Multiplier-Free Band-Selectable Digital Filters

Arnaud Santraine[1], Sébastien Leprince[1], and Fred Taylor[2]

[1] ESIEE, Paris, France; [2] University of Florida, USA

ABSTRACT

DSP applications, such as digital filters, are often multiply-accumulate (MAC) intensive. Furthermore, many applications require a deeply embedded low-power, high-speed solutions. Their physical implementation continues to be a challenge. A MAC-free bandpass and bandstop digital filter design paradigm is presented that can meet demanding performance and packaging requirements. Furthermore, the filter technology can be readily assimilated into modern ASIC and FPGA-enabled designs.

1. INTRODUCTION

High-end digital filter algorithms are notoriously MAC-intensive and therefore present a number of real-time implementation challenges. To overcome this barrier, wireless communication engineers have looked for relief in the form of the ubiquitous digital downconverter, or channelizer [1,2]. Channelizers are a core infrastructure technology that accepts signals at a high data rate and export a baseband signal at a decimated rate. The channelizer, shown in Figure 1, consists of:

- a front-end *data acquisition system* (analog anti-aliasing filter and ADC running at a rate f_s Sa/s),
- a *digital mixer* and attendant *direct digital synthesizer* (DDS) that heterodynes a desired subband down to DC,
- a *multiplier-less* N^{th}-order cascade integrator-comb (CIC) filter consisting of integrators and M-delay comb filter (see insert). The CIC filter accepts data at a rate f_s Hz and exports data at a decimated R rate $f_{out} = f_s/R$. The magnitude frequency response of a CIC filter has a $\sin(x)/x$ envelope, and
- a low-order programmable FIR, running at the decimated rate, that shapes CIC output spectrum.

Currently commercial channelizers are manufactured and distributed by Intel, National, Texas Instruments, Intersil, and others. They are intrinsically lowpass, multi-rate, digital filters that extract information from a specific subband by heterodyning (mixers and DDS) a signal down to baseband. Heterodyning, unfortunately, adds to system cost, complexity, and power dissipation. In addition, commercial data rates are typically bounded below 100-300MHz. As a result, channelizer-enabled solutions must often include multiple IF sections in order to translate high-frequency signals down to the channelizer data rate. Each IF stage can add an additional \$10-\$100 to the final solution [3]. It is therefore highly desired to develop new embeddable signal processing agent that can implement band-selective filters at high-speeds and low-complexities.

2. CHANNELIZERS

The N^{th}-order CIC filter, shown in Figure 1, is defined by a transfer function given by:

$$H_{CIC}(z) = (1-z^{-S})^N/(1-z^{-1})^N \qquad 1.$$

where S=RM, R is the decimation index, and M is comb filter delay. The transfer function is that of N cascaded S-sample moving average FIRs. The N^{th}-order CIC filter possesses N-poles at DC (i.e., z=1.0), and S-zeros of multiplicity N located on the unit circle at z=$e^{j2\pi k/S}$, $k\in[0,S)$. The N zeros located at z=1 cancel an equal number of zeros at the same location, resulting in a filter having a high DC gain of a lowpass filter. Pole-zero cancellation is insured because all the filter coefficients are exactly unity. Furthermore, since all CIC filter coefficients are ternary valued $\{0,\pm1\}$, no general-purpose MAC unit is required to implementing a filter. Because of this, CIC filter can operate at high data rates with limited complexity.

The dynamic range requirements of an N^{th} order CIC filter are defined in terms of a worst case gain G=$(S)^N$, which occurs at DC. In practice, the run-time dynamic range requirements of a CIC system can easily exceed 64-bits in practice. Since a CIC is multiplier-free, the tangible effects of a high internal gain are wide internal data paths and extended precision 2's complement adders.

3. CIC-ENABLED BANDPASS FILTER

Unfortunately, today's programmable bandpass, or bandstop filter design methodologies invariably lead MAC-bound solutions, limiting their effective bandwidth and adding to their complexity. In concept, a bandpass filter having a CIC-structure could be realized by moving

0-7803-8484-9/04/\$20.00 ©2004 IEEE

the pole-zero cancellation points to a location on the periphery of the unit circle ($z=e^{j\theta}$) other than DC ($z=e^{j0}=1.0$). Unfortunately, this would require a filter of the form:

$$H(z)=(1-z^{-RM})^N/(z^2+\alpha z+\beta)^N. \quad 2.$$

This resulting filter has non-unity coefficients and therefore assumes an additional MAC penalty that can deny its use in mobile (low-power) applications. If, however, bandpass or bandstop filters could be defined in the context the MAC-free CIC filter, then a potentially viable embeddable solution may result.

Historically, pole locations are defined in terms of the roots of an M^{th}-order polynomial $\Psi_i(z)$ having real valued coefficients a_i where:

$$\Psi_i(z)=a_0+a_1z^{-1}+\ldots+a_Mz^{-M} \quad 3.$$

For example, to place poles at $z=e^{j2\pi/8}$ would require a 2[nd] order polynomial of the form $P(z) = 1+1.848\, z^{-1}+z^{-2}$, where the coefficients are assumed to be known to infinite precision. There are two problems with this paradigm. First it is MAC intensive. Secondly, finite wordlength effects will invariably result in incomplete pole-zero cancellation, introducing both performance and stability problems. Fortunately, a solution does exist and it is based upon the polynomial manipulation schemes used in algebraic coding theory [4]. The data found in Table 1 examines the polynomial studied in Equation 3 for ternary valued coefficients (i.e., $a_i \in \{0, \pm 1\}$), insuring a MAC-free filter design. Continuing, let $\Psi_j(z)$ be generated as:

$$\Psi_j(z)=(1-z^{-i})/\Pi\,\Psi_i(z);\ \forall i<j \quad 4.$$

where i is relatively prime to j and $\Psi_j(z)$) has poles residing on the periphery of the unit circle in the z-plane (i.e., $z=e^{j2k\pi/S}$). Some of these polynomials are tabled in [4] and again Table 1, along with their critical frequencies. For example, the roots of Ψ_3 are located at normalized frequencies $2\pi/3$ and $4\pi/3$, and correspond to real frequencies $f=\pm f_s/3$. The number of roots located on the periphery of the unit circle is given by the Euler "phi" function $\phi(i)$. Replacing the integrators, shown in Figure 1 (i.e., $H_i(z)=1/(1-z^{-1})$), with recursive filters $H_i(z)=1/\Psi_i$, defines a bandpass filter whose poles are located on the unit circle with a transfer function given by:

$$H_i(z)=(1-z^{-S})^N/(\Psi_j)^N \quad 5.$$

where filter $\Psi_i(z)$ has only ternary valued coefficients, and S is an integer multiple of i. Since filter coefficients in both the feedforward and feedback paths are ternary valued, exact pole-zero cancellation can be guaranteed at the filter's critical frequency.

For illustrative purposes, consider the filters found in Table 1 that are also divisible by 12 (shown as shaded entries), with magnitude frequency responses shown in Figure 2. The filter $H_i(z)$ has pole-zero cancellations of multiplicity N, at $z=e^{j\theta}$, for some θ. Each realized filter has a $|\sin(x)/x|^N$ magnitude frequency responses centered about the critical frequencies given in Table 1. The nulls are defined by the zeros of the sin(x)/x and are located on $\theta=2k\pi/S$ radian centers, except where poles reside.

There are several design parameters that can be used to adjust the sensitivity and frequency selectivity of the CIC-enabled bandpass filters. The filter's bandwidth and center frequencies are established by $S=MR$ which defines the number of unit circle zeros of multiplicity N (see Figure 3). The depth of the stopband, and steepness of the filter skirt, are primarily influenced by the order parameter N. Increasing any of these parameters will, however, increase the internal worst-case gain given by $G=(S)^N$. For example, for $S=MR=48$ and $N=3$, worst case internal gain is $G=(48)^3 \leq 2^{17}$ which means that adders must have 17-bits of additional "headroom" (extended precision). The maximum side lobe gain is given by $H(e^{j(\varphi+\theta)})=2^N/|(1-e^{j(3\pi/S)})|^N$ and the ratio of the maximum filter gain to maximum side lobe gain is given by:

$$\Delta G=S^N|(1-e^{j(3\pi/S)})|^N/2^N \quad 6.$$

For $S>>1$, which is the typical case, Equation 6 can be approximated to be:

$$\Delta G=(3\pi)^N/2^N=(1.5\pi)^N \quad 7.$$

Refer to Figure 2 which reports a filter design for $N=3$. Based on Equation 7, the differential gain between the main lobe and maximum side lobe is approximately 40 dB which is shown to be the case.

3. MODES OF OPERATION

For wireless applications, the bandpass channelizer can replace existing digital downconverters as well as eliminate the need for mixers and DDS systems. The bandpass CIC-enabled filter can operate in a *critical*, *over*, or *undersampled* mode. The first two regimes are easily realized by direct implementation of Equation 5. Oversampling requires the use of an IF ADCs (e.g., Analog Devices AD9870). IF ADCs have analog sample and hold circuits that operate at speeds much higher than the ADC digital sample rate [5]. For example, a 2.4 GHz IEEE 802.11a OFDM system has an information bandwidth of 20 MHz, consisting of 64 0.3125 MHz subbands, of which 52 only are used. The "rule-of-thumb" undersampled rate would be 50 Sa/s, sending the 2.4 GHz carrier down to a baseband frequency (2.4 GHz) mod(50 MHz)=0 Hz. Information, coded in a subband near the center line located at 2.4 GHz is thereby translated down to baseband for processing by a back-end signal processor. The analogy sample and hold circuits of the ADC, however, would need to operate at a much higher RF or IF frequency.

To illustrate the mechanics of an undersampled CIC bandpass filter, consider then the data shown in Figure 4 which is based on the dual passband filter displayed in Figure 3 clocked at 12 MHz. Assuming that the higher subband is of interest, decimating the filter output by R=12, will result in undersampling with the subbands centered about 1 MHz and 5 MHz aliased down to DC. A low-order digital filter operating at the ADC rate is used to attenuate energy originally residing in the 1 MHz subband. The CIC-bandpass filter output is then the subband originally located about 5 MHz. This process can be generalized.

4. BANDSTOP FILTERING

The developed bandpass CIC filter reported in this paper has transfer function given by $H_i(z)=(1-z^{-S})^N/(\Psi_i)^N$ (Equation 5). The filter is also linear phase having a group delay $\tau_g = N(S-1)/2$. The *complement* version of $H_i(z)$, denoted $G_i(z)$, is given by [6].

$$H_i(z)+G_i(z)=1 \qquad 7.$$

In particular, if $H_i(z)$ is a highly selective bandpass filter, then $G_i(z)$ is a highly selective "notch" filter. A bandstop version of the CIC-bandpass filter shown in Figure 3 is reported in Figure 5.

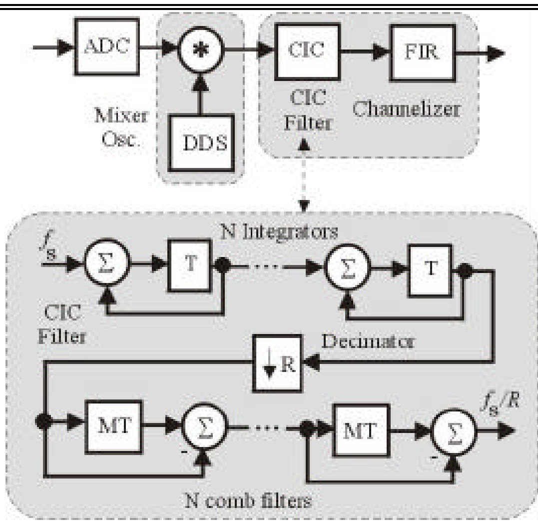


Figure 1: Conventional signal processing heterodyne solution involving a channelizer (a.k.a., digital down converter). The N^{th} order channelizer consists of N integrators and N delay M comb filter, separated by a decimate-by-R circuit.

2. SUMMARY

A design methodology is presented for programmable high-speed low-complexity bandpass and bandstop digital filters. The filters gain their performance advantage by eliminating the need for multipliers. Since the realized filters are multiplier-free, they are excellent candidates for embedding into ASIC or FPGA -centric designs.

5. REFERENCES

1. Hogenauer, E. R., "An Economical Class of Digital Filter for Decimation and Interpolation, IEEE Trans. of ASSP, April, 1981.
2. Chester, D., "Digital IF Filter Technology for 3G Systems," IEEE Communications Magazine, Feb. 1999.
3. Buchanan, D., "Performance of an IF Sampling ADC in Receiver Applications," International IC-China Conf., 2002.
4. Garcia, A., Meyer-Base, U., and Taylor, F., "Pipelined Hogeneauer CIC Filters Using Field Programmable Logic and Residue Number System," IEEE ICASSP Conf., Seattle, May, 1999.
5. Kester, W., "Undersampling Applications," Analog Devices, June, 2002.

Table 1: Ternary-valued polynomials, roots, and critical frequencies (θ_1 positive baseband frequency, θ_2 positive baseband frequency)

Ψ_i	$a=[a_0, a_1, a_2, \ldots a_k]$	θ_1	θ_2	ϕ
Ψ_1	[1,-1]	0	2π	1
Ψ_2	[1,1]	$2\pi/2$	$2\pi/2$	1
Ψ_3	[1,1,1]	$2\pi/3$	$4\pi/3$	2
Ψ_4	[1,0,1]	$2\pi/4$	$6\pi/4$	2
Ψ_5	[1,1,1,1,1]	$2\pi/5$ $4\pi/5$	$8\pi/5$ $6\pi/5$	4
Ψ_6	[1,-1,1]	$2\pi/6$	$10\pi/6$	2
Ψ_7	[1,1,1,1,1,1,1]	$2\pi/7$ $4\pi/7$ $6\pi/7$	$12\pi/7$ $10\pi/7$ $8\pi/7$	6
Ψ_8	[1,0,0,0,1]	$2\pi/8$ $5\pi/8$	$14\pi/8$ $10\pi/8$	4
Ψ_9	[1,0,0,1,0,0,1]	$2\pi/9$ $4\pi/9$ $8\pi/9$	$16\pi/9$ $14\pi/9$ $10\pi/9$	6
Ψ_{10}	[1,-1,1,-1,1]	$2\pi/10$ $6\pi/10$	$18\pi/10$ $14\pi/10$	4
Ψ_{11}	[1,1,1,1,1,1,1,1,1,1,1]	$2\pi/11$ $4\pi/11$ $6\pi/11$ $8\pi/11$ $10\pi/11$	$20\pi/11$ $18\pi/11$ $16\pi/11$ $14\pi/11$ $12\pi/11$	10
Ψ_{12}	[1,0,-1,0,1]	$2\pi/12$ $10\pi/12$	$22\pi/12$ $14\pi/12$	4

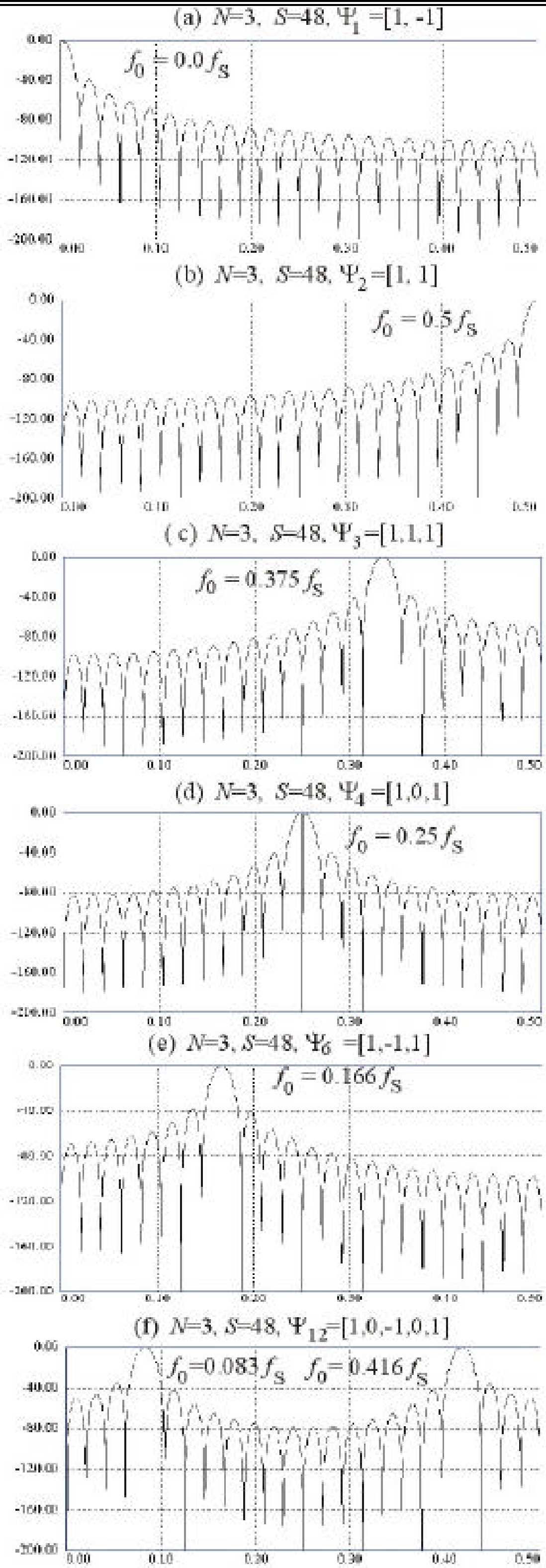
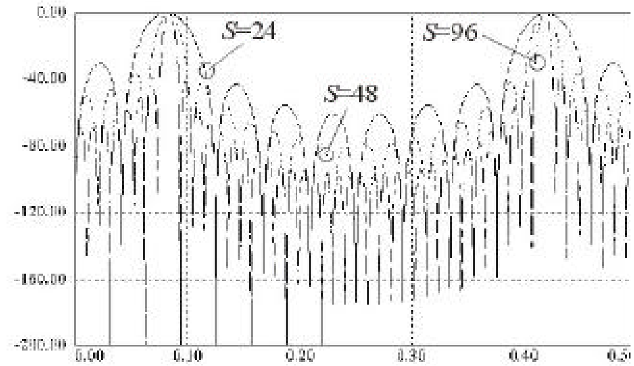


Figure 2: Examples of selected CIC-enabled bandpass filters having N=3 and S=48.



6.

Figure 3: Frequency selectivity for CIC-enabled filter $H_{12}(z)$ using $\boldsymbol{\Psi}_{12}$=[1,0,-1,0,1], N=3, and S=24, 48, and 96.

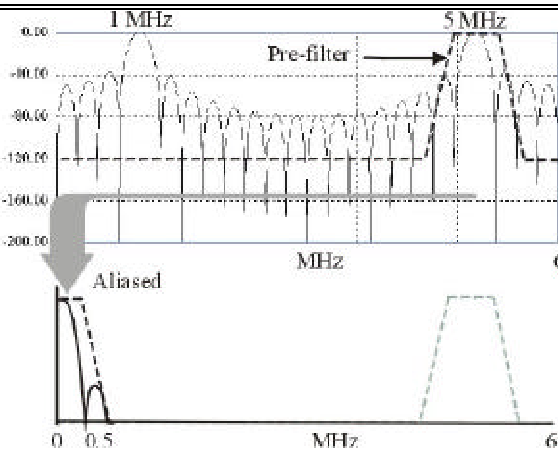


Figure 4: Example of pre-filtering used in undersampling application.

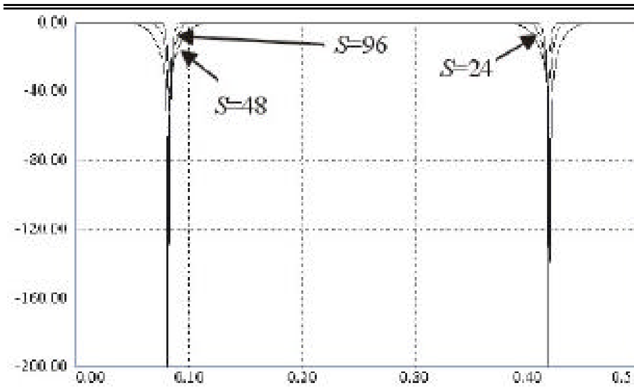


Figure 5: Conversion of the bandpass filter, shown in Figure 3, to a stopband filter.