SIGNAL ACQUISITION FOR COOPERATIVE TRANSMISSIONS IN MULTI-HOP AD-HOC NETWORKS

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ABSTRACT

Cooperative transmission schemes in multi-hop networks have the advantage of energy efficiency and increased network coverage. Realizing these advantages, however, require a compatible physical layer. In this paper we consider the detection of a signal transmitted by multiple cooperative nodes. Exploiting the structure of the network, we formulate the problem, and propose a generalized likelihood ratio detector. We compare the performance of the proposed detector with two others: estimator-correlator and a genie aided detector (provides a performance benchmark). The genie aided detector assumes the knowledge of certain network parameters which may be unknown during reception. Simulations show that the proposed detector performs reasonably close to the genie aided one, while being considerably better than the estimator-correlator.

1. INTRODUCTION

We consider a network with *cooperative broadcasting*. That is, an initiator (source) starts a transmission session, and the nodes who hear the packet decode and retransmit. The retransmissions are done *simultaneously*, even though they may not be symbol synchronized. The retransmissions continue until every node who can hear the others retransmits at least once.

This cooperation mechanism, and others, have received considerable attention recently [1]-[4]. Node cooperation has been shown to be beneficial for reaching far distances (an application potentially important in sensor networks) [2], improving the energy efficiency [3], and increasing network coverage [4].

Advantages of cooperation, however, can not be realized, unless the network is supported by feasible and reliable physical layer functions. The nodes who would like to help out other nodes' transmissions always face the problem of whether they should contribute, and if so, at what time. This decision is critical, because if a node prematurely decides to relay a poorly received signal, it may retransmit erroneously decoded symbols. At the other extreme, if the nodes behave too conservatively, then fewer nodes transmit simultaneously and this decreases the average signal-to-noise ratio (SNR) of the combined signal. Also, the signal propagates using smaller hops, which results in increased delay.

In this work, we address the *signal acquisition* problem in a cooperative network. We use the term "acquisition" to describe the detection of existence and timing of a known signal in noise. The detection has a direct effect on relaying decisions, since the nodes relay only if they detect a signal. The problem of signal acquisition has been extensively studied for point-to-point links [5]. What makes the cooperative transmission different is that simultaneous (but, asynchronous) transmission of the same message creates a special inter-symbol interference (ISI) in the received signal. In this work, we design a detector that particularly exploits the structure of this special form of *multipath*, resulting from the asynchronous retransmission of multiple relays.

In the following sections, first we explain our broadcasting technique and give the expressions for the received and transmitted signals, and then we derive an equivalent discrete-time channel model. The channel statistics are found, and the detection problem is formulated. In Section 5, we introduce the generalized maximum likelihood ratio detector. In Section 6, we discuss two other detectors and provide simulation results.

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2. TRANSMISSION POLICY

Transmission session is initiated by a single source. Each packet contains three parts: (i) Guard Period (ii) Training Period (iii) Data. The training portion of the packet is used for signal acquisition and channel estimation. The nodes that can detect the signal from the source are called first level nodes. After detecting the presence of the signal, first level nodes decode and re-transmit the same packet. That is, the nodes in the n^{th} level can not detect the presence of the signal until after the nodes in the $(n-1)^{th}$ level transmit. The guard period is included at the beginning of each packet in order to prevent any interference between the nodes in the same level.

The received signal at a node can be considered as multiple replicas of the same signal, so that the channel can be modelled as a multi-path channel. We assume the channel is time varying from packet to packet, but constant during a single packet transmission. The time varying nature of the channel is due to many reasons, one of which is the frequency differences between the oscillators of each node. This, in first approximation introduces a time-varying phase shift in the received signal at each node.

In the rest of the paper, we'll deal with one-shot transmissions, and we'll assume that the packet contains only training since in this paper we deal with signal acquisition only. We will also assume that during the training sequence, the carrier offset effect is negligible and can be modelled simply as a phase offset. The basic model developed in this work will be used in future papers to handle the data detection.

3. TRANSMITTED/RECEIVED SIGNAL MODELS

In our derivations, we analyze a network that contains a single source and nodes which are distributed randomly in an area. Let $c[n]$ be the n^{th} sample in the packet of length M. Let $p(t)$ be the pulse shape and θ_{ik} be the phase shift between i^{th} and k^{th} node due to frequency drifts. Let τ_k be the time instant that k^{th} node starts transmission and d_{ik}, α_{ik} are the distance and fading coefficient between i^{th} and k^{th} nodes, respectively. Also, the speed of light is denoted by v_c, and path-loss exponent is denoted by β. T is the symbol interval.

The transmitted signal at the i^{th} node is,

$$s^{(i)}(t) = \sum_{k=1}^{M} c[k]p(t - \tau_i - kT),$$

where we'll refer to τ_i as the relaying time of i^{th} node. The source node is indexed by zero. After bandpass filtering, the received signal at the i^{th} node can be written as

$$r^{(i)}(t) = \sum_{k \in S_i} \frac{\alpha_{ik} e^{j\theta_{ik}}}{d_{ik}^{\beta}} \sum_{n=0}^{M-1} c[n]p(t-\tau_k-\frac{d_{ik}}{v_c}-nT)+w^{(i)}(t),$$

where $w^{(i)}(t)$ is the additive white Gaussian noise (AWGN), and S_i is the index set of nodes whose transmissions the i^{th} node can hear. After sampling at the Nyquist rate $r^{(i)}[n] = r^{(i)}(nT)$,

$$r^{(i)}[n] = \sum_{m=0}^{M-1} c[m] \sum_{k \in S_i} \frac{1}{d_{ik}^{\beta}} \alpha_{ik} e^{j\theta_{ik}} p_k^{(i)}[n-m] + w^{(i)}[n] \quad (1)$$

where

$$p_k^{(i)}[n-m] := p(nT - \tau_k - \frac{d_{ik}}{v_c} - mT) \quad (2)$$

Let's define channel coefficients as,

$$g^{(i)}[n] = \sum_{k \in S_i} \frac{\alpha_{ik} e^{j\theta_{ik}}}{d_{ik}^{\beta}} p(nT - \tau_k - \frac{d_{ik}}{v_c}). \quad (3)$$

Then the received signal is,

$$r^{(i)}[n] = g^{(i)}[n] * c[n] + w^{(i)}[n].$$

From now on, we'll include $e^{j\theta_{ik}}$ inside fading coefficients: $\tilde{\alpha}_{ik} = \alpha_{ik} e^{j\theta_{ik}}$.

4. CHANNEL STATISTICS

Since we consider a network of nodes that are randomly distributed, τ_k, and d_{ik} are considered as random parameters that depend on the network topology. We assume that $\tilde{\alpha}_{ik}$'s are independent of each other and independent of τ_k, d_{ik}. Also we assume that $\tilde{\alpha}_{ik}$'s are complex circular Gaussian zero mean and unit variance.

Using Eqn. 3, the mean of the n^{th} channel coefficient is,

$$E\{g^{(i)}[n]\} = \sum_{k \in S_i} E\{\tilde{\alpha}_{ik}\} E\{\frac{p(nT - \tau_k - \frac{d_{ik}}{v_c})}{d_{ik}^{\beta}}\} = 0$$

Using Eqn. 2, the covariance between l_1^{th} and l_2^{th} channel coefficients can be found as,

$$\begin{aligned} R_{gg}^{(i)}[l_1, l_2] &= E\{g_i[l_1] g_i^*[l_2]\} \\ &= \sum_{k1 \in S_i} \sum_{k2 \in S_i} E\{\frac{\tilde{\alpha}_{ik_1} \tilde{\alpha}_{ik_2}^*}{d_{ik_1}^{\beta} d_{ik_2}^{\beta}} p_{k_1}^{(i)}[l_1] p_{k_2}^{(i)}[l_2]\} \end{aligned}$$

Since $\tilde{\alpha}_{ik}$'s are independent, zero mean and unit variance,

$$R_{gg}^{(i)}[l_1, l_2] = \sum_{k \in S_i} E\{\frac{1}{d_{ik}^{2\beta}} p_k^{(i)}[l_1] p_k^{(i)}[l_2]\}$$

By simple manipulation, we can obtain,

$$p_k^{(i)}[l_1] p_k^{(i)}[l_2] = \int \delta(t - \tau_k - \frac{d_{ik}}{c}) p(l_1 T - t) p(l_2 T - t) dt.$$

This trick lets us combine the random parameters all together, i.e.

$$\varphi^{(i)}(t) := E\{\frac{1}{d_{ik}^{2\beta}} \delta(t - \tau_k - \frac{d_{ik}}{v_c})\}. \quad (4)$$

Then the covariance is,

$$R_{gg}^{(i)}[l_1, l_2] = \sum_{k \in S_i} \int \varphi^{(i)}(t) p(l_1 T - t) p(l_2 T - t) dt. \quad (5)$$

When we consider the detection of the signal in the form Eqn. 1, we face a *chicken-egg dilemma*. The received signal at the i^{th} node depends on the relaying times, τ_k, of nodes in S_i, and the relaying times of these nodes depend on the detection method we choose. Hence, in principle we can not provide a model for the received signal unless we specify the detection policy but, at the same we can not derive an optimal detection policy unless we have a model for the receiver signal. To overcome this problem, we broke the coupling between the relaying times and detection method by assuming an arbitrary but reasonable statistical model for the relaying times. We assume that the processing times are large compared to the propagation times.

Let q_i be the level that i^{th} node is located. In the following, we assume the relaying times (τ_k) are uniform with mean μ_{q_k}, that depends on the level k^{th} node is located, and variance σ^2, a constant. This assumption is valid when the propagation delays are insignificant compared to the processing time. Let $\triangle = \sqrt{3}\sigma$. Note that we assume $\mu_{q_k} - \triangle$ is large enough that the causality is maintained in the system, that's a node can't transmit before receiving the packet. The probability density function (PDF) for τ_k is,

$$f(\tau_k; \mu_{q_k}) = \begin{cases} \frac{1}{2\triangle} & \mu_{q_k} - \triangle \le \tau_k \le \mu_{q_k} + \triangle \\ 0 & otherwise \end{cases}$$

We assume that the received signal at the i^{th} node is due to the transmitted signal from the nodes that are located in the q_{i-1}^{th} level. Then given that the i^{th} node is located in the q_i^{th} level, τ_k, and d_{ik} are independent. Assuming $\tau_k >> \frac{d_{ik}}{v_c}$, Eqn. 4 reduces to,

$$\begin{aligned} \varphi(t)^{(i)} &= E\{\frac{1}{d_{ik}^{2\beta}}\} E\{\delta(t - \tau_k)\} \\ &= \begin{cases} \frac{1}{2\triangle} E\{\frac{1}{d_{ik}^{2\beta}}\} & |t - \mu_{q_i}| \le \triangle \\ 0 & otherwise \end{cases} \end{aligned}$$

And Eqn. 5 reduces to

$$R_{gg}^{(i)}[l_1,l_2] = \frac{1}{2\triangle}\sum_{k\in S_i} E\{\frac{1}{d_{ik}^{2\beta}}\}\Psi^{(i)}(l_1,l_2;\mu_{q_i},\triangle)$$

where

$$\Psi^{(i)}(l_1,l_2;\mu_{q_i},\triangle) = \int_{\mu_{q_i}-\triangle}^{\mu_{q_i}+\triangle} p(l_1T-t)p(l_2T-t)dt$$
$$= \int_{-\triangle}^{\triangle} p(l_1T-t-\mu_{q_i})p(l_2T-t-\mu_{q_i})dt.$$

Assume that μ_{q_i} is a multiple of T (sampling interval). Define

$$\overline{\Psi}(l_1,l_2;\triangle) = \int_{-\triangle}^{\triangle} p(l_1T-t)p(l_2T-t)dt. \tag{6}$$

Then $\Psi^{(i)}(l_1,l_2;\mu_{q_i},\triangle) = \overline{\Psi}(l_1-\frac{\mu_{q_i}}{T}, l_2-\frac{\mu_{q_i}}{T};\triangle)$ and,

$$R_{gg}^{(i)}[l_1,l_2] = \sum_{k\in S_i}\frac{1}{2\triangle}E\{\frac{1}{d_{ik}^{2\beta}}\}\overline{\Psi}(l_1-\frac{\mu_{q_i}}{T}, l_2-\frac{\mu_{q_i}}{T};\triangle).$$

Also define,

$$h^{(i)}[l] = g^{(i)}[l+D_i],$$

where $D_i := \frac{\mu_{q_i}}{T}$ is the delay parameter for the i^{th} node. Then,

$$R_{hh}^{(i)}[l_1,l_2] = \sum_{k\in S_i}\frac{1}{2\triangle}E\{\frac{1}{d_{ik}^{2\beta}}\}\overline{\Psi}(l_1,l_2;\triangle) = \kappa_i\overline{\Psi}(l_1,l_2;\triangle)$$

where

$$\kappa_i = \sum_{k\in S_i}\frac{1}{2\triangle}E\{\frac{1}{d_{ik}^{2\beta}}\}. \tag{7}$$

If we assume nodes' locations are identically distributed then $\kappa_i = \frac{|S_i|}{2\triangle}E\{\frac{1}{d_{ik}^{2\beta}}\}$, where $|S_i|$ is the cardinality of set S_i.

Note that $\overline{\Psi}(l_1,l_2;\triangle)$ does not depend on node index i since $\triangle$ is assumed to be constant throughout the network. As we will see next, this remarkable property is helpful in setting up a simplified structure for our detector. In the following sections, we'll assume channel coefficients of the i^{th} node are circular complex Gaussian with zero mean, and covariance $\Sigma_{h^{(i)}} = \kappa_i\Sigma$. Indeed, when we consider large, dense networks, this is a valid assumption due to central limit theorem, since the number of replicas of the signal that a node receives from its neighbors are large. Refer to Fig. 1 for an example.

5. PROPOSED SIGNAL ACQUISITION METHOD

Note that in this section we drop superscripts that show dependence on the node. And $\mathcal{CN}(0,\Sigma)$ is used to denote circular complex Gaussian random vectors, with zero mean and covariance Σ. We assume that the channel length is known at the node which can be calculated by using Eqn. 6.

The detection method operates with a sliding window of length $N := M+L-1$ (M is the length of training sequence, L is the channel length) on the received signal, i.e. the detection is performed on a block of N samples at a time. The received signal is

$$r[n] = g[n]*c[n]+w[n],$$

where $g[n]=h[n-D]$, and D is the time-delay of the sequence.

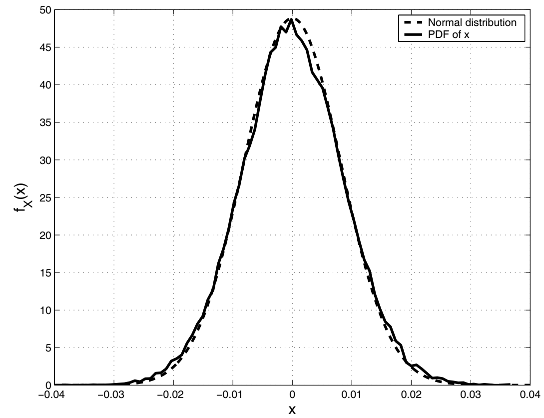

Fig. 1. Comparison of PDF of x ($\Re\{h[0]\}$) with the normal density having the same mean and variance as x. Network consists of 10 nodes uniformly distributed in a circle of radius 10m with the distance of destination node to the source node being 20m.

Let $\mathbf{r}_k$ be the received signal at the k^{th} window, and $\mathbf{h} = [h(0)\dots h(L-1)]^T$. Then we express $\mathbf{r}_k$ as,

$$\mathbf{r}_k = \mathbf{C}(D_k)\mathbf{h}+\mathbf{w}$$

where $\mathbf{C}(D_k)$ is a $N\times L$ matrix such that first D_k rows are zero. Let C_{ij} be the $(i,j)^{th}$ of $\mathbf{C}(D_k)$, then

$$C_{ij}(D_k) = \begin{cases} c(i-j-D_k) & 0\le i-j-D_k\le M-1 \\ 0 & otherwise \end{cases}$$

Notice that since we work with a window of data, D_k ranges from $1-N$ to N. $D_k=0$ corresponds to the case where the received signal in the window contains the entire training sequence, and $D_k=N$ corresponds to the case where the received signal is noise only. Negative D_k values corresponds to the case where part of the packet lies in the previous window. Assume that signal is detected at the K^{th} window, then

$$D = K-1+D_K.$$

Let's define our hypotheses as follows,

$$\begin{aligned} H_0 &: \mathbf{r}_k = \mathbf{w} \\ H_1 &: \mathbf{r}_k = \mathbf{C}(D_k)\mathbf{h}+\mathbf{w},\ \mathbf{D}_k\in\{1-N,\dots,N-1\} \end{aligned}$$

where $\mathbf{w}\sim\mathcal{CN}(0,\sigma_0^2\mathbf{I})$ and $\mathbf{h}\sim\mathcal{CN}(0,\kappa\Sigma)$. The density function of the received signal under hypothesis H_1 given $D_k=\ell$ (for $\ell=0\dots N-1$) is,

$$p_1(\mathbf{r}_k;D_k=\ell,\kappa) = \frac{1}{\pi^N|\Sigma_1(\ell)|}e^{-\mathbf{r}_k^H\Sigma_1^{-1}(\ell)\mathbf{r}_k}$$

where $\Sigma_1(\ell) = \kappa\mathbf{C}(\ell)\Sigma\mathbf{C}(\ell)^H+\sigma_0^2\mathbf{I}$. And the density of $\mathbf{r}_k$ under H_0 is,

$$p_0(\mathbf{r}_k) = \frac{1}{\pi^N\sigma_0^{2N}}e^{\frac{-\mathbf{r}_k^H\mathbf{r}_k}{\sigma_0^2}}.$$

We use generalized likelihood ratio test (GLRT) [6] as the detection method, and the likelihood ratio $L(\mathbf{r}_k)$ is,

$$L(\mathbf{r}_k) = \max_{D_k,\kappa>0} L(\mathbf{r}_k,D_k,\kappa) = \max_{D_k,\kappa>0}\frac{p_1(\mathbf{r}_k;D_k,\kappa)}{p_0(\mathbf{r}_k)}.$$

For a given threshold, Th, the detector $\delta(\mathbf{r}_k)$ is defined as,

$$\delta(\mathbf{r}_k) = \begin{cases} 1 & L(\mathbf{r}_k) \geq Th \\ 0 & L(\mathbf{r}_k) < Th \end{cases}$$

Maximizing the likelihood ratio over D_k is just a search over a discrete finite set; on the other hand maximizing over κ is a continuous optimization problem. We found upper and lower bounds on κ and quantized the interval to do the search.

As a positive outcome of the proposed detector, we can obtain the maximum likelihood estimate of κ, which gives some information about the network; e.g., κ is proportional to the number of neighboring nodes in the case of identically distributed nodes (Eqn. 7).

6. SIMULATIONS

We deal with a single node receiving packets from N nodes uniformly distributed in a circular area of radius R. These nodes' relaying times are uniformly distributed between 0 and $2\triangle$. The pulse shape is rectangular of width T. The realizations of channel coefficient are generated through Eqn. 3, and actual κ is calculated from Eqn. 7. Uniformly generated delays, $D < M + L - 1$, are introduced and packets are generated under each hypothesis. In this section we compare the performance of the proposed detector (Eqn. 8) with the following methods.

Genie aided detector: This detector knows what the actual delay, D_{act}, and κ_{act} are.

$$\delta_{Genie}(\mathbf{r}_k) = \begin{cases} 1 & \frac{p_1(\mathbf{r}_k; D_{act}, \kappa_{act})}{p_0(\mathbf{r}_k)} \geq Th \\ 0 & \frac{p_1(\mathbf{r}_k; D_{act}, \kappa_{act})}{p_0(\mathbf{r}_k)} < Th \end{cases}$$

Estimator-Correlator detector: This detector produces a least squares estimate of the channel, and correlates the estimate with the received signal (this is essentially a deterministic version of the estimator-correlator in [6] Sec 5.3). Let $\mathbf{C}_0 := C(D_k)|_{D_k=0}$.

$$\delta_{EstCorr}(\mathbf{r}_k) = \begin{cases} 1 & T(\mathbf{r}_k) \geq Th \\ 0 & T(\mathbf{r}_k) < Th \end{cases}$$

where $T(\mathbf{r}_k) = \mathbf{r}_k^H \mathbf{C}_0 [\mathbf{C}_0^H \mathbf{C}_0]^{-1} \mathbf{C}_0^H \mathbf{r}_k$.

The curves showing the probability of detection versus false alarm for all methods are given in Fig. 2. The proposed detector's performance lies close to the genie aided detector, and is much better than the estimator-correlator detector. As a second set-up, for the same simulation parameters and $D > M + L - 1$, we present the performance of sliding window by plotting the rate of detection versus estimated delay parameter (Fig. 3). When the threshold is high, the detection is concentrated around the actual delay as expected. Note that the detection is stopped once the signal is detected. Hence for low threshold, the concentration of estimated delay is around a value that is lower than the actual delay.

7. CONCLUSION

We formulated the signal detection problem in cooperative transmission, derived the channel statistics and proposed a GLRT detector. Then we compared the proposed detector with a genie aided one, which provides performance benchmark, and with the estimator-correlator. We plan to study the effect of detection on overall network performance as our future work.

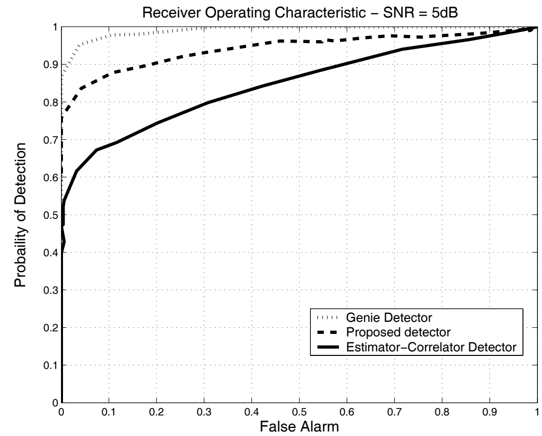

Fig. 2. Network consists of 15 nodes uniformly distributed in a circular area with radius, $R = 20m$. Simulation parameters: $\triangle = 1, T = 1, \kappa_{act} = 5.2 \times 10^{-6}, M = 16, L = 3$.

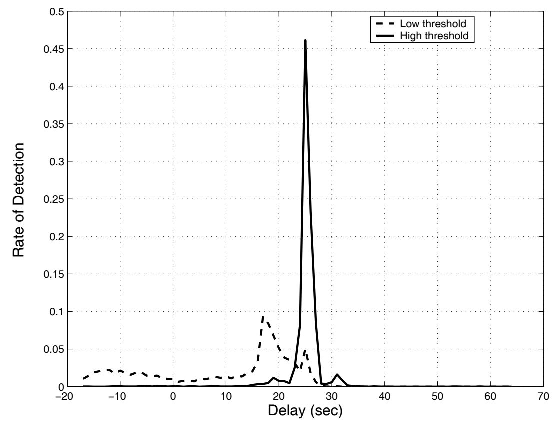

Fig. 3. Rate of detection versus the estimated delay where actual delay is 25 sec.

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