Structure-Based Water-Filling Algorithm in Multipath MIMO Channels¹

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Abstract—Recent advances in information theory show that employing multiple antennas at both sides of a wireless channel promises enormous capacity potential. With perfect knowledge of channel state information (CSI) at the transmitter, eigenbeamforming is the optimal coding scheme to exploit this potential. However, in non-stationary wireless environments, high complexity on multiple-input-multiple-output (MIMO) channel tracking and large amounts of CSI feedback render such an approach impractical. In this letter, by exploiting the wireless multipath channel structure characterized by the path delays and the path directions-of-departure/arrival, a new space-time transmit scheme which employs a structure-based water-filling algorithm is proposed.

I. INTRODUCTION

Employing multiple antennas at both sides of a wireless channel is considered an effective way to improve the performance of a wireless communication system. Information theory of this perspective studied in [1] provides measures of significant increase in system capacity. Several space-time architectures [2], [3] were proposed to exploit this potential. With channel state information (CSI) at the transmitter, the space-time eigen-beamforming approach adopted in [3] is claimed to be the optimum coding scheme in terms of system capacity. However, eigenbeams are usually fast time-varying. High complexity on multiple-input-multiple-output (MIMO) channel tracking and large amounts of CSI feedback render such an approach impractical.

Alternatively, coding schemes looking into space-time channel structure should be more practical and robust. In [4], [5], spatially correlated fading channels are considered in investigating the effects of the spatial channel structure on the capacity of a wireless MIMO communication system. On the other hand, the frequency-selective channels in most of the research works on space-time coding so far are modeled as independently-fading time taps with no channel structure. Current and upcoming indoor wireless applications usually adopt higher carrier frequencies, e.g., 5GHz for wireless LAN 802.11a and even up to 60GHz in the future wireless standards, for obtaining more channel bandwidth and higher data throughputs. In such indoor wireless environments, measurement results [6], [7] clearly demonstrate strong multipath channel structures, which are highly dependent on signal propagation environments. Illustrated in Fig. 1 are the descriptions of the channel structures in two different environments: from one room to another room, and from the corridor to a room [7]. For instance, in Fig. 1(a), the power polar plot at the receiver in Lab 1 shows that the reflected path coming from the lab doors contributes the majority of the multipath energy. Measurement results also indicate that the path directions-ofdeparture/arrival (DODs/DOAs) of these dominant paths have a strong dependence on the path delays. Unlike the wireless channels in either urban or rural outdoor environments, in which strong clustering effects cause signal dispersion in time, the indoor wireless channel is characterized by reflections rather than by diffused scattering. Wireless channel measurements in office buildings also confirm that the variations of the dominant path delays are little and the variations of the dominant path DODs/DOAs are confined within limited angle spreads. In this letter, by exploiting such channel structures characterized by the path delays and the path DODs/DOAs, a new space-time transmit scheme which employs a structurebased water-filling algorithm is proposed. Outage capacity as the measure of achievable performance is evaluated by Monte Carlo methods to confirm the performance advantage of the proposed transmit scheme.

II. STRUCTURED MULTI-PATH MIMO CHANNELS

As illustrated in Fig. 1, a wireless channel can be aptly modeled by its dominant paths, each representing the majority energy of a path cluster. If we assume $M_{\rm T}$ transmit antennas and $M_{\rm R}$ receive antennas, the baseband signals arriving at the receive antenna array through the l^{th} dominant path can be expressed as

$$\mathbf{r}_{l}(t) = \mathbf{a}_{\mathrm{R}}(\theta_{\mathrm{R},l}) \cdot \beta_{l} \cdot \sum_{k=1}^{M_{\mathrm{T}}} [\mathbf{a}_{\mathrm{T}}(\theta_{\mathrm{T},l})]_{k} \tilde{s}_{k,l}(t) + \mathbf{n}(t), \quad (1)$$

where $\mathbf{a}_{\mathrm{R}}(\theta_{\mathrm{R},l})$ is the receive array steering vector of the l^{th} dominant path arriving from the path DOA $\theta_{\mathrm{R},l}$; β_l is the complex fading amplitude of the l^{th} dominant path, which is assumed to be a complex Gaussian random process; $[\mathbf{a}_{\mathrm{T}}(\theta_{\mathrm{T},l})]_k$ is the k^{th} entry of the transmit array steering vector corresponding to the path DOD $\theta_{\mathrm{T},l}$ of the l^{th} dominant path; $\mathbf{n}(t)$ is the additive complex white Gaussian noise; and $\tilde{s}_{k,l}(t)$ is the transmit baseband signal of the l^{th} path at the k^{th} transmit antenna element. With a linear modulation scheme, the transmit baseband signal $\tilde{s}_{k,l}(t)$ can be written as $\tilde{s}_{k,l}(t) = \sum_n g(t - nT - \tau_l) s_n^k$, where T denotes the symbol period; g(t) is the pulse-shaping function; τ_l is the propagation

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delay of the l^{th} path; and s_n^k is a space-time-coded symbol transmitted through the k^{th} transmit antenna element at each time instant n.

To simplify notation, the discrete-time pulse $g_l(n) \triangleq g(nT - \tau_l)$ is adopted as the sampled pulse-shaping function for the l^{th} path. If the summation of the support length of g(t)and the maximum path delay is V symbols long, we define \mathbf{G}_l as an $(N+V-1) \times N$ Toeplitz matrix with $[g_l(n) \ 0 \cdots 0]$ as its first row, and with $[g_l(n) \cdots g_l(n + V - 1) \ 0 \cdots 0]^T$ as its first column, where $(\cdot)^T$ denotes the matrix transpose operation. After sampling with symbol rate at the receiver, we can express the signals \mathbf{r} at the receive antenna array for all the L dominant paths as

$$\mathbf{r} = \sum_{l=1}^{L} \beta_l \left(\underbrace{(\mathbf{a}_{\mathrm{R}}(\boldsymbol{\theta}_{\mathrm{R},l}) \mathbf{a}_{\mathrm{T}}^{\mathrm{T}}(\boldsymbol{\theta}_{\mathrm{T},l}))}_{M_{\mathrm{R}} \times M_{\mathrm{T}}} \otimes \underbrace{\mathbf{G}_l}_{(N+V-1) \times N} \right) \cdot \underbrace{\mathbf{f}}_{(M_{\mathrm{T}}N) \times 1} \cdot \underbrace{\mathbf{s}}_{1 \times 1} + \mathbf{n}, \qquad (2)$$

where a codeword data burst of N time samples are considered; $\mathbf{r} = [r_n^1 \cdots r_{n+N+V-1}^1 \cdots r_n^{M_{\mathrm{R}}} \cdots r_{n+N+V-1}^{M_{\mathrm{R}}}]^T$; \otimes denotes the Kronecker product; $\mathbf{f} = [f_1^1 \ f_2^1 \cdots f_N^1 \ f_1^2 \cdots f_N^{M_{\mathrm{T}}}]^T$ is the transmit space-time weight vector with s being the input data and with $s_n^k = f_n^k \cdot s$; and \mathbf{n} is the additive complex white Gaussian noise stacked with the noise samples in the same way as the receive signal \mathbf{r} .

III. SPACE-TIME TRANSMIT SCHEME

In this section, by exploiting only the slow-varying part of the CSI that describes the multipath channel structure, we propose a new structure-based water-filling transmit scheme. The partial CSI considered here includes path DOAs $\{\theta_{R,l}\}$, path DODs $\{\theta_{T,l}\}$, path delays $\{\tau_l\}$, and the number of dominant path clusters L, all of which can be obtained through either TST-MUSIC in [8] or some of the references therein. Note that the influence of the fast time-varying complex fading amplitudes $\{\beta_l\}$ is averaged out and excluded. Also note that, based on the principle of channel reciprocity, at each transceiver, $\{\theta_{R,l}\}, \{\tau_l\}$, and L can be estimated at the receiver and used respectively as $\{\theta_{T,l}\}, \{\tau_l\}$, and L at the transmitter. The only feedback CSI required from the other end of the communication link is the path DODs $\{\theta_{T,l}\}$. Since what interests us is the path clusters that dominate the channel structure, channel measurements indicate that L is about 2-4 and the amount of the CSI feedback in the proposed algorithm is very limited. As for the conventional optimum transmit scheme [3], the algorithm constantly tracks the eigenbeams, which are fast time-varying. To obtain information of eigenbeams, the MIMO CSIs need to be estimated/tracked very frequently and then sent back to the transmitter through some feedback channels. As compared to the proposed algorithm, the conventional optimum approach is impractical because of its high complexity in tracking the fast-varying CSI and its large amount of CSI feedback.



Fig. 1. Indoor wireless measurements of the wireless channel characterized by the dominant paths [7]. (a) Propagation from one room to another room. (b) Propagation from the corridor to a room.

A. Receive Energy Maximization

From the transmitter's viewpoint, it intuitively intends to deliver as much energy to the receiver as possible. Additionally, it is observed in [9] that the average receive energy plays a crucial role in system performance. Therefore, let us consider receive energy maximization as our first-stage temporary goal now. From (2), the noise-free receive signal energy at the receiver, $|| \hat{\mathbf{r}} ||^2$, can be expressed as

$$\| \hat{\mathbf{r}} \|^{2} = \| \sum_{l=1}^{L} \beta_{l} \Big((\mathbf{a}_{\mathrm{R}}(\theta_{\mathrm{R},l}) \mathbf{a}_{\mathrm{T}}^{T}(\theta_{\mathrm{T},l})) \otimes \mathbf{G}_{l} \Big) \mathbf{f}_{s} \|^{2} .$$
(3)

Let $\mathbf{Q}_l \triangleq (\mathbf{a}_{\mathrm{R}}(\theta_{\mathrm{R},l})\mathbf{a}_{\mathrm{T}}(\theta_{\mathrm{T},l})^T) \otimes \mathbf{G}_l$. By taking expectation on *s* in (3), we can rewrite the array receive energy $\parallel \tilde{\mathbf{r}} \parallel^2$ as

where σ_s^2 is the variance of s, tr(·) denotes the trace of the argument matrix, and (·)* denotes the conjugate transpose operation. We then obtain the mean of the array receive energy $|| \tilde{\mathbf{r}} ||^2$ as

$$\mathcal{E}\{\| \tilde{\mathbf{r}} \|^2\} = \mathcal{E}\{\beta^* \mathcal{Q}\beta\} = \mathcal{E}\{\operatorname{tr}(\mathcal{Q}\beta\beta^*)\} \triangleq \operatorname{tr}(\mathcal{Q}\mathbf{R}_\beta), \quad (6)$$

where \mathbf{R}_{β} is the covariance matrix of β and $\mathcal{E}(\cdot)$ denotes the expectation operation.

To maximize the receive signal energy, we consider the following constrained problem

$$\mathbf{f}_{\text{opt}} = \operatorname{Arg} \operatorname{Max}_{\mathbf{f}} \operatorname{tr}(\mathcal{Q}\mathbf{R}_{\beta})$$
(7)
subject to $\operatorname{tr}(\sigma_s^2 \mathbf{f} \mathbf{f}^*) = E_s,$

where \mathbf{f}_{opt} is the optimum space-time weight vector for the design criterion, and the total transmit power $tr(\sigma_s^2 \mathbf{f} \mathbf{f}^*)$ is constrained to E_s . Next, by using a Lagrange multiplier λ , we define a quadratic expression $L(\mathbf{f})$ as

$$L(\mathbf{f}) = \operatorname{tr}(\boldsymbol{\mathcal{Q}}\mathbf{R}_{\boldsymbol{\beta}}) - \lambda \big(\operatorname{tr}(\sigma_s^2 \mathbf{f} \mathbf{f}^*) - E_s\big).$$
(8)

Let ρ_{ij} denotes the $(i, j)^{th}$ entry of \mathbf{R}_{β} , we can then rewrite $L(\mathbf{f})$ as

$$L(\mathbf{f}) = \sum_{i} \sum_{j} \rho_{ij} \cdot \sigma_s^2 \cdot \operatorname{tr}(\mathbf{Q}_i \mathbf{f} \mathbf{f}^* \mathbf{Q}_j^*) - \lambda(\operatorname{tr}(\sigma_s^2 \mathbf{f} \mathbf{f}^*) - E_s).$$
(9)

Next, differentiating $L(\mathbf{f})$ with respect to \mathbf{f} , we have

$$\frac{\partial L(\mathbf{f})}{\partial \mathbf{f}} = \sum_{i} \sum_{j} \rho_{ij} \cdot \sigma_s^2 \cdot (\mathbf{Q}_j^* \mathbf{Q}_i \mathbf{f}) - \lambda \cdot \sigma_s^2 \cdot \mathbf{f}.$$
 (10)

Let $\partial L(\mathbf{f})/\partial \mathbf{f} = 0$. Therefore,

$$\sum_{i} \sum_{j} \rho_{ij} (\mathbf{Q}_{j}^{*} \mathbf{Q}_{i}) \mathbf{f} = \lambda \mathbf{f}.$$
(11)

Since $\sum_{i} \sum_{j} \rho_{ij} (\mathbf{Q}_{j}^{*} \mathbf{Q}_{i})$ is Hermitian, \mathbf{f}_{opt} is thus the dominant eigenvector of $\sum_{i} \sum_{j} \rho_{ij} \cdot (\mathbf{Q}_{j}^{*} \mathbf{Q}_{i})$.

B. Structure-Based Water-Filling

Up to now, we have identified the best space-time eigenbeam, or the best subchannel, for the transmitter to transfer signal energy to the receiver. However, in the M_TN -dimensional signal space seen at the transmitter, if we transmit all the data through a single-dimensional eigenbeam, either the MIMO system will have a very limited throughput, or serious signal collision will be unavoidable. Parallel transmission is obviously an indispensable feature in the design of a MIMO system in order to maximize the data throughput. From the transmitter's viewpoint again, the *orthogonal* eigenvectors of (11) represent parallel subchannels to allocate energy for parallel data transmission. Therefore, we propose a heuristic water-filling solution for these subchannels as follows:

By applying eigen-decomposition to $\sum_{i} \sum_{j} \rho_{ij}(\mathbf{Q}_{j}^{*}\mathbf{Q}_{i})$ in (11),

$$\sum_{i} \sum_{j} \rho_{ij}(\mathbf{Q}_{j}^{*}\mathbf{Q}_{i}) = \mathbf{U} \operatorname{diag}(\lambda_{1}, \lambda_{2}, \cdots, \lambda_{M_{\mathrm{T}}N})\mathbf{U}^{*}, \quad (12)$$

we impose the water-filling solution based on $\{\lambda_i\}$ and then obtain the space-time weight vectors as

$$\mathbf{f}_{i} = \sqrt{p_{i}} \mathbf{u}_{i} \quad \text{with } p_{i} = \left(\nu - \frac{\sigma_{n}^{2}}{\lambda_{i}}\right)^{+}, \tag{13}$$
$$i = 1, \cdots, M_{\mathrm{T}}N, \sum_{i} p_{i} = \frac{E_{s}}{\sigma_{s}^{2}},$$

where p_i is the energy assigned to the i^{th} eigenmode; \mathbf{u}_i is the i^{th} column of U; ν is a constant chosen so that the total transmit energy, $\sigma_s^2 \sum_i p_i$, is equal to E_s ; σ_n^2 is the noise variance; and $(\cdot)^+$ is an operator that produces zero when its argument is negative. Note that the first-stage temporary goal pursued in Section 3-A provides us with a tool to identify the partial-CSI-based eigenbeams seen at the transmitter.

It should also be noted that, based on the criterion of maximizing the mutual information between the channel input signal and the channel output signal, [3], [4], [5] each also suggests similar but different water-filling solutions for MIMO systems. If we assume constant path fading amplitudes and ignore all the space-time structures, the proposed water-filling solution degenerates exactly to the solution provided by [3]. If we ignore both the temporal channel structure and the receive spatial channel structure, the proposed solution degenerates exactly to the solution degenerates exactly to the solution growided by [4]. Although both the transmit and the receive spatial channel structures are considered in [5], the water-filling solution provided in [5] not only ignores the temporal channel structure, but it is also too complex for real-time implementation.

C. Performance Measure

With $M_T N$ orthogonal transmit carriers $\{\mathbf{f}_i\}$ available at the transmitter, $M_T N$ data symbols $\{s_i\}$ can be transmitted simultaneously. Therefore, $\mathbf{f} \cdot s$ in (2) becomes

$$\mathbf{s} = \sum_{i} \mathbf{f}_{i} \cdot s_{i},\tag{14}$$

where the variance of s_i is σ_s^2 for all *i*. Assume that the input data $\{s_i\}$ are Gaussian and independent. The transmit signal vector, s, is then Gaussian with its covariance matrix being

$$\mathbf{R}_{\mathbf{s}} = \sigma_s^2 \mathbf{U} \cdot \operatorname{diag}(p_1, \cdots, p_{M_{\mathrm{T}}N}) \cdot \mathbf{U}^*.$$
(15)

With perfect CSI, $\mathbf{H} = \sum_{l=1}^{L} \beta_l ((\mathbf{a}_{\mathrm{R}}(\theta_{\mathrm{R},l}) \mathbf{a}_{\mathrm{T}}(\theta_{\mathrm{T},l})^T) \otimes \mathbf{G}_l)$, known at the receiver, the mutual information between the channel input signal and the channel output signal is given by

$$C = \log_2 \det(\mathbf{H}^* \mathbf{R}_{\mathbf{n}}^{-1} \mathbf{H} \cdot \sigma_s^2 \mathbf{U} \operatorname{diag}(p_1, \cdots, p_{M_{\mathrm{T}}N}) \cdot \mathbf{U}^* + \mathbf{I}_{M_{\mathrm{T}}N}), \qquad (16)$$

where $\mathbf{R_n}$ is the space-time noise covariance matrix. Outage capacity $C_{\text{out}}(p)$,

$$\operatorname{Prob}\{ C < C_{\operatorname{out}}(p) \} = p, \tag{17}$$

as the measure of the achievable performance is evaluated with Monte Carlo methods in the next section. We focus on the 10% outage capacity, i.e., $C_{\text{out}}(0.1)$.

IV. NUMERICAL EXPERIMENTS

Through computer simulation, we evaluate the link performance of the proposed structure-based water-filling transmit scheme. In the simulation scenarios, the communication system is equipped with a five-element transmit ULA (uniform linear array) and a five-element receive ULA, the antenna elements in both of which are spaced half a wavelength apart. We generate one hundred representative channels with



Fig. 2. Capacity complementary cumulative distribution functions of various space-time transmit schemes with the input data temporal block size N being 2 symbols.

different combinations of path DODs, path DOAs, and path delays, each with 10,000 channel path fading amplitude realizations. In each wireless multipath channel, there are four dominant paths, and the fading amplitudes are assumed to be independent among the paths. The sampled pulse-shaping function $g(nT-\tau_l)$ is a raised-cosine pulse with symbol period 1.8 μ s and roll off factor 1. The duration of the pulse-shaping function is truncated to six symbols long.

Shown in Figs. 2 and 3 are the complementary cumulative distribution functions (CCDF) of the channel capacities for the communication link employing different space-time transmit schemes. Three algorithms are tested, denoted by perfect CSI, partial CSI, and no CSI at the transmitter. The case with perfect CSI is the system employing the optimum eigen-beamforming solution [3]. The case with partial CSI represents the proposed system. The case with no CSI denotes a system where the transmit signal is Gaussian distributed with covariance $\frac{E_s}{M_TN}$ I, i.e. the total transmit power E_s is evenly distributed among the M_TN signal symbols $\{s_i\}$. The input data block sizes considered in Figs. 2 and 3 are N = 2 and N = 3, respectively. The SNRs, defined as $\frac{E_s}{N_0}$, in both Figs. 2 and 3 are 5 dB, 10 dB, and 15 dB.

In Figs. 2 and 3, we observe that, for over 90% of the channels, the capacities of the proposed transmit scheme at SNR=15 dB are 0.8 and 1.6 bits, respectively, less than the optimum transmit scheme employing eigen-beamforming based on the perfect CSI at the transmitter. Compared with the transmit scheme without CSI at the transmitter, by exploiting the partial CSI, the proposed transmit scheme has 3.1 bits and 4.4 bits capacity advantage at SNR=15 dB, respectively, in Figs. 2 and 3. Furthermore, it is also observed that, at a higher SNR, there is a larger 10%-outage-capacity gap between the proposed transmit scheme and the one without CSI at the transmitter. Another interesting observation worth pointing out is that, by increasing the block size of the input data vector, the



Fig. 3. Capacity complementary cumulative distribution functions of various space-time transmit schemes with the input data temporal block size N being 3 symbols.

10%-outage-capacities of all the transmit schemes increase. That is due to the increase of the number of the effective transmit eigenmodes.

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