DECISION FEEDBACK DETECTION AND CHANNEL TRACKING FOR SPACE-TIME BLOCK CODED TRANSMISSION SYSTEMS OVER TIME-VARYING CHANNELS

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ABSTRACT

This paper proposes a new decision feedback decoding scheme for Alamouti-based space-time block coding (STBC) transmission over time-selective fading channels. In wireless channels, time-selective fading effects arise mainly due to Doppler shift and carrier frequency offset. Modelling the time-selective fading channels as the first-order Gauss-Markov processes, we use recursive algorithms such as Kalman filtering, LMS and RLS algorithms for channel tracking. The proposed scheme consists of the symbol decoding stage and channel tracking algorithms. Computer simulations confirm that the proposed scheme shows the better performance and robustness to time-selectivity.

1. INTRODUCTION

Alamouti discovered a remarkable space-time block coding (STBC) scheme for transmission with two transmit antennas achieves full diversity gains using a linear maximum-likelihood (ML) decoder [1]. Alamouti's STBC has been adopted in several wireless standards such as IS-136, WCDMA, and CDMA-2000. Most STC schemes rely on accurate channel estimation, which may require the insertion of many pilot symbols when the channels are highly time-varying. While differential STC (DSTC) schemes have been developed for slowly time-varying channels [2]. Moreover, double differential STC (DDSTC) offers a simple and robust means of handling channel time-selectivity but loses 6dB in performance [3].

Most existing STC schemes have been developed for flat fading channels. Different from [4], we consider here more realistic time-selective but frequency-flat fading channels. In wireless mobile communications, time selectivity is mainly caused by Doppler shifts and carrier frequency offsets, which are jointly independent. Information theoretic results have been shown that the first-order Gauss-Markov random processes provides a accurate model for time-selective fading channels, and, therefore, this channel models will be adopted in this paper. The problem of channel tracking for STBC was also investigated in [4].

In this paper, we investigate the impact of time-selective fading channels on the performance of the transmit-diversity scheme proposed by Alamouti. We propose a new decision feedback detection scheme for Alamouti-based STBC transmission over time-selective fading channels. We model time-selective fading channels as the first-order Gauss-Markov processes. We then apply recursive algorithms (LMS, RLS, Kalman filtering) to track the channel variations and decode the transmitted symbols with diversity gains. A detaied description of how to link the symbol decoding stage with the channel tracking stage is also presented. Simulation results confirm that the proposed scheme shows the better performance and robustness to time-selectivity. Most notations are standard: vectors and matrices are boldface small and capital letters, respectively; the matrix transpose and the Hermitian are denoted by $(\cdot)^T$ and $(\cdot)^H$, respectively; $E[\cdot]$ is the statistical expectation.

2. SYSTEM AND CHANNEL MODEL

Consider a wireless system equipped with two transmit antennas and one receive antenna as shown in Fig. 1, where the information symbols $s(n)$ are transmitted using Alamouti's space-time block encoder. Different from previous work [1] where the channels are assumed flat fading, we consider time-selective but frequency-flat fading channels. Denote by $h_i(n),\ i = 1, 2$, the time-selective fading channel from the ith transmit antenna to the receive antenna. At the receive antenna, the two successive received samples $y(2n)$ and $y(2n+1)$ are given by

$$\begin{aligned} y(2n) &= h_1(2n)s(2n) + h_2(2n)s(2n+1) + w(2n) \\ y(2n+1) &= -h_1(2n+1)s^*(2n+1) + h_2(2n+1)\cdot \\ &\quad s^*(2n) + w(2n+1) \end{aligned} \tag{1}$$

where the additive noise $w(n)$ is complex Gaussian distributed with zero-mean and variance $\sigma_w^2/2$ per dimension. We assume that data, channels, and noise are jointly independent. Among various channel model, the information theoretic results in [5] have shown that the first-order Gauss-Markov process provides a accurate model for time-selective fading channels and, therefore, will be adopted henceforth. The dynamics of the channel state $h_i(n)$ are modeled by

$$h_i(n) = \alpha h_i(n-1) + v_i(n) \tag{2}$$

where the $v_i(n)$ is the white complex Gaussian with zero-mean and covariance $\sigma_v^2/2$ per dimension and is statistically independent of $h_i(n-1)$. Parameter $\alpha \in [0, 1]$ is the fading correlation coefficient that characterizes the degree of time variations; small α models fast fading and large α corresponds to slow fading. The first-order Gauss-Morkov model is parameterized by the fading correlation coefficient α, which depends on the channel Doppler spread, and can be accurately obtained in [5]. In wireless mobile communications, channel time-varying characteristics arise

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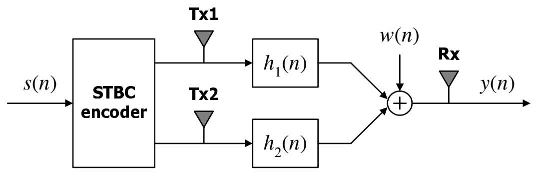

Fig. 1. Space-time block coded transmission diagram.

mainly due to Doppler shifts arising from relative motion between the transmitter and the receiver, and the carrier frequency offsets due to the transmitter-receiver oscillators' mismatch. Denote by f_o the carrier frequency offset and by T_s the symbol duration. We can factorize $h_i(n)$ into

$$h_i(n) = \bar{h}_i(n) e^{j2\pi f_o T_s n}. \tag{3}$$

where $\bar{h}_i(n)$ and $e^{j2\pi f_o T_s n}$ account for the Doppler and the carrier frequency offset effects, respectively. Assume that $h_i(n)$ is complex Gaussian distributed with zero-mean and unit-variance, we know that

$$\sigma_v^2 = 1 - |\alpha|^2, \;\; \alpha = E[h_i(n) h_i^*(n-1)]. \tag{4}$$

According to the Jakes' model [6], time-varying channel $\overline{h}_i(n)$ is zero-mean Complex Gaussian process, and has time-autocorrelation properties governed by the Doppler rate $f_d T_s$ as in

$$E[\bar{h}_i(n) \bar{h}_i^*(n-1)] = J_0(2\pi f_d T_s) \tag{5}$$

where $J_0(\cdot)$ is the zeroth-order Bessel function of the first kind and f_d denotes the maximum Doppler shift. Recalling (4), the α is related to f_d, T_s and f_o as following

$$\alpha = J_0(2\pi f_d T_s) e^{j2\pi f_o T_s}. \tag{6}$$

3. PROPOSED DECISION FEEDBACK DETECTOR AND CHANNEL TRACKING ALGORITHMS

The receiver observations $y(2n)$ and $y(2n+1)$ corresponding to the two symbol periods are given by

$$\mathbf{y}(n) = \mathbf{H}(n)\mathbf{s}(n) + \mathbf{w}(n) \tag{7}$$

where, $\mathbf{y}(n) = [y(2n)\; y^*(2n+1)]^T$; $\mathbf{s}(n) = [s(2n)\; s(2n+1)]^T$; $\mathbf{w}(n) = [w(2n)\; w(2n+1)]^T$; and the channel matrix

$$\mathbf{H}(n) = \begin{bmatrix} h_1(2n) & h_2(2n) \\ h_2^*(2n+1) & -h_1^*(2n+1) \end{bmatrix}. \tag{8}$$

Because of the white Gaussian noise, the joint maximum-likelihood (ML) detector choose the pair of symbol $\mathbf{s}(n)$ to minimize

$$\| \mathbf{y}(n) - \mathbf{H}(n)\mathbf{s}(n) \|^2 . \tag{9}$$

To decode $\mathbf{s}(n)$, the space-time block decoder is designed by forming the two consecutive output sample vector, $\mathbf{z}(n) = [z(2n)\; z(2n+1)]^T$, as

$$\mathbf{z}(n) = \mathbf{H}^H(n)\mathbf{y}(n). \tag{10}$$

Based on the definition (8), it follows by

$$\mathbf{R}(n) = \mathbf{H}^H(n)\mathbf{H}(n) = \begin{bmatrix} \rho_1(n) & \epsilon(n) \\ \epsilon^*(n) & \rho_2(n) \end{bmatrix} \tag{11}$$

where, $\rho_1(n) = |h_1(2n)|^2 + |h_2(2n+1)|^2$, $\rho_2(n) = |h_1(2n+1)|^2 + |h_1(2n)|^2$, and $\epsilon(n) = h_1^*(2n)h_2(2n) - h_1^*(2n+1)h_2(2n+1)$. Using (11), we know that

$$\mathbf{z}(n) = \begin{bmatrix} \rho_1(n) & 0 \\ 0 & \rho_2(n) \end{bmatrix} \mathbf{s}(n) + \begin{bmatrix} 0 & \epsilon(n) \\ \epsilon^*(n) & 0 \end{bmatrix} \mathbf{s}(n) + \mathbf{H}^H(n)\mathbf{w}(n). \tag{12}$$

The first part in (12) contains the maximum ratio combined signals from the two transmit antennas whereas the second part contains inter-symbol-interference (ISI) on the off-diagonal elements caused by time-selective channels.

Bit-error-rate (BER) performance analysis of the detector (12) is possible for a given constellation under perfect channel knowledge. We first obtain from (12),

$$\begin{aligned} z(2n) \; = \; & \rho_1(n)s(2n) + \epsilon(n)s(2n+1) \\ & + h_1^*(2n)w(2n) + h_2(2n+1)w^*(2n+1). \end{aligned} \tag{13}$$

Treating the interference as noise and after some mathematical manipulation about $h_1(2n)$ and $h_2(2n+1)$, we compute the instantaneous SNR $\gamma(n)$ as following

$$\gamma(n) = \frac{0.5\rho_1^2(n)E_s}{[\rho_1(n)\sigma_v^2 + \{\kappa(n) - \rho_1(n)\}\sigma_v^4]E_s + \rho_1(n)\sigma_w^2} \tag{14}$$

where E_s denote the symbol energy of $s(n)$ and $\kappa(n) = |h_1(2n)|^2 \cdot |h_2(2n)|^2$. In deriving (14), we divided the transmit power by two for each transmit antenna because each symbol is transmitted twice. The derivation of (14) is given in Appendix. A similar equation can be obtained for $s(2n+1)$. Supposing that QPSK modulation is used, the BER $P_b(n)$ can be expressed as following

$$P_b(n) = Q\left(\sqrt{\gamma(n)}\right). \tag{15}$$

where $Q(x)$ is the Q-fuction defined as $Q(x) = \frac{1}{\sqrt{2}} \int_x^\infty e^{-t^2/2} dt$. We neglecting the fourth order term σ_v^4 (≈ 0). When $E_s \gg \sigma_w^2$ (high SNR), we observe from (14) that $P_b(n)$ does not increase with E_s but approaches an error floor given by

$$P_b(n) = Q\left(\sqrt{\frac{\rho_1(n)}{2\sigma_v^2}}\right). \tag{16}$$

In time-selective fading channels, Alamouti-based decoding scheme has an error floor caused by interference as shown in (12). In order to remove this error floor, we do not model the interference in (12) as noise, but treat it as ISI and propose the decision feedback detector, to decode $\mathbf{s}(n)$ from $\mathbf{y}(n)$, at the cost of smaller receiver complexity.

From (11), we know that $\mathbf{R}(n)$ is Hermitian, therefore, it has a unique Cholesky factorization of the form $\mathbf{R}(n) = \mathbf{G}^H(n)\mathbf{G}(n)$, where $\mathbf{G}(n)$ is lower triangular with real diagonal element. With $\mathbf{H}(n)$ defined by (8), we can verify easily that

$$\mathbf{G}(n) = \frac{1}{\sqrt{\rho_2(n)}} \begin{bmatrix} \rho_0(n) & 0 \\ \epsilon^*(n) & \rho_2(n) \end{bmatrix}$$

where $\rho_0(n) = |h_1(2n)h_1^*(2n+1) + h_2(2n)h_2^*(2n+1)|$. Multiplying both vectors in (9) by the unitary matrix $\left[\mathbf{H}(n)\mathbf{G}^{-1}(n)\right]^H$, we find that the ML detector can be equivalent choose $\mathbf{s}$ to minimize

$$\| \mathbf{x}(n) - \mathbf{G}(n)\mathbf{s}(n) \|^2 . \tag{17}$$

Substituting (7), we find that the output $\mathbf{x}(n)$ is related to $\mathbf{s}(n)$ by

$$\mathbf{x}(n) = [\mathbf{H}(n)\mathbf{G}^{-1}(n)]^H \mathbf{y}(n) = \mathbf{G}(n)\mathbf{s}(n) + \mathbf{n}(n) \tag{18}$$

where the white Gaussian noise $\mathbf{n}(n)$ has the same statistics as $\mathbf{w}(n)$. The decision feedback detector uses a decision about $s(2n)$ to help make a decision about $s(2n+1)$. Because the channel model $\mathbf{G}(n)$ is lower triangular, there is no interference from $s(2n+1)$ to $x(2n)$, and thus a suboptimal decision $\hat{s}(2n)$ can be found by quantizing $x(2n)$ as following.

$$x(2n) = \frac{\rho_0(n)}{\sqrt{\rho_2(n)}}\, s(2n) + n(2n). \tag{19}$$

Then, assuming this decision is correct, the contribution from $s(2n)$ in $x(2n+1)$ can be recreated and subtracted off, allowing the receiver to determine the decision $\hat{s}(2n+1)$ by quantizing the resulting difference D as following

$$\begin{aligned} D \;&=\; x(2n+1) - \frac{\epsilon^*(n)}{\sqrt{\rho_2(n)}}\, \hat{s}(2n) \\ &=\; \sqrt{\rho_2(n)} s(2n+1) + n(2n+1). \end{aligned} \tag{20}$$

Let us analyze the BER performance of the proposed decision feedback detector. The BER is thus again of the form $P_1(n) = Q(\sqrt{\gamma_1(n)})$ and $P_2(n) = Q(\sqrt{\gamma_2(n)})$ for QPSK modulation, where $\gamma_1(n)$ and $\gamma_2(n)$ are the effective instantaneous SNR for (20) and (21), respectively:

$$\gamma_1(n) = \frac{\rho_0^2(n)E_s}{2\rho_2(n)\sigma_w^2}, \;\; \gamma_2(n) = \frac{\rho_2(n)E_s}{2\sigma_w^2} . \tag{21}$$

Let us express the average BER of the proposed decision feedback detector. Combining, we can obtain the average BER $P_b(n)$ as following

$$P_b(n) = \frac{1}{2}\left[Q\left(\sqrt{\gamma_1(n)}\right) + Q\left(\sqrt{\gamma_2(n)}\right)\right]. \tag{22}$$

To attain the proposed decoding scheme, accurate estimates of the channel must be available at the receiver. The estimated channel at time n will be written in vector form as following

$$\hat{\mathbf{h}}(n|n) = [\hat{h}_1(n|n) \;\; \hat{h}_2(n|n)]^T \tag{23}$$

where $\hat{\mathbf{h}}(n|m)$ is the predicted channel at time n based on the observation at time m. Let us define the state vector $\mathbf{h}(n) = [h_1(n) \;\; h_2(n)]^T$ and rewrite (2) to arrive at the state equation as following

$$\mathbf{h}(n) = \mathbf{A}\mathbf{h}(n-1) + \mathbf{v}(n) \tag{24}$$

where $\mathbf{A} = \text{diag}(\alpha, \alpha)$ and $\mathbf{v} = [v_1(n) \;\; v_2(n)]^T$. At time n, the received signal $y(n)$ is given by

$$y(n) = \bar{\mathbf{s}}^T(n)\mathbf{h}(n) + w(n) \tag{25}$$

where $\bar{\mathbf{s}}^T(n) = [s(n) \;\; s(n+1)]^T$ when n is even and $\bar{\mathbf{s}}^T(n) = [-s^*(n+1) \;\; s^*(n)]^T$ when n is odd. Assuming that $\mathbf{A}$ is known from a preceding training mode and assuming the vector of the most recent available decision $\hat{\mathbf{s}}(n)$ to be equal to the true $\mathbf{s}(n)$. In the training mode, the receiver knows the transmitted symbols, whereas in the decision-directed mode, the decoded symbols replace the information symbol. We will focus on the decision-directed mode and assume that initial channel estimates are available by using techniques developed in [7]. In the decision-directed mode, the prediction of the channels may not be accurate. In this case, we can use the previous estimates and tentative channel prediction can be expressed as following

$$\begin{aligned} \hat{\mathbf{h}}(2n|2n-1) \;&=\; \alpha\hat{\mathbf{h}}(2n-1|2n-1) \\ \hat{\mathbf{h}}(2n+1|2n-1) \;&=\; \alpha^2\hat{\mathbf{h}}(2n-1|2n-1) \end{aligned} \tag{26}$$

that are initialized by $\mathbf{h}(1|1)$, which is obtained during the training mode.

The receiver can use the Kalman filter to track the channel variations $\mathbf{h}(n)$. The algorithm may be summarized as following

$$\begin{aligned} &\hat{\mathbf{h}}(n|n-1) = \mathbf{A}\hat{\mathbf{h}}(n-1|n-1) \\ &\mathbf{M}(n|n-1) = \mathbf{A}\mathbf{M}(n-1|n-1)\mathbf{A}^T + \mathbf{Q} \\ &\mathbf{K}(n) = \frac{\mathbf{M}(n|n-1)\bar{\mathbf{s}}(n)}{\sigma_w^2 + \bar{\mathbf{s}}(n)^T\mathbf{M}(n|n-1)\bar{\mathbf{s}}(n)} \\ &\hat{\mathbf{h}}(n|n) = \hat{\mathbf{h}}(n|n-1) + \mathbf{K}(n)[y(n) - \bar{\mathbf{s}}^T(n)\hat{\mathbf{h}}(n|n-1)] \\ &\mathbf{M}(n|n) = [\mathbf{I} - \mathbf{K}(n)\bar{\mathbf{s}}^T(n)]\mathbf{M}(n|n-1) \end{aligned} \tag{27}$$

where the estimator of $\mathbf{h}(n)$ based on $\{y(n)\}_{i=0}^m$ is $\hat{\mathbf{h}}(n|m)$, $\mathbf{M}(n|n-1)$ is the one-step minimum prediction mean-square error (MSE) at time n, and $\mathbf{M}(n|n)$ is the minimum MSE (MMSE) at time n. $\mathbf{K}(n)$ is the Kalman gain. Moreover, we can use the well-known adaptive algorithms for channel variation tracking. LMS and RLS algorithms are summarized as following, respectively.

$$\begin{aligned} e(n) \;&=\; y(n) - \bar{\mathbf{s}}^T(n)\hat{\mathbf{h}}(n|n-1) \\ \hat{\mathbf{h}}(n|n) \;&=\; \hat{\mathbf{h}}(n|n-1) + \mu e(n)\bar{\mathbf{s}}(n) \end{aligned} \tag{28}$$

RLS algorithm is summarized as following

$$\begin{aligned} e(n) \;&=\; y(n) - \bar{\mathbf{s}}^T(n)\hat{\mathbf{h}}(n|n-1) \\ \mathbf{K}(n) \;&=\; \frac{\lambda^{-1}\mathbf{P}(n-1)\bar{\mathbf{s}}^T(n)}{1 + \lambda^{-1}\bar{\mathbf{s}}^T(n)\mathbf{P}(n-1)\bar{\mathbf{s}}(n)} \\ \mathbf{P}(n) \;&=\; \lambda^{-1}\mathbf{P}(n-1) - \lambda^{-1}\mathbf{K}(n)\bar{\mathbf{s}}^T(n)\mathbf{P}(n-1) \\ \hat{\mathbf{h}}(n|n) \;&=\; \hat{\mathbf{h}}(n|n-1) + e(n)\mathbf{K}(n). \end{aligned} \tag{29}$$

4. SIMULATION RESULTS

In all simulations, we use the same parameters as shown in [4] for fair comparison with the scheme therein. We use (3) to generate $h_i(n)$. The generation of $\bar{h}_i(n)$ follows the Jakes model [6] with the parameters f_d and T_s corresponding to a carrier frequency of 1.9 GHz, a mobile speed of 250 km/h, and a transmission rate of 144 kb/s. QPSK modulation is considered. We simulate the performance of our proposed detection scheme using BER as performance index which we average over 5000 channel and noise realizations for each SNR point. Moreover, in order to avoid divergence in recursive algorithms, similar to [4], we insert one pilot symbol every 12 symbols which introduces 8% bandwidth efficiency loss.

In order to show the importance of channel tracking in time-selective fading channels, we test channel tracking algorithms for channel variations, and compare the BER performane with and

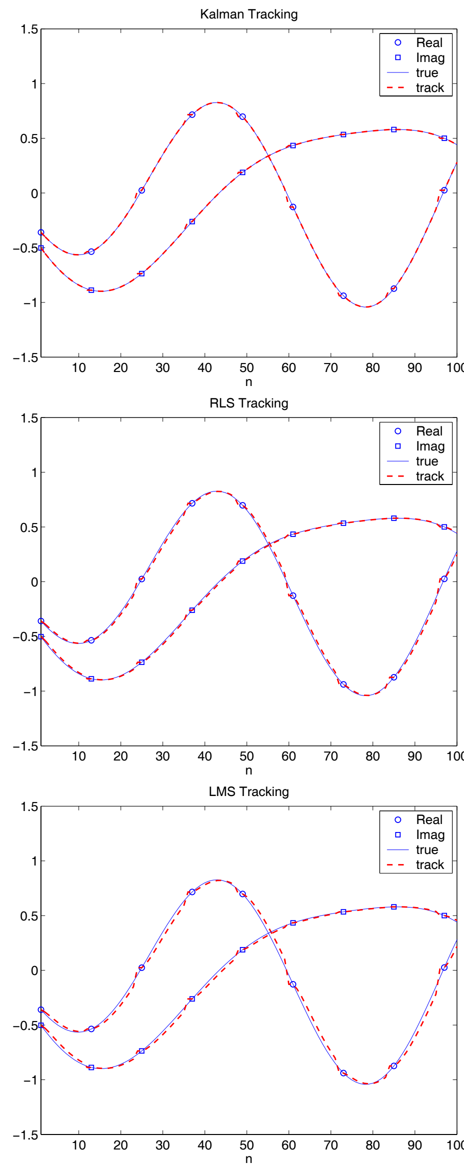

Fig. 2. True and estimate of channel variations.

without channel tracking, when the carrier frequency offset f_o = 1000 Hz. Fig. 2 shows the true channels and their corresponding tracked values in the case of time-selective channels. We observe that RLS algorithm gives good tracking results, LMS algorithm can find the channel but is loosing the tracking. Moreover, it is observed that Kalman filtering yields excellent tracking results. Fig. 3 confirms that channel tracking improves the BER performance. As compared to the performance when channels are perfectly known, the results in Fig. 3 implies that even small channel tracking errors could induce performance loss, because channel tracking errors cause ISI between the two transmit antennas for the Alamouti-based decoding scheme. However, decision feedback detection scheme shows the better performance than Alamouti-based decoding scheme.

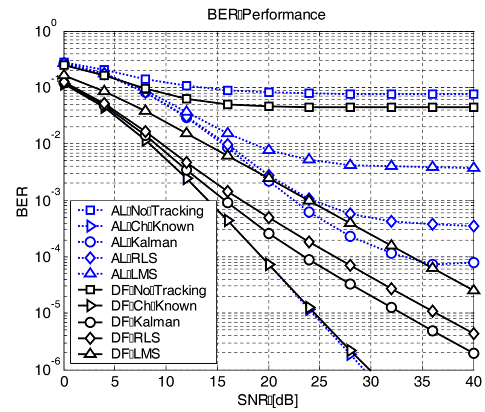

Fig. 3. BER improvement with channel tracking for the decision feedback detection scheme (AL and DF denote the Alamouti-based and our decision feedback detection scheme).

5. CONCLUSION

We proposed a new decision feedback detection scheme for Alamouti transmit diversity scheme in time-selective fading channels. Modelling time-selective channels as the first-order Gauss-Markov processes, several recursive algorithms have been employed to track their time variations. It has been shown by simulations that good channel tracking performance improves the BER performance and our proposed detection scheme has better performance than the Alamouti-based scheme.

6. REFERENCES

[1] S. M. Alamouti, “A simple transmit diversity technique for wireless communications,” *IEEE J. Select. Areas Commun.*, vol. 16, no. 8, pp. 1451-1458, Oct. 1998.

[2] V. Tarokh and H. Jafarkhani, “A differential detection scheme for transmit diversity,” *IEEE J. Select. Areas Commun.*, vol. 18, no. 7, pp. 1169-1174, July 2000.

[3] Z. Liu, G. B. Giannakis, and B. L. Huges, “Double differential space-time block coding for time-varying fading chanenls,” *IEEE Trans. Commun.*, vol. 49, no. 9, pp. 1529-1539, Sept. 2001.

[4] Z. Liu, X. Ma, and G. B. Giannakis, “Space-time coding and Kalman filtering for time-selective fading channels,” *IEEE Trans. Commun.*, vol. 50, no. 2, pp. 183-186, Feb. 2002.

[5] H. Wang and P. Chang, “On the verifying the first-order Markovian assumption for a Rayleigh fading channel model,” *IEEE Trans. Veh. Technol.*, vol. 45, pp. 353-357, May 1996.

[6] W. C. Jakes, *Microwave Mobile Communication*. New York, NY: Wiley, 1974.

[7] N. Al-Dhahir, et al., “Space-time processing for broadband wireless access,” *IEEE Commun. Mag.*, vol. 40, no. 9, pp. 136-142, Sept. 2002.

[8] S. Haykin, *Adaptive Filter Theory*, 4th ed. Elglewood Cliffs, NJ: Prentice-Hall, 2002.