QUASI-ORTHOGONAL SPACE-TIME BLOCK CODED TRANSCEIVER SYSTEMS OVER FREQUENCY SELECTIVE WIRELESS FADING CHANNELS

Lei He *Hongya Ge*

Center for Communications and Signal Processing Research
Department of Electrical and Computer Engineering
New Jersey Institute of Technology, Newark, NJ 07102
Email: lxh4862@njit.edu

ABSTRACT

In this work, we propose a new block time-reversal conjugate based quasi-orthogonal space-time block encoding and decoding (QO-STBC) scheme for a wireless system with four transmit antennas in frequency selective fading channels. The proposed scheme effectively combats channel dispersion and frequency selectivity due to multipath, yet still provides full diversity gain. We also proposed a maximum likelihood (ML) solution, with a simple decoupled structure, to decode the block time-reversal conjugate QO-STBC symbols at low complexity for channels with short delay.

1. INTRODUCTION

Among many proposed space-time block code (STBC) designs, the *orthogonal* STBC (the Alamouti code with 2 transmitters) provides full-rate and full diversity along with a fast ML decoding scheme with *linear complexity* given the knowledge of channel [1][2]. However, there is no full rate complex *orthogonal* code design (with *linear complexity*) for higher dimensional systems with multiple transmitters beyond 2. Recently proposed quasi-orthogonal STBC (QO-STBC) for systems with 4 transmitters [3] delivers a full coding rate but only achieves half of diversity gain. It has been observed that the performance and diversity gain of the QO-STBC can be improved through a *constellation rotation* [4]. In [5], we proposed a fast ML detection scheme for QO-STBC system, and proved the full diversity gain of the constellation rotated QO-STBC. Hence, the constellation rotated QO-STBC becomes another important STBC scheme that simultaneously provides full-rate and full-diversity.

It should be pointed out that the above mentioned work all assumed that there is no inter-symbol interference in the received data. That is, a non-dispersive block fading narrowband channel model was assumed. This assumption is not valid for multipath wireless fading channels of time dispersive and frequency selective nature. In [6][7], a clever combination of time-domain filtering, and time reversal conjugate operation was used to convert a 2×1 multiple-input single-output (MISO) system composed of 2 frequency selective channels into 2 equivalent single-input single-output (SISO) system with a combined frequency selective channel. After the conversion, a standard SISO equalization is applied to detect 2 independent information streams from transmitters.

In this work, we successfully extended such technique to block based QO-STBC transceiver system in combating channel dispersion and frequency selectivity. We propose a new block based QO-STBC transceiver design scheme for wireless systems over frequency selective fading channels. In the proposed transceiver scheme, we used a combination of time-domain filtering, and time reversal conjugate operation to convert the 4×1 MISO system over 4 independent frequency selective fading channels into 2 pairs equivalent 2×1 MISO sub-systems with frequency-selective channels. Within each pair of MISO system, two independent data streams are jointly decoded using the Viterbi Algorithm (VA).

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2. NOTATION AND SYSTEM MODEL

The input/output (I/O) relation in a SISO system can be represented, in discrete time notation, as,

$$\begin{aligned} y(t) &= H(z)\,\{\,x(t)\,\} \\ &= h_0\,x(t) + h_1\,x(t-1) + \cdots + h_{L_a}x(t-L_a)\,, \end{aligned}$$

where $H(z) = \sum_{l=0}^{L_a} h_l\, z^{-l}$ is a FIR channel filter of length $(L_a + 1)$.

Similarly, for a MISO system, we use a polynomial row vector to represent FIR channel filters; for a single-input-multiple-output (SIMO) system, we use a polynomial column vector to represent FIR channel filters, and for a multiple-input-multiple-output (MIMO) system, we use a polynomial matrix to represent the FIR filters between all possible transceiver pairs.

The conjugate reciprocal version of a SISO channel filter $H(z)$ is defined as,

$$H^*(1/z^*) \triangleq (h_0^* + h_1^* z + \cdots + h_{L_a}^* z^{L_a}),$$

which corresponds to a time reversal conjugate operation on the original impulse response. Similarly, the conjugate reciprocal versions of the MISO, SIMO and MIMO filters are defined according to the same time-reversal conjugate operation on their impulse responses of the corresponding matrix filters, respectively.

In this work, we considered a MISO system with N transmitters and 1 receiver. The received data sequence from such a system is,

$$r(t) = \sum_{k=1}^{N} H_k(z)\{\, s_k(t)\,\} + n(t) \qquad (1)$$

where $H_k(z) = \sum_{l=0}^{L_k} h_{k,l} z^{-l}$ is the (L_k+1)-ray multipath channel between the the k-th transmitter and the receiver; $s_k(t)$ is the symbol sequence transmitted from the k-th transmitter. The additive noise $n(t)$ is zero-mean complex Gaussian distributed, i.e. $n(t) \sim \mathcal{CN}(0, 1/\text{SNR})$.

We further assume that the maximum of L_k's equals L. The norm of each channel filter $H_k(z)$ is normalized, i.e. $E\{\|\mathbf{h}_k\|\} = 1$ and $\mathbf{h}_k = [h_{k,0}\, h_{k,1} \cdots, h_{k,L_k}]^T$, $(k = 1, 2, \ldots, N)$. We also assume that the total transmission power is equally distributed among all transmit antennas, and the received data in (1) is normalized so that the reception SNR is equivalently present in the noise power.

3. QO-STBC FOR FREQUENCY SELECTIVE FADING CHANNELS

3.1. Quasi-orthogonal space-time block code

The QO-STBC code for a system with 4 transmitters was proposed by Jafarkhani in [3]. Given a block of 4 symbols to be transmitted by 4 transmitters over 4 consecutive time slots over a block flat fading channel, the QO-STBC is defined by the following transmission code matrix,

$$\mathbf{C}(\mathbf{s}) = \begin{bmatrix} s_1 & s_2 & s_3 & s_4 \\ -s_2^* & s_1^* & -s_4^* & s_3^* \\ -s_3^* & -s_4^* & s_1^* & s_2^* \\ \underbrace{s_4}_{\mathbf{c}_1} & \underbrace{-s_3}_{\mathbf{c}_2} & \underbrace{-s_2}_{\mathbf{c}_3} & \underbrace{s_1}_{\mathbf{c}_4} \end{bmatrix} = \begin{bmatrix} \mathbf{C}_1 & \mathbf{C}_2 \\ -\mathbf{C}_2^* & \mathbf{C}_1^* \end{bmatrix}. \qquad (2)$$

Each row of the above transmission matrix denotes symbols simultaneously sent through four transmit antennas within a given transmission slot. Note that only the subspace $\langle \mathbf{c}_1, \mathbf{c}_4 \rangle$ is orthogonal to the subspace $\langle \mathbf{c}_2, \mathbf{c}_3 \rangle$, where $\mathbf{c}_i$ denotes the i-th column of the code matrix $\mathbf{C}(\mathbf{s})$. Hence, the STBC in (2) is called quasi-orthogonal (QO) code.

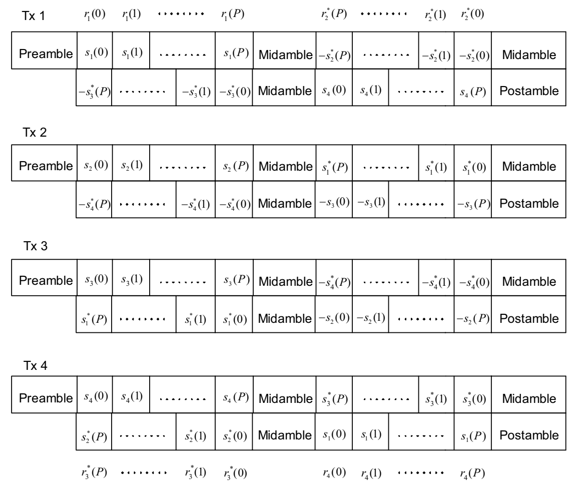

Fig. 1. Transmission mechanism of the proposed block based QO-STBC.

3.2. Block based QO-STBC Scheme

In this work, we consider a block of $4(P+1)$ digitally modulated symbols to be transmitted by 4 transmitters and subsequently received by 1 receiving antenna over frequency selective fading channels. For the block based transmission over a multipath fading channel, a *time reversal conjugate technique* is used to embeded the quasi-orthogonality into the STBC blocks. Essentially, we first decimate a block of symbols, $\{s(t)\}_{t=1}^{4(P+1)}$, into 4 parallel sub-blocks, $\{s_1(t)\}_{t=0}^{P}$, $\{s_2(t)\}_{t=0}^{P}$, $\{s_3(t)\}_{t=0}^{P}$ and $\{s_4(t)\}_{t=0}^{P}$. These 4 sub-blocks of symbols and/or their time reversal conjugate versions are to be transmitted through 4 transmitters according to the space-time scheduling outlined in the transmission code matrix in (2). To avoid the inter-block interference, preamble and midambles of length L are inserted into the QO-STBC data streams prior to transmission. Detailed data transmission mechanism using the proposed block based QO-STBC is shown in figure 1.

Mathematically, such a space-time transmission scheme over frequency selective channels can be described by the following model after removing preamble/midambles and time reversal conjugating the corresponding sub-blocks of received data according to the block based QO-STBC mechanism. That is, the properly arranged received data,

$$\mathbf{r}(t) = \mathbf{H}(z)\{\mathbf{s}(t)\} + \mathbf{n}(t), \qquad (3)$$

where $\mathbf{H}(z)$ is a polynomial matrix of the form,

$$\begin{bmatrix} H_1(z) & H_2(z) & H_3(z) & H_4(z) \\ H_2^*(1/z^*) & -H_1^*(1/z^*) & H_4^*(1/z^*) & -H_3^*(1/z^*) \\ H_3^*(1/z^*) & H_4^*(1/z^*) & -H_1^*(1/z^*) & -H_2^*(1/z^*) \\ H_4(z) & -H_3(z) & -H_2(z) & H_1(z) \end{bmatrix}$$

and $\mathbf{s}(t) = [s_1(t)\ s_2(t)\ s_3(t)\ s_4(t)]^T$.

3.3. ML detection of block based QO-STBC symbols

Using an elementary matrix operation $\mathbf{J}$, we can rewrite the properly arranged received data model in (3) as

$$\mathbf{r}(t) = \widetilde{\mathbf{H}}(z)\{\widetilde{\mathbf{s}}(t)\} + \mathbf{n}(t),$$

where $\widetilde{\mathbf{s}}(t) = \mathbf{J}\,\mathbf{s}(t) = \begin{bmatrix} \widetilde{s}_1 & \widetilde{s}_2 & \widetilde{s}_3 & \widetilde{s}_4 \end{bmatrix}^T$ and $\widetilde{\mathbf{H}}(z) = \mathbf{H}(z)\,\mathbf{J}^T = \begin{bmatrix} \widetilde{\mathbf{h}}_1 & \widetilde{\mathbf{h}}_2 & \widetilde{\mathbf{h}}_3 & \widetilde{\mathbf{h}}_4 \end{bmatrix}^T$, with $\widetilde{s}_{1/4} = s_1 \pm s_4$; $\widetilde{s}_{2/3} = s_2 \pm s_3$; $\widetilde{\mathbf{h}}_{1/4} = \mathbf{h}_1 \pm \mathbf{h}_4$; $\widetilde{\mathbf{h}}_{2/3} = \mathbf{h}_2 \pm \mathbf{h}_3$; with $\mathbf{h}_i(z)$ is the i-th column of the polynomial matrix $\mathbf{H}(z)$.

Note the important fact that the polynomial channel matrix $\widetilde{\mathbf{H}}(z)$ here becomes *orthogonal* in the sense that $\mathbf{G}(z) = \widetilde{\mathbf{H}}^H(z)\widetilde{\mathbf{H}}(z)$ is a diagonal matrix. That is,

$$\mathbf{G}(z) = diag\,\{G_1(z), G_2(z), G_1(z), G_2(z)\},$$

where

$$\begin{aligned} G_1(z) &= 2\left(H_1^*(1/z^*) + H_4^*(1/z^*)\right)\left(H_1(z) + H_4(z)\right) \\ &+ 2\left(H_2^*(1/z^*) - H_3^*(1/z^*)\right)\left(H_2(z) - H_3(z)\right) \end{aligned}$$

and

$$\begin{aligned} G_2(z) &= 2\left(H_1^*(1/z^*) - H_4^*(1/z^*)\right)\left(H_1(z) - H_4(z)\right) \\ &+ 2\left(H_2^*(1/z^*) + H_3^*(1/z^*)\right)\left(H_2(z) + H_3(z)\right) \end{aligned}$$

Therefore, we obtained the interesting decoupled results after a matrix channel matched filtering. That is,

$$\widetilde{\mathbf{r}}(t) = \widetilde{\mathbf{H}}^H(z)\{\mathbf{r}(t)\} = \mathbf{G}(z)\{\widetilde{\mathbf{s}}(t)\} + \mathbf{v}(t) \qquad (4)$$

where the noise $\mathbf{v}(t) = \widetilde{\mathbf{H}}^H(z)\{\mathbf{n}(t)\}$, is found to be spatially (inter-block-wise) uncorrelated, since the spectrum of $\mathbf{v}(t)$ is given by

$$\begin{aligned} \mathbf{R}_{\mathbf{vv}}(z) &= \sum_m E\left[\mathbf{v}(t)\,\mathbf{v}^H(t-m)\right] z^{-m} \\ &= \mathbf{H}^H(z)\,\mathbf{R}_{\mathbf{nn}}(z)\,\mathbf{H}(z) \\ &= \sigma_n^2 \mathbf{G}(z). \end{aligned}$$

From (4), we can see that two pairs of symbol blocks ($\{s_1(t)\}$, $\{s_4(t)\}$) and ($\{s_2(t)\}$, $\{s_3(t)\}$) are decoupled, hence, can be ML decoded independently. The noise is spatially (inter-block-wise) *uncorrelated* and temporally (intra-block-wise) colored. Therefore, each sub-blocks of data $\widetilde{\mathbf{r}}(t)$ can be whitened individually for subsequential processing.

The well known spectral decomposition fact can be observed in the polynomials $G_i(z) = \widetilde{G}_i(z)\widetilde{G}_i^*(1/z^*)$, $(i = 1,\ 2)$. The whitening filter for the i-th information block can be designed as $\left(\widetilde{G}_i(z)\right)^{-1}$. And the whitening filter bank for decoupled information streams is simply the following diagonal polynomial matrix,

$$\left(\widetilde{\mathbf{G}}(z)\right)^{-1} = diag\left\{\widetilde{G}_1^{-1}(z), \widetilde{G}_2^{-1}(z), \widetilde{G}_1^{-1}(z), \widetilde{G}_2^{-1}(z)\right\}.$$

At the output of whitening filter we have,

$$\begin{aligned} \underline{\mathbf{r}}(t) &= \left(\widetilde{\mathbf{G}}(z)\right)^{-1}\{\widetilde{\mathbf{r}}(t)\} \\ &= \widetilde{\mathbf{G}}^*(1/z^*)\{\widetilde{\mathbf{s}}(t)\} + \widetilde{\mathbf{v}}(t), \end{aligned}$$

where $\widetilde{\mathbf{v}}(t)$ is WGN vector and the variance of each element is σ_n^2.

The ML sequence estimation (MLSE) can then be performed on the above data set using the Viterbi algorithm.

3.3.1. Diversity Discussion

Given the knowledge of channel state information (CSI), the probability of transmitting sequence of $\mathbf{c} = (s_1(t),\ s_4(t))$ and deciding in favor of sequence of $\mathbf{e} = \left(s_1'(t),\ s_4'(t)\right)$ at the decoder can be bounded by the following [8],

$$P\left(\mathbf{c} \to \mathbf{e}|\mathbf{H}(z)\right) \le \exp(-d^2(\mathbf{c}, \mathbf{e})/2\sigma_n^2),$$

where σ_n^2 is the variance of the noise; and

$$\begin{aligned} d^2(\mathbf{c}, \mathbf{e}) &= norm(\widetilde{G}_1^*(1/z^*)\{\Delta_1(t)\})^2 \\ &+ norm(\widetilde{G}_4^*(1/z^*)\{\Delta_4(t)\})^2 \end{aligned} \qquad (5)$$

We define $\Delta_1(t) = (s_1(t) + s_4(t)) - (s_1'(t) + s_4'(t))$ and $\Delta_4(t) = (s_1(t) - s_4(t)) - (s_1'(t) - s_4'(t))$ as error sequences.

Assume that SNR is high enough so that error propagation can be ignored. The symbol errors occur independently, hence, the pairwise error probability (PEP) of the t_0-th symbol is bounded by

$$P\left(\mathbf{c}(t_0) \to \mathbf{e}(t_0)|\mathbf{H}(z)\right) \le \exp(-d^2(\mathbf{c}(t_0), \mathbf{e}(t_0))/2\sigma_n^2)$$

where

$$\begin{aligned} & d^2(\mathbf{c}(t_0), \mathbf{e}(t_0)) \\ =\ & norm(\widetilde{G}_1^*(1/z^*)\{\Delta_1(t_0)\})^2 \\ & + norm(\widetilde{G}_4^*(1/z^*)\{\Delta_4(t_0)\})^2 \\ =\ & \sum_{l=0}^{L}\Big[\left(|h_{1,l} + h_{4,l}|^2 + |h_{2,l} - h_{3,l}|^2\right)|\Delta_1(t_0)|^2 \\ & + \left(|h_{1,l} - h_{4,l}|^2 + |h_{2,l} + h_{3,l}|^2\right)|\Delta_4(t_0)|^2\Big] \end{aligned} \qquad (6)$$

If $h_{k,l}$'s are complex Gaussian distributed and both $\Delta_1(t_0)$ and $\Delta_4(t_0)$ are non-zero, an order of $4(L+1)$ diversity gain is obtained. This is due to the fact that the l-th ray multipath channel related quantities $\left(|h_{1,l} + h_{4,l}|^2 + |h_{2,l} - h_{3,l}|^2\right)$ and $\left(|h_{1,l} - h_{4,l}|^2 + |h_{2,l} + h_{3,l}|^2\right)$ are χ^2 distributed with 4 degrees of freedom.

4. SIMULATION RESULTS

In the computer simulated experiments, we transmit a block containing $4(P+1) = 120$ QPSK modulated data symbols.

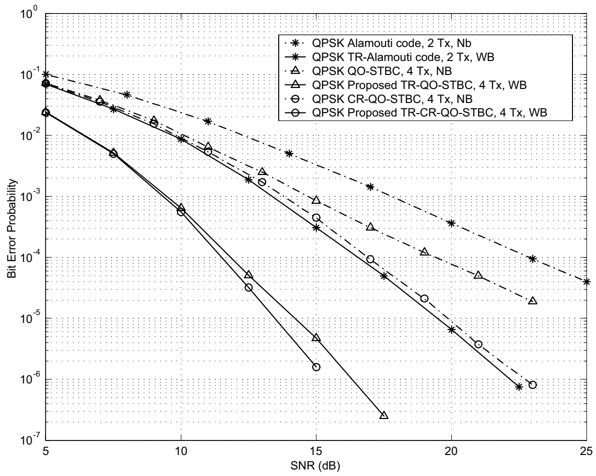

Fig. 2. BER versus SNR performance of time-inversal space-time block coding schemes in frequency selective channels compared with performance of space-time block coding schemes in non-frequency selective channels.

The system under consideration has multiple transmitters and one receiver. We considered a QO-STBC system with $N = 4$ transmitters in a frequency selective fading channel. As a performance reference, we also considered the Alamouti O-STBC system with $N = 2$ transmitters in a frequency *non-selective* fading channel. The block faded frequency selective channels are modeled as a 2 tap FIR filter with a delay of 1 symbol duration time; and the values of the tap coefficients are i.i.d. complex Gaussian distributed.

Figure 2 shows the performance of STBC transceiver systems using the constellation rotated and non-rotated QO-STBC (4×1 systems) in frequency selective environment, as well as the O-STBC (the Alamouti code, 2×1 system) in frequency non-selective environment. The same curve slopes at high SNRs indicate the same order of diversity for the corresponding encoding schemes. As expected, the constellation rotated QO-STBC shows better diversity property than the un-rotated quasi-orthogonal code. Figure 2 proves that block based time reversal conjugate space-time block coding technique can not only be used for *orthogonal* STBC, such as Alamouti code [6][7], but also be used for *quasi-orthogonal* scheme to obtain diversity gain from frequency selective channels. Compared with the narrowband scheme, the time reversal conjugate constellation rotated and non-rotated QO-STBC achieve SNR gains about 8 dB and 5 dB, repectively.

5. CONCLUSIONS

Applying the time reversal conjugate technique in combination with the QO-STBC schdeuling, we proposed a block based QO-STBC scheme to effective combat channel dispersion and frequency selective fading, and obtained diversity gain. The ML detector of the time-reversal QO-STBC can be constructed by a simple linear filter bank for space domain decoupling, a whitening filter bank and a vector MLSE detector. The coding rate of the time reversal QO-STBC is $L_B/4L$, where L_B is length of the data block. The encoding complexity is very low. The decoding complexity is determined by the computational complexity of vector MLSE detector, which increases exponentialy with the channel delay time. When channel dealy time is short, the ML detection can be accomplished at low complexity. As channel delay time is long, the decoding complexity can be reduced by using some sub-optimum detector instead of the optimum MLSE detector.

6. REFERENCES

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