in [-0.5, 0.5[, \quad \text{for } p = 0, 1, 2, \ldots, P \quad (13)$$

relative to the sampling interval T_S. For integer-ratio oversampling this set will remain constant for all symbol indices m. Moreover, the set $\{\mu_{mp}\}_{p=0}^{P}$ will generally contain duplicate values of which only the subset consisting of all unique values must be processed to obtain the required outputs. With rational oversampling ratios T/T_S, instead, each delay value d_p relative to the symbol interval T will map into a cyclic sequence of delay values $\{\mu_{mp}\}_m$ relative to the sampling interval T_S, for consecutive symbol indexes m. The length of the cycle can be solved as q_m from

$$\min_{q_m} \left\{ q_m = n\frac{T}{T_S} \right\}, \quad \text{for } (n, q_m) \in \mathbb{Z}^+ \quad (14)$$

Furthermore, with irrational oversampling ratios there will be an infinite sequence of distinct values, $\{\mu_{mp}\}_m$, corresponding to a single delay value d_p, for consecutive symbol indexes m, i.e., $q_m \to \infty$.

With FIR filters it does not matter whether the delay values map one-to-one or one-to-many, because each output sample evaluated at a desired delay is independent of any previously processed output samples and their delay values. For recursive (IIR) filters, however, the mapping is significant. Whenever the delay value (variable filter coefficients) of a given recursive filter is changed, the output signal will become corrupted by transient effects [9]. This makes it desirable to dedicate individual feedback loops for each of the required delay values $\{\mu_{mp}\}$ relative to the sampling interval T_S. More delay values will then mean more feedback branches. The number of distinct delay values $\{\mu_{mp}\}$ depends on the OSR as in (14) and choice of the delay grid $\{d_p\}_{p=0}^{P}$ in (12).

3. TIMING ADJUSTMENT USING GATHERING STRUCTURES

In this section, we show how the proposed generalized symbol synchronization scheme for arbitrary oversampling ratios can be implemented using either IIR or FIR gathering structures for FD filters [2]. The proposed implementations allow to efficiently obtain several symbol estimates at different delay values, as required by the LLF in (9) and in (12).

3.1. IIR Allpass Structure

This subsection presents how the fractional delay required by the generalized symbol synchronization scheme can be implemented utilizing IIR allpass gathering structures [10]. IIR filters often offer lower implementation complexity than their equivalent FIR counterparts [11] and allpass filters have exactly unity magnitude by definition, so that they provide an ideal FD technique for applications, where stability problems may arise. The transfer function of an IIR allpass filter with variable real coefficients is given by

$$H(\mu, z) = \frac{z^{-1} A(\mu, z^{-1})}{A(\mu, z)} \quad (15)$$

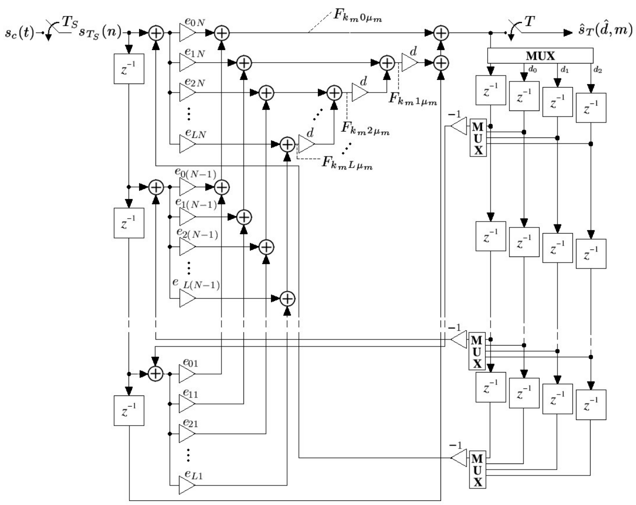

Fig. 1. *Gathering structure with multiplexed output history for IIR allpass FD filters. The equations $F_{k_m l \mu_m}$ used in ML timing estimation are pointed out.*

where

$$A(\mu, z) = 1 + \sum_{n=1}^{N} a_n(\mu) z^{-n} = 1 + \sum_{l=0}^{L} \left(\sum_{n=1}^{N} e_{ln} z^{-n} \right) \mu^l \quad (16)$$

The filter coefficients $\{a_n\}_{n=1}^{N}$ of the IIR allpass filter structure appear both in the feedforward branch and in the recursive feedback branch. Utilizing this symmetry inherent in allpass IIR filters their computational complexity can be reduced further by reusing the constant coefficients [10]. The fractionally delayed symbol estimates are obtained as

$$\hat{s}_T(d, m) = \hat{s}_{T_S}(\mu_m, k_m) \quad (17)$$

$$= s_{T_S}(k_m - N) + \sum_{l=0}^{L} F_{k_m l \mu_m} \mu_m^l \quad (18)$$

where the variable

$$F_{k_m l \mu_m} = \sum_{n=1}^{N} [s_{T_S}(k_m - N + n) - \hat{s}_{T_S}(\mu_m, k_m - n)] e_{ln} \quad (19)$$

is introduced merely for the purpose of conveniently illustrating the signal paths in Fig. 1.

For IIR filters, it is relevant whether the delay values in (13) map one-to-one or one-to-many, because each output sample evaluated at a desired delay is dependent on any previously processed samples and their delay values through feedback. Whenever the delay value (variable filter coefficients) of a given recursive filter is changed, the output signal will become corrupted by transient effects [9]. In order to avoid transients the filter output is multiplexed and the different constant delay values, $\{\mu_{mp}\}$, processed in separate feedback loops (see Fig. 1). The symbol estimates, however, at the estimated delay values $\hat{d}$ will suffer from transients, because only a single feedback loop is used for the output signal $\hat{s}_T(\hat{d}, m)$.

3.2. FIR Structure

Optionally, FIR FD filters can be used to process the symbol estimates. Several advantages exist in using an FIR filter, since it can be designed with exactly linear phase and the filter structure is always stable with quantized filter coefficients. Furthermore, FIR filters can be easily used in variable filter structures [12] since they do not suffer from transient problems.

The transfer function of a generic, causal, non-variable Nth-order FIR filter can be expressed as

$$H(z) = h_0 + h_1 z^{-1} + h_2 z^{-2} + \cdots + h_N z^{-N} = \sum_{n=0}^{N} h_n z^{-n} \quad (20)$$

in which the filter coefficients $\{h_n\}$ have fixed values. To make the FIR filter variable with an independent parameter μ, we express each filter coefficient as an Lth-order polynomial in μ:

$$h_n(\mu) = c_{0n} + c_{1n}\mu + c_{2n}\mu^2 + \cdots + c_{Ln}\mu^L = \sum_{l=0}^{L} c_{ln}\mu^l \quad (21)$$

Using Horner's rule similarly as with the IIR filter coefficients we can separate the variable coefficients μ, of the polynomially expressed filter coefficients, from the fixed-value coefficients and obtain

$$H(\mu, z) = \sum_{l=0}^{L} \left(\sum_{n=0}^{N} c_{ln} z^{-n} \right) \mu^l \quad (22)$$

A canonic implementation of FIR filters with polynomially approximated coefficients can be found in the form of a so-called *FIR gathering structure* [10] (or *generalized Farrow structure*). The fractionally delayed symbol estimates are obtained as

$$\hat{s}_T(d, m) = \sum_{l=0}^{L} F_{k_m l} \mu_m^l = \sum_{l=0}^{L} \left[\sum_{n=0}^{N} s_{T_S}(k_m - n) c_{ln} \right] \mu_m^l \quad (23)$$

Because FIR filters do not have recursion, $F_{k_m l}$ is now independent of the particular delay value μ_m.

4. EXAMPLES

In order to demonstrate how the proposed symbol synchronization scheme can be used together with different kinds of FD filters, we used it to measure the residual timing jitter of two FIR FD filter and two IIR FD filter synchronizers. The FIR filters measured were a second-order Lagrange [11] FD filter and a 7^{th}-order Interpolator II [13] FD filter. The measured IIR filters included a polynomially approximated first-order allpass Thiran [11] FD filter and a third-order allpass equiripple phase delay FD filter [11]. As was done in [4], we used OSR=2 and split the interval $d \in [-0.5, 0.5[$ (relative to the symbol interval) into two equal-length intervals. In order to maximize (9) for (10) the LLF was approximated in the two equal-length intervals using two separate third-order ($P = 3$) polynomials. The LLF (9) was measured over $M = 64$ QAM-64 symbols using root-raised cosine (RRC) matched filter pairs having roll-off factor $\alpha = 0.35$. The measurement results obtained under a signal-to-noise (SNR) ratio of 10 dB are shown in Fig. 2. Furthermore, in Fig. 3 we demonstrate how the proposed generalized symbol synchronization scheme allows us to measure the same filters using another oversampling ratio, namely OSR=3.5. The implementation complexities of the filters are given in Table 1.

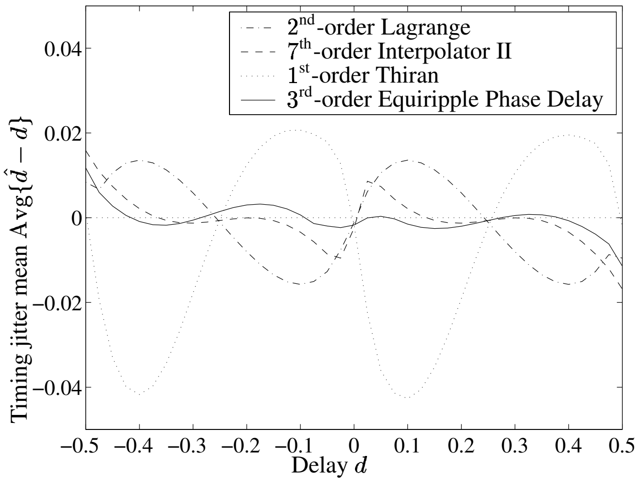

Fig. 2. *Timing jitter mean of the different FIR and IIR FD filters using* OSR=*2.*

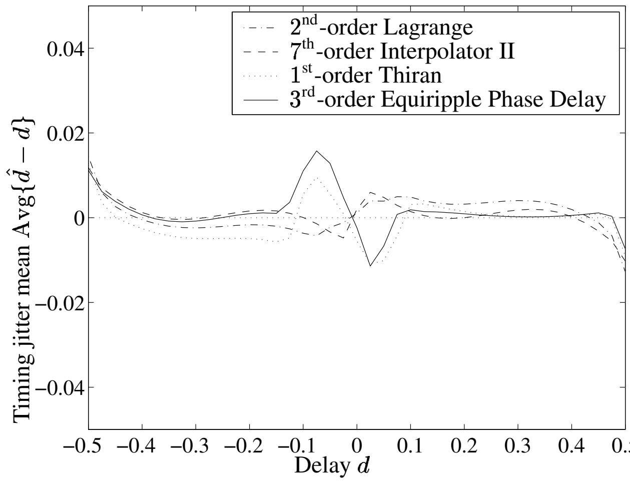

Fig. 3. *Timing jitter mean of the different FIR and IIR FD filters using* OSR=*3.5.*

Table 1. *Implementation complexity: Number of delay elements, number of variable multipliers, and number of constant coefficients required for implementing each FD filter. If the coefficient-symmetry of the Interpolator II [13] is exploited, only half of the constant multiplier coefficients are needed.*

FD Filter	N	L	z^{-1}	d	$\{c_{lk}, e_{lk}\}$
IIR			$2N$	L	$N(L+1)$
Thiran (polyn.)	1	2	2	2	3
Equiripple phase delay (ω_0=0.5)	3	3	6	3	12
FIR			N	L	$(N+1)(L+1)$
Lagrange	2	2	2	2	9
Interpolator II	7	3	7	3	32 (16 w/sym.)

5. CONCLUSIONS

A generalized synchronization scheme for polynomial IIR and FIR FD filters allowing arbitrary oversampling ratios was proposed. The technique was illustrated with example implementations of two FIR FD filters and two IIR FD filters running at two different oversampling ratios. It is left as a future research topic to analyze the exact performance of different fractional-delay filters and the reasons for the differences in results obtained with different oversampling ratios. Finally, the proposed scheme can be used also for performing comparison between different symbol timing estimation methods.

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