

# MIMO ARRAY DS-CDMA SYSTEM: A BLIND SPACE-TIME-DOPPLER ESTIMATION/RECEPTION

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## ABSTRACT

In this paper, a blind near-far resistant MIMO array receiver is proposed for time-varying asynchronous multipath DS-CDMA systems. Unlike many other MIMO works, the proposed receiver is built on *array-antenna* technology which is commonly used in applications such as the well-known 'smart antenna' system. Hence, a novel Space-Time-Doppler estimation algorithm is devised which provides the joint angle and delay, as well as the Doppler shift frequency estimation. With the estimator integrated as the front-end, the proposed MIMO array receiver requires no knowledge of the channel and is unsusceptible to near-far problems. Furthermore it is robust to channel estimation errors in the event of any unidentified or erroneous channel parameter.

## NOTATION

$A$	Scalar	$(.)^T$	Transpose
$\underline{A}$	Vector	$(.)^*$	Complex conjugate
$\mathbb{A}$	Matrix	$(.)^H$	Conjugate transpose
$\exp(\underline{A})$	Elemental exponential	$\lceil \cdot \rceil$	Roundup to integer
$\underline{0}_N$	$N$ -element zero vector	$\otimes$	Kronecker product
$\mathbb{I}_N$	$N \times N$ identity matrix	$\odot$	Hadamard product

## I. INTRODUCTION

The increased use of antenna elements at both ends of the transmission link, giving rise to Multiple Input and Multiple Output (MIMO) systems, is one of the viable and promising means of enabling the types of data rates and capacities needed for wireless internet and multimedia services. However a vast majority of the reported works on MIMO systems often adopts the assumption of multiple independent antenna elements [1,2], referred herein as *multiple-antenna* system. This is different from an *array-antenna* system where a number of antenna elements form an array system of a given geometry. The powerful effect of *array-antenna* system, for instance the 'smart antennas' technology lies in beamforming in a particular desired direction. By harnessing the spatio-temporal properties of the channel provided by the *array-antenna*, an extra layer of co-channel interference cancellation and new ways for handling unwanted channel effects can be developed.

Unlike most MIMO researches which require the full knowledge of the channel, this work proposes a blind near-far resistant MIMO array receiver for time-varying asynchronous multipath DS-CDMA system. The proposed receiver is

This work is supported by the EPSRC research grant GR/R08148/01, with Dr A. Manikas as the principal investigator.

applicable in both the data rate or diversity maximisation transmission schemes. Although a single spreading code is being assigned to each user in the work here, the proposed formulation can also be easily adapted to the code assignment strategies employed in typical CDMA BLAST system [1]. Through the incorporation of the *array-antenna* system, a novel subspace-based Space-Time-Doppler estimation algorithm is devised which provides the joint angle and delay, as well as the Doppler shift frequency estimation. The multipath Doppler spread, which is often regarded as one of the detrimental factors in degrading the performance of existing receivers, is being employed in the proposed receiver to provide an additional domain of diversity [3]. The resulting receiver, with its front-end integrated with the channel estimator, is also robust against any unidentifiable or erroneous channel parameter resulted in the estimation process.

## II. SYSTEM MODEL

Consider an  $M$ -user asynchronous DS-CDMA multiple-input multiple-output system, with each user having  $L$  transmitting antenna elements. Two transmission schemes can be employed over the MIMO channels: (i) data rate maximisation scheme or (ii) diversity maximisation scheme. The former scheme is performed by demultiplexing a high bit-rate input signal source into multiple data streams and each data stream is then transmitted simultaneously using different transmit antenna. The latter scheme, on the other hand, creates multiple copies of the same input signal source and transmits using multiple antennas to maximise the diversity advantages in fading channels. In addition, a unique spreading code is assigned to each user to be applied across its transmitting elements. Hence the modulating information signal due to the  $j^{\text{th}}$  transmitting antenna element of the  $i^{\text{th}}$  user may be written, as a function of the symbol index  $n$  and chip index  $p$ , as

$$m_{ij}(t) = \sum_{n=-\infty}^{+\infty} a_{ij}[n] \sum_{p=0}^{\mathcal{N}_c-1} \alpha_i[p] c(t - nT_{cs} - pT_c) \quad (1)$$

where  $\{a_{ij}[n] \in \pm 1, \forall n \in \mathcal{Z}\}$  is the  $i^{\text{th}}$  user's data symbol transmitted from its  $j^{\text{th}}$  antenna element,  $T_{cs}$  is the channel symbol period,  $\{\alpha_i[p] \in \pm 1, p = 0, 1, \dots, \mathcal{N}_c - 1\}$  corresponds to the  $i^{\text{th}}$  user's pseudo-noise spreading sequence of period  $\mathcal{N}_c = T_{cs}/T_c$ , and  $c(t)$  denotes the chip pulse-shaping waveform of duration  $T_c$ .

Suppose the transmitted signal from the  $j^{\text{th}}$  antenna element of  $i^{\text{th}}$  user arrives at the receiver via  $K_{ij}$  multipaths. For a

receiver employing an antenna array of  $N$  sensors, its corresponding spatial array manifold vector of the  $k^{\text{th}}$  path due to the  $j^{\text{th}}$  transmitting element of the  $i^{\text{th}}$  user can be expressed as

$$\underline{S}(\theta_{ijk}) = \exp(-j \cdot [\underline{r}_x, \underline{r}_y, \underline{r}_z] \cdot \underline{k}_{ijk}) \quad (2)$$

with  $[\underline{r}_x, \underline{r}_y, \underline{r}_z] \in \mathcal{R}^{N \times 3}$  defining the Cartesian coordinates of the *array-antenna* geometry and  $\underline{k}_{ijk}$  is the wavenumber vector pointing towards the azimuth direction  $\theta_{ijk}$ . By defining  $\underline{S}_{ijk} \triangleq \underline{S}(\theta_{ijk})$ , the net baseband received signal vector, in the presence of additive isotropic white Gaussian noise, can be written explicitly as

$$\underline{x}(t) = \sum_{i=1}^M \sum_{j=1}^L \sum_{k=1}^{K_{ij}} \underline{S}_{ijk} \beta_{ijk} \exp(j2\pi f_{ijk} t) m_{ij}(t - \tau_{ijk}) + \underline{n}(t) \quad (3)$$

where  $\underline{n}(t)$  is the complex white Gaussian noise vector;  $\beta_{ijk}$  and  $\tau_{ijk}$  are respectively the complex fading coefficient and path delay for the  $k^{\text{th}}$  path associated with the  $j^{\text{th}}$  transmitting element of the  $i^{\text{th}}$  user, whilst  $\exp(j2\pi f_{ijk} t)$  signifies its corresponding Doppler shift with  $f_{ijk} = v_{ijk} F_c / c$  (where  $v_{ijk}$  is the velocity of the motion,  $F_c$  is the carrier frequency, and  $c$  is the speed of light) being the Doppler shift frequency.

Taking the multipath delay spread to lie within the range  $[0, T_{cs})$ , the  $N$ -dimensional signal vector  $\underline{x}(t)$ , received from the *array-antenna* output, is then sampled at a rate of  $1/T_s$  (where  $T_s = T_c/q$  is the sampling period with  $q$  being the oversampling factor) and passed through a bank of  $N$  tapped-delay lines (TDL), each of length  $2qN_c$ . Upon concatenating the contents of the TDLs, the  $2qN_c$ -dimensional discretised signal vector is thus formed and read for every  $T_{cs}$  with the  $n^{\text{th}}$  observation interval represented as

$$\underline{x}[n] = [\underline{x}_1[n]^T, \underline{x}_2[n]^T, \dots, \underline{x}_N[n]^T]^T \quad (4)$$

where  $\underline{x}_u[n]$  is the  $2qN_c$ -dimensional discretised output frame from the  $u^{\text{th}}$  TDL for  $u = 1, 2, \dots, N$ . However due to the lack of synchronisation, the content of each TDL contains contributions from not only the current but also the previous and next symbols. To model such contributions, let's first introduce the following Spatio-Temporal ARray (STAR) manifold vector [4] by modifying Eqn. (2) as

$$\underline{h}_{ijk} = \underline{S}_{ijk} \otimes \mathbb{J}^{l_{ijk}} \underline{c}_i \quad (5)$$

where  $l_{ijk} = \lceil \tau_{ijk} / T_s \rceil$  is the discretised multipath delay; the matrix  $\mathbb{J}$  (or  $\mathbb{J}^T$ ) is a  $2qN_c \times 2qN_c$  time down-shift (or up-shift) operator matrix given as follows

$$\mathbb{J} = \begin{bmatrix} \mathbb{0}_{2qN_c-1}^T & 0 \\ \mathbb{I}_{2qN_c-1} & \mathbb{0}_{2qN_c-1} \end{bmatrix} \quad (6)$$

and  $\underline{c}_i$  is related to the  $i^{\text{th}}$  user's spreading sequence defined as

$$\underline{c}_i = \sum_{p=0}^{N_c-1} \alpha_i[p] \cdot \mathbb{J}^{p/q} \underline{c} \quad (7)$$

with the vector  $\underline{c}$  being the oversampled chip-level pulse shaping function  $c(t)$  padded with zeros at the end, i.e.

$$\underline{c} = [c(0), c(T_s), \dots, c((q-1)T_s), \mathbb{0}_{q(2N_c-1)}^T]^T \quad (8)$$

Having specified the STAR manifold vector, its discretised Doppler effect can be easily included by extending Eqn. (5) to obtain the Doppler-STAR manifold vector due to the  $k^{\text{th}}$  path from the  $j^{\text{th}}$  transmitting element of the  $i^{\text{th}}$  user, given as

$$\underline{h}_{ijk}[n] = \underline{S}_{ijk} \otimes (\mathbb{J}^{l_{ijk}} \underline{c}_i \odot \underline{F}_{ijk}[n]) \quad (9)$$

with the time-varying Doppler component  $\underline{F}_{ijk}[n]$  modelled as

$$\underline{F}_{ijk}[n] = \begin{bmatrix} 1 \\ \exp(j2\pi f_{ijk} T_s) \\ \exp(j2.2\pi f_{ijk} T_s) \\ \vdots \\ \exp(j2.(2qN_c-1)\pi f_{ijk} T_s) \end{bmatrix} \exp(j2n\pi f_{ijk} T_{cs}) \quad (10)$$

By taking into account the previous, current and next symbol contributions as  $\underline{a}_{ij}[n] = [a_{ij}[n-1], a_{ij}[n], a_{ij}[n+1]]^T$ , the discretised representation of the received signal vector  $\underline{x}[n]$ , incorporating the Inter-Symbol Interference (ISI), Multiple-Access Interference (MAI) and co-code interference constituents, can hence be expressed as follows

$$\underline{x}[n] = \sum_{i=1}^M \sum_{j=1}^L [\mathbb{H}_{ij}^{\text{prev}}[n] \beta_{ij}, \mathbb{H}_{ij}[n] \beta_{ij}, \mathbb{H}_{ij}^{\text{next}}[n] \beta_{ij}] \underline{a}_{ij}[n] + \underline{n}[n] \quad (11)$$

where  $\underline{n}[n]$  is the sampled noise vector and

$$\begin{aligned} \mathbb{H}_{ij}[n] &= [\underline{h}_{ij1}[n], \underline{h}_{ij2}[n], \dots, \underline{h}_{ijK_{ij}}[n]] \in \mathcal{C}^{2qN_c \times K_{ij}} \\ \mathbb{H}_{ij}^{\text{prev}}[n] &= (\mathbb{I}_N \otimes \mathbb{J}^{qN_c}) \mathbb{H}_{ij}[n] \in \mathcal{C}^{2qN_c \times K_{ij}} \\ \mathbb{H}_{ij}^{\text{next}}[n] &= (\mathbb{I}_N \otimes \mathbb{J}^{qN_c}) \mathbb{H}_{ij}[n] \in \mathcal{C}^{2qN_c \times K_{ij}} \end{aligned}$$

### III. SPACE-TIME-DOPPLER ESTIMATION & RECEPTION

In order to perform an estimation of the channel parameters pertaining to the desired user, a computationally efficient algorithm is devised which requires only the knowledge of the desired user's spreading code sequence. The proposed algorithm, unlike the one in [4], does not need a restoration of the desired signal subspace dimensionality. By exploiting the Doppler structural property on the signal data vector, it is noted that the covariance matrix  $\mathbb{R}_{xx}$  of the discretised signal vector in Eqn. (11) will preserve and provide a basis for the desired signal subspace. This algorithm can be applied for any paths originated from the same source arriving from the same direction (co-directional) or arriving at the same time (co-delay); and even in scenario whereby the paths originated from the different sources of the desired user arriving in the same space and time domains (co-located in space and time). However, for paths of the same source having the same Doppler shift frequencies, singularity in  $\mathbb{R}_{xx}$  will occur. This special case cannot be resolved for a general array geometry but for a uniform linear array where spatial smoothing can be overlaid on top of  $\mathbb{R}_{xx}$  to form the

smoothed covariance matrix  $\mathbb{R}_{\text{Smooth}}$  [5]. Having obtained the covariance matrix, the multipaths' space-time channel parameters can then be found jointly by minimising the following MUSIC-type cost function, given as

$$\xi_{\text{ST}}(\theta, l) = \frac{(\underline{S}(\theta) \otimes \mathbb{J}^l \underline{\mathbf{c}}_D)^H \mathbb{E}_{\text{n,ST}} \mathbb{E}_{\text{n,ST}}^H (\underline{S}(\theta) \otimes \mathbb{J}^l \underline{\mathbf{c}}_D)}{(\underline{S}(\theta) \otimes \mathbb{J}^l \underline{\mathbf{c}}_D)^H (\underline{S}(\theta) \otimes \mathbb{J}^l \underline{\mathbf{c}}_D)} \quad (12)$$

where  $\underline{S}(\theta) \otimes \mathbb{J}^l \underline{\mathbf{c}}_D$  is the STAR manifold vector with  $\underline{\mathbf{c}}_D$  being related to the spreading code sequence of the desired  $\mathcal{D}^{\text{th}}$  user as shown in Eqn. (7), and  $\mathbb{E}_{\text{n,ST}}$  is the matrix with columns the noise eigenvectors of  $\mathbb{R}_{\text{Smooth}}$ .

Now by denoting  $\widehat{\mathbb{B}}_{ij} \triangleq [\widehat{\mathbf{b}}_{ij1}, \widehat{\mathbf{b}}_{ij2}, \dots, \widehat{\mathbf{b}}_{ijK_{ij}}]$ , the STAR manifold vector due to the desired user can be reconstructed based on the estimated space-time channel parameters as

$$\widehat{\mathbb{B}}_D = [\widehat{\mathbb{B}}_{D1}, \widehat{\mathbb{B}}_{D2}, \dots, \widehat{\mathbb{B}}_{DL}] \quad (13)$$

In order to suppress the contributions from the MAI and ISI interferences, the received discretised signal vector  $\underline{x}[n]$  in Eqn. (11) is thus passed through a novel space-time multipath filter bank to yield

$$\underline{y}[n] = \mathbb{L}^H \cdot \underline{x}[n] \quad (14)$$

where  $\mathbb{L}$  is the multipath filter bank based on the orthogonal projection of the interference subspace, i.e.

$$\mathbb{L} = \mathbb{P}_{\widehat{\mathbb{B}}_{\text{intf}}}^\perp \widehat{\mathbb{B}}_D \left( \widehat{\mathbb{B}}_D^H \mathbb{P}_{\widehat{\mathbb{B}}_{\text{intf}}}^\perp \widehat{\mathbb{B}}_D \right)^{-1} \quad (15)$$

where  $\mathbb{P}_{\mathbb{K}}^\perp = \mathbb{I} - \mathbb{K}(\mathbb{K}^H \mathbb{K})^{-1} \mathbb{K}^H$  is the complementary projection operator of the matrix  $\mathbb{K}$ ; and  $\widehat{\mathbb{B}}_{\text{intf}} = [(\mathbb{I}_N \otimes (\mathbb{J}^T)^{qN_c}) \widehat{\mathbb{B}}_{\text{com}}, \widehat{\mathbb{B}}_{\text{com}}^{\text{P}}, (\mathbb{I}_N \otimes \mathbb{J}^{qN_c}) \widehat{\mathbb{B}}_{\text{com}}]$  in which  $\widehat{\mathbb{B}}_{\text{com}} = [\widehat{\mathbb{B}}_1, \widehat{\mathbb{B}}_2, \dots, \widehat{\mathbb{B}}_M]$  is the estimated composite channel parameters and  $\widehat{\mathbb{B}}_{\text{com}}^{\text{P}}$  is the composite matrix  $\widehat{\mathbb{B}}_{\text{com}}$  with the exclusion of the matrix  $\widehat{\mathbb{B}}_D$ .

The output of the filter bank, comprising of  $\mathcal{O} = \sum_{j=1}^L K_{Dj}$

branches, is however contaminated with its corresponding Doppler component. This Doppler effect is dominated by the last term on the right-hand side of Eqn. (10) since its temporal variation within a symbol period is comparatively much smaller. Thus for a burst of  $d$  data symbols being processed by the *array-antenna* system, its Doppler component due to the  $k^{\text{th}}$  path from the  $j^{\text{th}}$  transmitting element of the desired  $\mathcal{D}^{\text{th}}$  user can be defined and modelled in terms of its more significant term as  $\underline{\Phi}(f_{\mathcal{D}jk}) \triangleq [1, \Phi^1(f_{\mathcal{D}jk}), \Phi^2(f_{\mathcal{D}jk}), \dots, \Phi^{(d-1)}(f_{\mathcal{D}jk})]^T$  with  $\Phi(f_{\mathcal{D}jk}) = \exp(j2\pi f_{\mathcal{D}jk} T_{cs})$ . To extract these Doppler parameters, the filter bank output is first pre-processed to remove the phase reversals introduced by the space-time filtered  $n^{\text{th}}$  symbol,

$$\underline{y}_{\text{pre}}[n] = \underline{y}[n] \odot \underline{y}[n] \quad \text{for } n = 1, 2, \dots, d \quad (16)$$

With that, the Doppler shift frequencies can then be obtained by performing one-dimensional search for each of the branches (i.e. br1, br2, ..., br $\mathcal{O}$ ) over the frequency range  $f_{\text{min}} \leq f \leq f_{\text{max}}$ :

$$\xi_{\text{D}}(f) = \frac{\underline{\Phi}(2f)^H \mathbb{E}_{\text{n,D}} \mathbb{E}_{\text{n,D}}^H \underline{\Phi}(2f)}{\underline{\Phi}(2f)^H \underline{\Phi}(2f)} \quad (17)$$

where  $\mathbb{E}_{\text{n,D}}$  has columns the noise eigenvectors associated with the second order statistics of each pre-processed filter output branch in Eqn. (16).

Now having found the Doppler shift frequencies, its Doppler effect at the filter bank output in Eqn. (14) is compensated as follows

$$\underline{z}[n] = \widehat{\underline{\Phi}}[n]^* \odot \underline{y}[n] \quad (18)$$

where  $\widehat{\underline{\Phi}}[n] = [\Phi^n(\widehat{f}_{\text{br1}}), \Phi^n(\widehat{f}_{\text{br2}}), \dots, \Phi^n(\widehat{f}_{\text{br}\mathcal{O}})]^T$  is the Doppler compensator vector.

For MIMO systems employing the diversity maximisation transmission scheme, the compensated filter bank outputs in Eqn. (18) can then be simply combined to realise the decision statistic for the  $n^{\text{th}}$  symbol of the desired user

$$b[n] = \underline{w}^H \cdot \underline{z}[n] \quad (19)$$

where  $\underline{w}$  is the combining weight vector obtained using the principal eigenvector of the autocorrelation matrix of Eqn. (18).

On the other hand, for MIMO system with the employment of the data rate maximisation transmission scheme, the compensated filter bank output needs to be partitioned to differentiate those branches belonging to each of the  $L$  transmitting elements of the desired user. This can be done by passing the compensated output in Eqn. (18), following the decision device, to the *Branch Identification Process*, whereby a cross-correlation is performed with each of the  $\mathcal{O}$  branches. The correlated output is then compared with a prespecified threshold value and aggroup together when it exceeds the threshold. Having segregated the branches, the outputs belonging to each of the  $L$  transmitting elements can then be subsequently combined using Eqn. (19). Note that it is not necessary to assign each and every of the  $\mathcal{O}$  branches to all the  $L$  transmitting elements of the desired user. If the channel parameters of any particular branch is erroneous (incorrect channel estimation) or unidentified (incomplete channel estimation), the *Branch Identification Process* will leave that branch unassigned, thus inducing robustness to the receiver.

#### IV. PERFORMANCE ANALYSIS

Several representative examples are presented in this section to highlight the key benefits of introducing *array-antenna* technology in typical MIMO systems. Consider a uniform  $N = 5$  element linear array of half-wavelength spacing operating in the presence of  $M = 3$  co-channel DS-CDMA users, each having  $L = 2$  transmitting antenna elements, employing the data rate maximisation transmission scheme. Each user is assigned a unique Gold sequence of length  $N_c = 31$  with rectangular chip pulse-shaping. The chip rate is set at  $1/T_c = 1.2288$  Mchips/s with a carrier frequency of  $F_c = 2$  GHz. The array is assumed to collect  $d = 200$  data symbols for processing with a chip-rate sampler of  $q = 1$ .

Let's take user 1 as the desired user having an SNR of 20dB; while the rest of the interferers each constituting an interference ratio of 20dB (i.e. near-far problem). All 3 users are assumed to have 10 multipaths each, with their parameters as listed in Table I. The Doppler spread is set at 200 Hz which corresponds to a maximum speed of 108 km/h. By partitioning the array into 2 overlapping 4-element subarrays for spatial smoothing, it is seen from Fig. 1 that all the 10 multipaths, associated with the two transmitting elements of the desired user, can be identified/estimated successfully using the proposed algorithm. Its Doppler shift frequencies, be it identical or zero, can also be correctly estimated as shown by the Doppler spectrum plotted in Fig. 2. Notice that the algorithm can still operate even when the desired user's paths are co-located in both (i) the code and space domains, (ii) the code and time domains, or (iii) the space and time domains. In addition to that, the number of multipaths that can be resolved by the algorithm is also not constrained by the number of antennas available in the array.

The multipaths belonging to each of the two transmitting elements of the desired user are then singled out by applying the *Branch Identification Process* to the compensated filter bank output. Its cross-correlation with the first output branch is as illustrated in Table II. Fig. 3 depicts the performance of the proposed Doppler-STAR receiver as compared with a conventional ST decorrelating detector and 2D RAKE receiver, with the latter two assuming full knowledge of the channel. The proposed receiver is thus seen to be more tolerant with the multipath Doppler spread, whilst the decorrelating detector and RAKE receiver deteriorate drastically at the onset of the spread.

## V. CONCLUSIONS

A blind MIMO array receiver based on a computationally efficient near-far resistant channel estimation technique is presented for time-varying asynchronous multipath DS-CDMA system. Without the need of exhausting the number of spreading codes available, the proposed receiver is applicable in either the data rate maximisation or diversity maximisation scheme. The potential benefit of incorporating the *array-antenna* system is demonstrated by the near-far resistant Space-Time-Doppler estimation algorithm which does not require the need of any power control or the knowledge of the channel, as are readily assumed in most MIMO systems.

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User 1 with code vector $\alpha_1$				User 2 with code vector $\alpha_2$				User 3 with code vector $\alpha_3$			
(Ant, Path)	$\theta_{1jk}$	$l_{1jk}$	$f_{1jk}$	(Ant, Path)	$\theta_{2jk}$	$l_{2jk}$	$f_{2jk}$	(Ant, Path)	$\theta_{3jk}$	$l_{3jk}$	$f_{3jk}$
$(j,k)=(1,1)$	$40^\circ$	$8T_c$	$30\text{Hz}$	$(j,k)=(1,1)$	$30^\circ$	$10T_c$	$-90\text{Hz}$	$(j,k)=(1,1)$	$20^\circ$	$15T_c$	$40\text{Hz}$
$(j,k)=(1,2)$	$50^\circ$	$18T_c$	$100\text{Hz}$	$(j,k)=(1,2)$	$70^\circ$	$25T_c$	$60\text{Hz}$	$(j,k)=(1,2)$	$60^\circ$	$4T_c$	$-80\text{Hz}$
$(j,k)=(1,3)$	$70^\circ$	$25T_c$	$-160\text{Hz}$	$(j,k)=(1,3)$	$80^\circ$	$20T_c$	$170\text{Hz}$	$(j,k)=(1,3)$	$60^\circ$	$7T_c$	$-10\text{Hz}$
$(j,k)=(1,4)$	$90^\circ$	$18T_c$	$100\text{Hz}$	$(j,k)=(1,4)$	$80^\circ$	$21T_c$	$30\text{Hz}$	$(j,k)=(1,4)$	$60^\circ$	$10T_c$	$5\text{Hz}$
$(j,k)=(1,5)$	$100^\circ$	$12T_c$	$0\text{Hz}$	$(j,k)=(1,5)$	$100^\circ$	$10T_c$	$-110\text{Hz}$	$(j,k)=(2,1)$	$100^\circ$	$3T_c$	$180\text{Hz}$
$(j,k)=(2,1)$	$60^\circ$	$15T_c$	$150\text{Hz}$	$(j,k)=(1,6)$	$110^\circ$	$11T_c$	$-100\text{Hz}$	$(j,k)=(2,2)$	$110^\circ$	$20T_c$	$-2\text{Hz}$
$(j,k)=(2,2)$	$90^\circ$	$5T_c$	$-80\text{Hz}$	$(j,k)=(2,1)$	$80^\circ$	$5T_c$	$-60\text{Hz}$	$(j,k)=(2,3)$	$130^\circ$	$11T_c$	$110\text{Hz}$
$(j,k)=(2,3)$	$90^\circ$	$20T_c$	$100\text{Hz}$	$(j,k)=(2,2)$	$90^\circ$	$5T_c$	$-60\text{Hz}$	$(j,k)=(2,4)$	$130^\circ$	$12T_c$	$-50\text{Hz}$
$(j,k)=(2,4)$	$120^\circ$	$10T_c$	$0\text{Hz}$	$(j,k)=(2,3)$	$120^\circ$	$28T_c$	$150\text{Hz}$	$(j,k)=(2,5)$	$140^\circ$	$8T_c$	$90\text{Hz}$
$(j,k)=(2,5)$	$130^\circ$	$15T_c$	$-120\text{Hz}$	$(j,k)=(2,4)$	$150^\circ$	$25T_c$	$0\text{Hz}$	$(j,k)=(2,6)$	$160^\circ$	$18T_c$	$-170\text{Hz}$

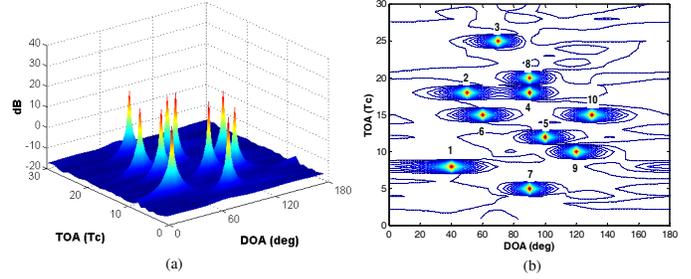


Fig. 1: Space-time spectrum of the desired user 1.

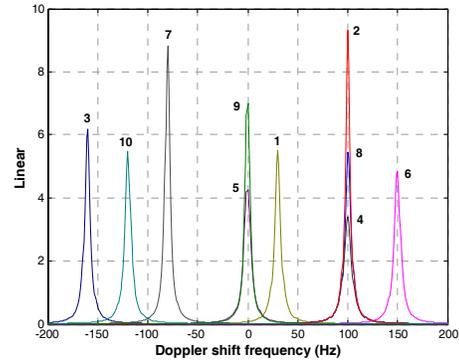


Fig. 2: Doppler shift frequency spectrum of the desired user 1.

(Ant, $j$ , Path $k$ )	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)
Correlation	1.000	0.999	0.999	1.000	0.999	0.025	0.025	0.024	0.025	0.024

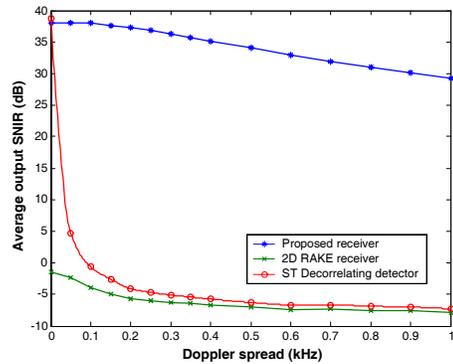


Fig. 3: Output SNIR performance versus Doppler spread.