Constrained Eigenfilter Design without Specified Transition Bands

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***Abstract*— The design of FIR filter with constraints in frequency domain and/or time domain is considered. We further considered the design specification without explicitly specified transition band. A constrained eigenfilter is proposed to design FIR filter with various design constraints, and without transition band specification. We have suggested the possible design tradeoff between transition band bandwidth and the ripple size of the filter. The proposed algorithm can design filters with optimal tradeoff between transition band bandwidth and the peak constrained ripple size. The eigenfilter formulation further allows the filter design specification to incorporate time domain constraints. Various design examples are presented to illustrate the versatility of the digital filter obtained by the proposed filter design method.**

I. Introduction

Linear phase FIR filters with equiripple passband and/or stopband magnitude response are widely used. Parks-McClellan algorithm (PM) and it's variant that make use of the Remez exchange algorithm are popular techniques to design filters with equiripple magnitude response. An iterative recursive least squares algorithm was developed in [8] to achieve Chebyshev or L_∞ approximation. The algorithm was subsequently applied to design FIR filters that are optimal in L_∞ and hence achieve equiripple response. It was shown to be able to solve certain FIR filter design problems that neither the Remez exchange algorithm nor least squares method [9] can.

One of such filter design problem is the peak constrained least squares filter, which is originally proposed by Adam [6]. Adams commented that both the L_2 and the L_∞ filter designs are inherently inefficient. Therefore, he proposed to minimize the L_2 filter design error subjected to a constraint on the L_∞ filter design error. The design criteria is shown to be effective because Adams found that the peak errors of the L_2 optimal filter can be significantly reduced with only a slight increases in the squares error. Similarly, the squares error of the L_∞ optimal filter can be reduced with only a slight increases in the peak error of the L_∞ optimal filter.

This paper discusses an iterative algorithm for designing constrained eigenfilters, which can be used to design digital filter with various constraints, including peak constrained least square filter. We will then show that the new iterative method allows users to make tradeoff between transition band bandwidth and ripple levels. It is noticed that with the same filter order, the lower the ripple level in L_∞ optimal sense, the wider the transition band bandwidth. This can be easily derived from the design rules proposed by Herrmann *et al* [10]. However, conventional filter design algorithm can only achieve optimal ripple level with a given transition band. Such design approach is not applicable in those design problems that have transition band bandwidth being a design parameter under a desired ripple size. It is the aim of this paper to propose a new method to design filter with optimal tradeoff between ripple size and transition band bandwidth where the transition band bandedges are design parameters. The proposed design method can design multiband filter as well as incorporate time domain constraint simultaneously, whereas there is no efficient design method in literature that can achieve all these goals.

To efficiently evaluate the performance of the proposed design method, the design method proposed by Selesnick *et al* [4], which is commonly used for designing constrained least squares FIR filters without specified transition band bandedges, is implemented and compared with the proposed method. Selesnick's method in [4] formulates the peak constrained least squares design problem as a constrained quadratic programming problem, which is solved by Lagrange multipliers with a multiple exchange algorithm that iteratively test the validity of the constraints by examining Kuhn-Tucker conditions. Matrix inversion is needed to solve the Lagrange multipliers which, in turn, are used to solve the constrained optimization problem. However, matrix inversion is numerically inefficient and unstable. This is especially true when the matrix size is large. In addition, Selesnick's method [4] and [5] cannot be used to design filters with simultaneous time and frequency domain constraints, because it does not converge for most cases. In reality, the Lagrange multiplier formulation is not that robust in bandpass filter design. It has been shown in [5] that the algorithm in [4] fails to converge when the constraints set suffered from the cycling problem.

This work described in this paper has been supported by the Research Grants Council of Hong Kong, China (Project no. CERG HKUST6173/00E).

II. Iterative Reweighted Eigenfilter

The amplitude response of a causal N-th order type-I linear phase FIR filter is expressed by [3]

$$A(\omega) = \sum_{n=1}^{M} 2h(M-n)\cos(\omega n) + h(M), \tag{1}$$

where $h(n)$ is the impulse response of the filter and $M = \frac{N}{2}$. The above experssion can be simplified as

$$A(\omega) = \mathbf{a}^t \mathbf{c}(\omega), \tag{2}$$

where the superscript t denotes matrix transpose, and

$$\begin{aligned} \mathbf{a} &= [\ a(0) \quad a(1) \quad \cdots \quad a(M)\ ]^t \\ &= [\ h(M) \quad 2h(M-1) \quad \cdots \quad 2h(0)\ ]^t, \end{aligned} \tag{3}$$

$$\mathbf{c}(\omega) = [\ 1 \quad \cos(\omega) \quad \cdots \quad \cos(M\omega)\ ]^t. \tag{4}$$

Similar expression can be written for an odd order or odd symmetric filter [3]. The filter design problem is to find a set of impulse response, $h(n)$, such that the associate amplitude response $A(\omega)$ approximates a given frequency response $D(\omega)$, which is the same as finding the coefficient $a(n)$ in (3) to satisfy the given approximation problem. Various optimization criterion can be applied to the problem. Vaidyanthan and Nguyen [2] defined the weighted L_2 error as

$$\begin{aligned} \epsilon &= \int_{\omega\in\mathcal{R}} W(\omega)\left(\frac{D(\omega)}{D(\omega_0)}A(\omega_0) - A(\omega)\right)^2 d\omega, \\ &= \int_{\omega\in\mathcal{R}} W(\omega)\left(\frac{D(\omega)}{D(\omega_0)}\mathbf{a}^t\mathbf{c}(\omega_0) - \mathbf{a}^t\mathbf{c}(\omega)\right)^2 d\omega, \\ &= \mathbf{a}^t\mathbf{P}\mathbf{a}. \end{aligned} \tag{5}$$

where

$$\mathbf{P} = \int_{\omega\in\mathcal{R}} W(\omega)\left(\frac{D(\omega)}{D(\omega_0)}\mathbf{c}(\omega_0) - \mathbf{c}(\omega)\right)\left(\frac{D(\omega)}{D(\omega_0)}\mathbf{c}(\omega_0) - \mathbf{c}(\omega)\right)^t d\omega,$$

is a real, symmetric, and positive definite matrix, $W(\omega)$ is a non-negative weighting function that controls the relative importance at frequency domain, and $\mathcal{R}$ is the spectral domain under concern, with $\mathcal{R} \subset [0, \pi]$, that includes all the passband and stopband regions but exclude all the transition band regions.

Obviously, ϵ is minimized when $\mathbf{a} = 0$. To avoid this trivial solution, the obtained filter response at a particular frequency ω_ℓ

in the passband is constrained to equal to the design specification d_ℓ. As a result, the filter design problem is formulated as the following,

$$\min_{\mathbf{a}} \mathbf{a}^t \mathbf{P} \mathbf{a} \quad \text{subject to} \quad \mathbf{a}^t \mathbf{c}(\omega_\ell)\mathbf{c}^t(\omega_\ell)\mathbf{a} = d_\ell. \tag{6}$$

The dc response ($\omega_\ell = 0$) is commonly choosen for d_ℓ in lowpass filter design. The solution vector $\mathbf{a}$ is given by the eigenvector of the matrix $\mathbf{P}$ corresponding to the smallest eigenvalue. Such filter is known as the eigenfilter in [1], [2].

III. A Novel Iterative Eigenfilter Design Algorithm

The error of L_2 optimal filter is shown in Fig.1(a). Observed from Fig.1(a), a peak constrained filter with ripple size fit within upper and lower bound (δ_U and δ_L) can be obtained by the addition of a second filter (say $\Delta\mathbf{h}$ as shown in Fig.1(b)) into it, such that $\Delta\mathbf{h}$ decreases the peak error of the given L_2 optimal filter, i.e.,

$$\mathbf{h}_{PCLS} = \mathbf{h}_{L_2} + \Delta\mathbf{h}, \tag{7}$$

where $\mathbf{h}_{PCLS}$ and $\mathbf{h}_{L_2}$ are the peak constrained least squares optimal filter and L_2 optimal filter respectively. To satisfy (7), $\Delta\mathbf{h}$ should has spectral response of $\Delta_{v_k}(j)$ at ω_j for all ω_j that is not bounded by δ_U and δ_L and zero at all other spectral location, where $\Delta_{v_k}(j)$ and ω_j are observed from Fig.1(a) and is defined as,

$$\begin{aligned} v_k(i) &= \epsilon_k(\omega_i), \\ \Delta_{v_k}(i) &= \begin{cases} [\delta_U - \epsilon_k(\omega_i)] & \text{if } \epsilon_k(\omega_i) > \delta_U \text{ and } \omega_i \in \Phi_k \\ [\epsilon_k(\omega_i) - \delta_L] & \text{if } \epsilon_k(\omega_i) < \delta_L \text{ and } \omega_i \in \Phi_k \end{cases} \end{aligned} \tag{8}$$

This paper proposes to use eigenfilter method for designing $\Delta\mathbf{h}$, where the squared error ($\Delta\epsilon_k$) is formulated as,

$$\Delta\epsilon_k = \Delta\mathbf{a}_k{}^t \mathbf{Q}_k \Delta\mathbf{a}_k, \tag{9}$$

$$\begin{aligned} \mathbf{Q}_k = & \sum_{\substack{j \text{ s.t} \\ \omega_j \in \Phi_k}} \left[\frac{\Delta_{v_k}(j)}{D(\omega_0)}\mathbf{c}(\omega_0) - \mathbf{c}(\omega_j) \right] \left[\frac{\Delta_{v_k}(j)}{D(\omega_0)}\mathbf{c}(\omega_0) - \mathbf{c}(\omega_j) \right]^t \\ & + \sum_{\substack{j \text{ s.t} \\ \omega_j \notin \Phi_k}} \mathbf{c}(\omega_j)\mathbf{c}(\omega_j)^t, \end{aligned} \tag{10}$$

$\omega_0 = \arg\max_{\omega_j}\{\Delta_{r_k}(j)\}$, $D(\omega_0) = \max\{\Delta_{r_k}(j)\}$ and $\Delta\mathbf{a}_k$ is half of the filter coefficient of $\Delta\mathbf{h}_k$ defined similar to (3). Since it is almost impossible to design filter with several spectral peaks and large regions with zero response. Fig.1(b) shows the actual spectral response of $\Delta\mathbf{h}$ obtained by eigenfilter design method with the design specification that we've just discussed. Obviously, adding $\Delta\mathbf{h}$ in Fig.1(b) to $\mathbf{h}_{L_2}$ will not reduce all the peak errors in Fig.1(a) and result in a $\mathbf{h}_{PCLS}$ that satisfies the given bound. To remedy this problem, we formulated the design problem of $\Delta\mathbf{h}$ in a recursive way such that we'll design a new $\Delta\mathbf{h}$ to compensate for any discrepancy until $\mathbf{h}_{PCLS}$ in (7) satisfies the design specification. The details of the iterative design method is described in the following.

1) Design $\mathbf{h}_{L_2}$ with the design specifications N, ω_c, δ_U and δ_L using eigenfilter approach and form $\mathbf{a}_0$ using (3). Set iteration index $k = 1$.
2) Label extremal frequencies and calculate $\Delta_{v_k}(i)$ by (8).
3) Use $\mathbf{Q}_k$ defined in (10) to compute Δa_k as the eigenvector corresponds to the minimum eigenvalue of $\mathbf{Q}_k$.
4) Update the filter coefficients (i.e. $\mathbf{a}_k$) by $\mathbf{a}_k = \Delta\mathbf{a}_k + \mathbf{a}_{(k-1)}$.
5) Stop when all the peaks are bounded within the upper and lower bound (δ_U and δ_L), or when $k > 100$; otherwise set $k = k+1$ and go to Step 2.

IV. Interpretations and Extensions

A first glance at the proposed algorithm looks like that it is a variant of the Remez exchange algorithm 1 in [7]. Although the proposed algorithm is similar to the Remez exchange algorithm 1 in [7], there are fundamental differences between Remez exchange algorithm and the proposed algorithm. For simplicity, we compared the PM algorithm, an implementation of Remez exchange algorithm for filter design, with the proposed algorithm. Firstly, the number of reference set frequencies in PM is fixed and does not change throughout the algorithm, whereas the number of constraint set frequencies does change and is generally smaller than the number used in the PM algorithm.

Secondly, it is interesting to note that there is no minimum δ_U and δ_L below which the proposed algorithm fails to converges. If δ_U and δ_L are taken to be small, then the transition band between the passband and stopband simply becomes wider. On the other hand, the transition band bandwidth is fixed in PM algorithm. As a result, there exists an optimal δ_U and δ_L. Beyond that, the PM algorithm does not converge. Therefore, the proposed algorithm offers a design tradeoff between δ_U and δ_L with the transition band bandwidth. Similar design tradeoff is also available in [4].

Compared to the design algorithm of [4], the proposed design method does not involve Lagrange multipliers which prohibits the non-convergent Lagrange multiplier formulation when incorporating other design constraints simultaneously (such as simultaneous frequency domain and time domain constraints). Indeed, the proposed design method inherits all the advantages of the eigenfilter formulation which can easily incorporate various design constraints simultaneously. Furthermore, the Lagrange multiplier formulation is not that robust in bandpass filter design. It has been shown in [5] that the algorithm in [4] fails to converge when the constraints set suffered from the cycling problem. It is in contrast to the eigenfilter formulation of the proposed design method that results in a versatile digital filter design method for multiband filter with various time and frequency design specifications. Design examples in the later part of this paper will demonstrate this fact.

The proposed design method does not preclude the specification of a transition band bandedges. If both the transition band bandedges and the ripple sizes are specified simultaneously. It is possible that no solution exists because the transition band cannot be arbitrary sharp. Note that here a distinction is being made between the cut-off frequency ω_c and the bandedges frequencies (e.g. $\omega_p \le \omega_c \le \omega_s$ in lowpass filter design).

Finally, noticed that the proposed algorithm can be initialized with different filters which will affects the convergence of the algorithm. We further proposed to initialize the algorithm with the best unconstrained L_2 filters, such that there is no ambiguity about the initial filter used in the iterative procedure.

V. Design Examples

A. Example 1 - Peak Constrained Eigenfilter

A linear phase lowpass filter design specification with order $N = 60$, cut-off frequency $\omega_c = 0.3\pi$, $W(\omega) = 1\ \forall\omega$, and the upper and lower bound constraints on the amplitude response are equals to $\delta_U(\omega) = -\delta_L(\omega) = 0.04 = -27.9588$ for $\omega \in [0, \omega_c]$, while $\delta_U(\omega) = \delta_L(\omega) = 0.02 = -33.98dB$ for $\omega \in (\omega_c, \pi]$ was designed using the proposed method. The proposed algorithm converges at the 7-th iteration. The resulting magnitude response is shown in Fig.2(a). At the same graph, the filter obtained by using the Selesnick's method in [4] is also shown.

The magnitude response of the linear phase lowpass filter desinged by Selesnick's method [4] with the same specification is shown in the same figure. This implies that the filter obtained by the proposed method can achieve the same performance as that obtained by Selesnick's method described in [4].

B. Example 2 - Constrained Equiripple Eigenfilter

In this design example, the design specification is the same as that in Example 1, but with a different upper and lower bound constraints on the amplitude response, where $\delta_U(\omega) = -\delta_L(\omega) =$

$0.008 = -41.94dB\ \forall\omega$, was designed using the proposed method. The proposed method converges in 18 iterations. The magnitude response of the designed filter is shown in Fig.2(b). In the same figure, the magnitude response of the linear phase lowpass filter designed by Selesnick's method [4] with the same design specification is also shown.

Notice that the spectral shape of the filter obtained by Selesnick's method almost overlaps with that of the proposed method. In this example, it showed that when the upper and lower bound constraints (δ_U and δ_L) on the amplitude response is low enough, the proposed method will achieve equiripple filter. Furthermore, the designed filter is L_∞ optimal, as it has the same spectral shape as the L_∞ optimal design by Selesnick's method in [4].

C. Example 3 - Transition Band bandedges Tradeoff

In this design example, a lowpass filter with the same specification as that in Example 1, but with a different upper and lower bound constraints on the amplitude response of the filter, where $\delta_U(\omega) = -\delta_L(\omega) = 0.001 = -60dB\ \forall\omega$, was designed using the proposed method. The resulting magnitude response is shown in Fig.2(c), which also includes a plot of the magnitude response of the filter designed in Example 2.

In this example, it showed that when the upper and lower bound (δ_U and δ_L) are further decreased, the transition band bandedges will be adjusted automatically by the proposed algorithm. Digital filters with various amplitude constraints are designed. The resulting transition band bandwidth is reflected by the stopband bandedges, which is the first frequency where the amplitude response of the digital filter equals to $\delta_L(\omega)$, is plotted in Fig.2(d) versus the stopband ripple size $\delta_U(\omega)$ or $\delta_L(\omega)$ for $\omega \in (\omega_c, \pi]$.

The plot in Fig.2(d) shows that the transition band bandwidth can be significantly reduces with only a slightly increases in the ripple size and vice versa. The tradeoff is efficient as similar curve but with different tradeoff can be found in [6].

D. Example 4 - Multiband Filters

1) Multiple Passband Filter: A filter with order $N = 60$, that has multiple passband at $\omega \in [0, 0.2\pi) \cup (0.4\pi, 0.6\pi) \cup (0.8\pi, \pi]$, and other design specification as $W(\omega) = 1\ \forall\omega$, the upper and lower bound constraints on the amplitude response of the filter equals to $\delta_U(\omega) = -\delta_L(\omega) = 0.02 = -33.98dB\ \forall\omega$ was designed using the proposed method. Fig.2(e) and 2(f) showed the magnitude response obtained at the 0-th and the 8-th iteration respectively. This example showed that the proposed design method can be used to design PCLS multiple passband filters without any modification, whereas Selesnick's Lagrange multiplier method [4] cannot be used to achieved the same result without modification [5].

2) Multiband Filter with Arbitrary Spectral Response Constraints: In this example, the design of a linear phase filter with order $N = 80$, that has multiple passband at $\omega \in [0, 0.1\pi) \cup (0.5\pi, 0.66\pi)$ is considered. The spectral weighting function $W(\omega) = 1\ \forall\omega$ and the amplitude response of the filter is constrained to satisfy the follows upper and lower bound constraints

$$\delta_U(\omega) = -\delta_L(\omega) \quad = \quad \begin{cases} 0.01 & 0 \le \omega \le 0.1 \\ 0.12\omega^2 & 0.1 < \omega \le 0.32 \\ 0.005 & 0.32 < \omega \le 0.5 \\ 0.01 & 0.5 < \omega \le 0.66 \\ 0.03 & 0.66 < \omega \le 1 \end{cases} . \tag{11}$$

The proposed method converges at the 25-th iterations with the above design specification, and the magnitude response of the obtained filter is shown in Fig.2(g).

To test the robustness of the proposed design method, the upper and lower bound constraints on the amplitude response of the filter at the stopband region in $\omega \in (0.66\pi, 1]$ is reduced to $\delta_U = -\delta_L = 0.0024$. The proposed method converges at the 44-th iteration with the above design specification, and the magnitude response of the obtained filter is shown in Fig.2(h). Because the ripple size is small enough, the magnitude response exhibits equiripple properties. At the same time, it can be observed that the transition band located around $\omega = 0.66\pi$ has a large bandwidth in Fig.2(h) than that observed in Fig.2(g). This example showed the automatic transition band bandwidth adjustment capability of the proposed algorithm with respect to the change in ripple size. Furthermore, noticed that the transition band bandwidth in other regions of the filter in Fig.2(h) remains the same as that in Fig.2(g). This demonstrated the effect on transition band bandwidth tradeoff with the ripple size is localized.

Finally, we can concluded from this example that the proposed filter design method can be used to design digital filter with arbitrary spectral response, which cannot be easily achieved by other method presented in literature [4], [5].

E. Example 5 - Nyquist Filter

Nyquist filter was designed to demonstrate the versatility of the proposed design method, because it has strict time domain response requirement in the resulting digital filter. The Nyquist filter is a K-th band filter with filter impulse response, $h(n)$, being zero for $n = \pm K,\ \pm 2K,\ \pm 3K,\ \cdots$. Furthermore, the bandedge frequencies ω_p and ω_s of the Nyquist filter satisfies the condition $\omega_p + \omega_s = 2\pi/K$.

Noticed that if we initialize the proposed design method with an arbitrary K-th band filter that does not satisfy the Nyquist filter design specification as mentioned above. The convergence of the algorithm will be very slow. This is because the additive filter $\Delta\mathbf{h}$ is not efficient in correcting the violation in time domain constraints than that of frequency domain constraints. Therefore, to design Nyquist filter, we initialize our algorithm with an arbitrary designed Nyquist filter. In our simulation, we choose to use the Nyquist filter design method, which was proposed by Vaidyanthan *et al* in [1], to design the initial filter. The design method is essentially the same as the proposed method but with the specification of the transition band. Once the initial filter is designed, the proposed design method will optimize the filter towards other design constraints as detailed in the following examples. Noticed that although the initial filter is designed with a specified transition band bandedges, the transition band bandwidth will be optimized with the proposed design method to provide optimal tradeoff with the specified ripple size.

1) Peak Constrained Nyquist Filter: The design of a Nyquist filter with order $N = 38$ and transition band bandedges at $\omega_p = 0.15\pi$ and $\omega_s = 0.25\pi$, such that $\omega_p + \omega_s = 0.4\pi = 2\pi/5$ is considered. The upper and lower bound constraints on the amplitude response of the filter under concern is $\delta_U = -\delta_L = 0.005$. The Nyquist criteria constrained the filter to have $h(n) = 0$ for $n = \pm 5,\ \pm 10,\ \pm 15,\ \cdots$. The proposed design method converges at the 33-th iteration with the above design specification, and the time domain and magnitude responses of the obtained filter is shown in Fig.2(i) and 2(j) respectively.

2) Equiripple Nyquist Filter: The design of the Nyquist filter with the same design specification as that in the above example (example 5.1) is considered, where the constraints on the amplitude response is lowered to $\delta_U = -\delta_L = 0.002$. The proposed design method converges at the 99-th iteration with the above design specification, and the time domain and magnitude response of the obtained filter is shown in Fig.2(k) and 2(ℓ) respectively.

Observed from Fig.2(k), the magnitude response of the filter exhibits equiripple property. Furthermore, the transition band bandwidth of the filter in Fig.2(k) is wider that that in Fig.2(i). This is due to the smaller ripple size of the design specification for the filter in Fig.2(k) and hence results in transition band bandwidth tradeoff with the ripple size. In addition, the resulting transition band bandedges are at $\omega_p = 0.125\pi$ and $\omega_s = 0.275\pi$, which satisfies $\omega_p + \omega_s = 2\pi/5$.

Both design examples demonstrated the ease in incorporating various design constraints into the proposed design method. It further showed the effectiveness of the proposed design method to design optimal filters that satisfy various design constraints.

VI. Conclusions

We have proposed a constrained eigenfilter design algorithm, which can be used to design, peak constrained least squares FIR filters with multiple passbands and various time and frequency constraints that other algorithms in literature cannot achieve. The algorithm has exploited the design of FIR filter without explicit specification of the transition bands. We showed that the proposed design method allows the tradeoff between ripple sizes and transition band bandwidth. Although we have not proven the convergence of the proposed algorithm in the paper, the algorithm is found to converge efficiently for the large amount of design examples considered. Indeed the algorithm converges rapidly for all the design examples presented in this paper.

The algorithm has shown to be very flexible in term of the design specifications. Both frequency domain and time domain design constraints can be efficiently incorporated into the proposed algorithm without compromising the performance of the proposed algorithm. Design examples are presented to demonstrate this fact.

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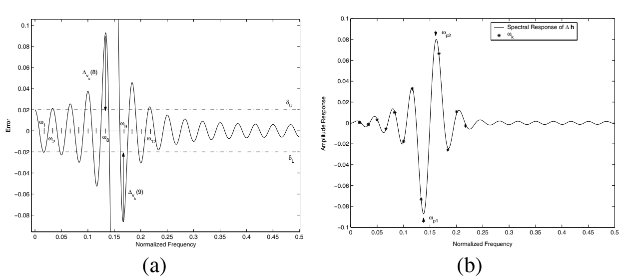

Fig. 1. (a) Actual error response of a lowpass filter at k-th iteration. (b) Spectral response of $\Delta\mathbf{h}$ at k-th iteration.

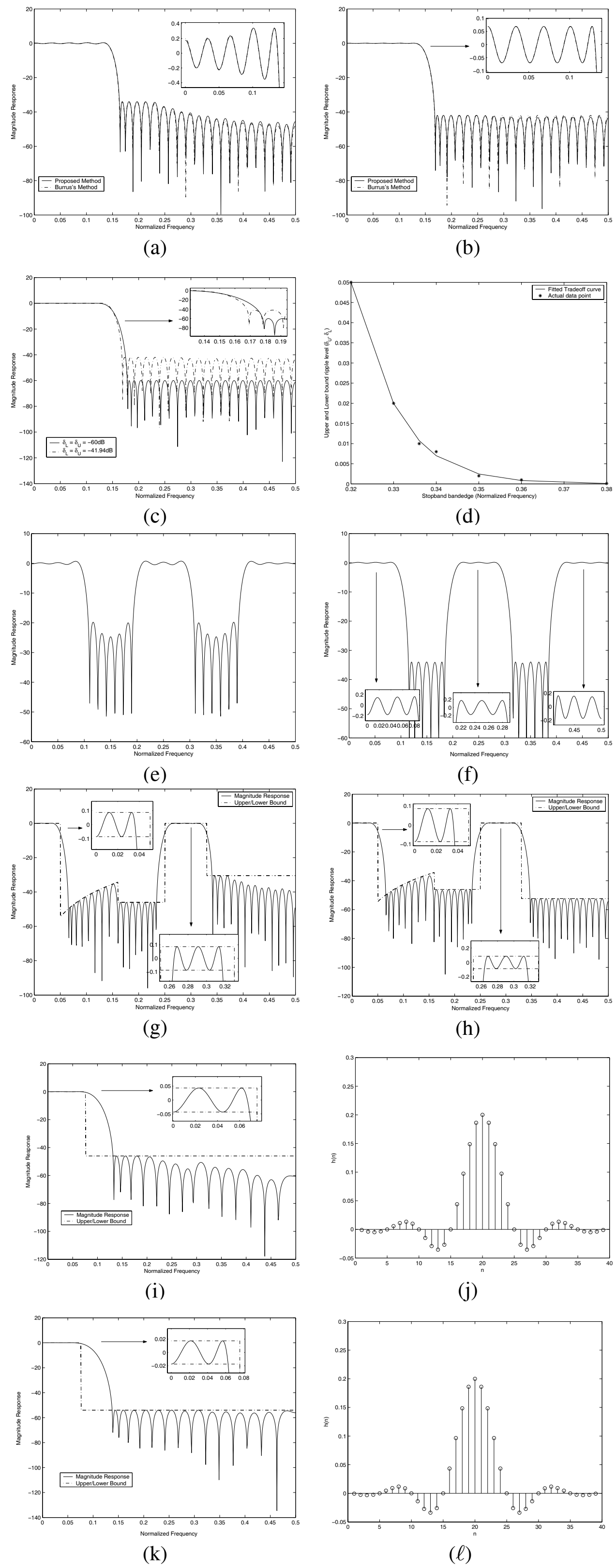

Fig. 2. Example 1 (a) Frequency response. Example 2 (b) Frequency response. Example 3 (c) Frequency response. (d) Bandedges tradeoff curve. Example 4.1 (e) Frequency response at 0-th iteration. (f) Frequency response at 8-th iteration Example 4.2 (g) Peak constrained stopband. (h) Equiripple stopband. Example 5.1 (i) Frequency response. (j) Impulse response. Example 5.2 (k) Frequency response. (ℓ) Impulse response.