BLIND ESTIMATION OF CHANNEL BERS IN A MULTI-RECEIVER NETWORK

Amitabh Dixit, Scott C. Douglas, and Geoffrey C. Orsak

Department of Electrical Engineering
Southern Methodist University
Dallas, Texas 75275 USA
Email: amitabh, douglas, gorsak@engr.smu.edu

ABSTRACT

In wireless communications, it is often desirable to merge bit decisions from multiple receivers to improve overall link performance. It is well known that in order to optimally fuse bit decisions from a network of receivers, precise knowledge of receiver bit error rates (BERs) is needed. This information, however, is rarely available in practice. In this work, we present an iterative procedure for blindly estimating receivers BERs to enable near optimal blind fusion of bit decisions in a multi-receiver network. We show that the solution of the estimation problem is a structured eigenvalue task and propose a modified power method procedure to perform it. We prove that the desired solution is a stable point of the algorithm and the algorithm is locally stable. Furthermore, we show via simulations that the technique results in excellent performance in nearly all practical operating scenarios.

1. INTRODUCTION

In wireless communications networks, one often wishes to merge data from various receivers to improve overall link performance. There are two well-known alternatives for doing this task: (1) combine sufficient statistics from each receiver using standard signal processing approaches, or (2) fuse raw bit decisions from each receiver into a final bit decision. Owing to its simplicity and utility in communications applications, we focus on the latter approach herein.

It is well known that in order to fuse bit decisions from multiple receivers, precise knowledge about the receiver BERs is required [1]. This information, however, is generally unavailable to us and moreover, in most wireless communications systems, this information is dynamically changing. Hence, to be able to implement bit fusion in practical systems, a low complexity algorithm with fast convergence is desired.

An algorithm was presented in [2] which blindly estimates the BER associated with each individual link in a multi-receiver network. Also, an empirical fusion algorithm was presented which uses BER estimates to blindly fuse bit decisions from multiple receivers. It was shown via simulations that the two completely blind algorithms working in tandem outperformed the best receiver (minimum BER receiver) in the network and the standard majority rule receiver for as few as 500 bit observations.

Although the algorithms in [2] achieve near optimal performance for reasonably small number of observations, the performance of the estimation algorithm can suffer drastically when sufficient observations are unavailable. This is mainly attributed to the fact that this algorithm utilizes only a subset of information (statistics) available and discards the rest. This discarded information can be critical specially when the observations are scarce.

In this work, we propose a new iterative algorithm that intelligently uses the set of statistics discarded in [2] to reduce the number of observations needed in order to form reliable BER estimates and also to lower the variance in the estimates. The price paid for using these extra statistics is that a closed form solution to the above estimation problem is no longer possible and an iterative procedure needs to be formulated. However, this price is worth paying in systems where fast convergence is desired. For example, for the 10 Mbps Ethernet having a 1 km maximum length specification, the minimum packet size is 200 bits. Similarly, the G723.1 codec and LPC10 codec can have packet sizes less than 500 bits. Finally, ATM packets are also less than 500 bits in size. In such systems, the algorithm in [2] would fail to give satisfactory performance.

This paper is organized as follows: In Section 2, we formalize the problem statement. In Section 3, we present a low complexity iterative algorithm that blindly estimates the receiver BERs. In Section 4, we analyze the local stability of the proposed iterative algorithm. In Section 5, we compare the performance of the proposed algorithm to that of algorithm in [2], and finally in Section 6, we conclude the paper.

2. PROBLEM STATEMENT

The problem setup shown in Fig. 1 depicts a network of N receivers where all receivers observe the same information through independent channels. Herein, we assume the following: (1) the receivers make independent bit decisions on their observations, (2) each channel is binary symmetric, and (3) the transmitted bits have equal *a-priori* probabilities.

Each receiver makes bit decisions based only on its observations and passes them onto a central processor called the fusion receiver. The fusion receiver combines the bit estimates from the individual receivers into one final bit decision according to a fusion rule.

If the BER of the i^{th} receiver is $P_e(i)$, then the optimal fusion rule can be expressed as

$$\sum_{i=1}^{N}(1-2\hat{b}_i)\ln\frac{P_e(i)}{1-P_e(i)} \underset{H_0}{\overset{H_1}{\gtrless}} 0$$

However, in practical systems, only the estimates of $P_e(i)$ available and the optimal fusion rule cannot be implemented. For such cases, the empirical fusion rule is obtained by simply replacing

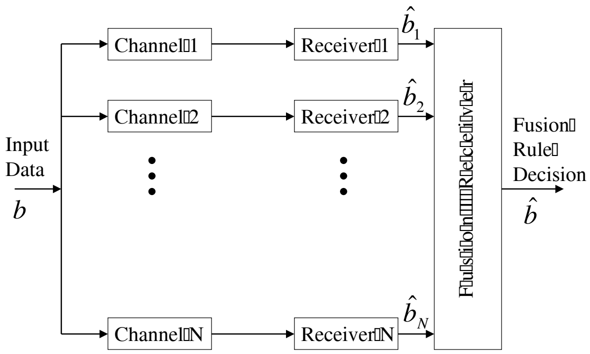

Fig. 1. Blind bit fusion setup.

$P_e(i)$ in the optimal fusion rule with its estimate $\widehat{P_e(i)}$ and is given as

$$\sum_{i=1}^{N}(1-2\hat{b}_i)\ln\frac{\widehat{P_e(i)}}{1-\widehat{P_e(i)}} \underset{H_0}{\overset{H_1}{\gtrless}} 0$$

Hence, a low complexity, fast converging algorithm for BER estimation is needed in order to implement the empirical fusion rule in practical systems.

3. THE MODIFIED POWER METHOD ALGORITHM

In this section, we present the modified power method procedure to blindly estimate receiver BERs. Let M be the total number of bits observed by each receiver and let $p_{ij} = 1 - P_e(i) - P_e(j) + 2P_e(i)P_e(j)$ be the probability that receiver i and receiver j make the same decision on a received bit. Then, an estimate α_{ij} of p_{ij} is simply the fraction out of the total of M times, the two receivers make the same decision i.e.

$$\alpha_{ij} = \frac{\sum_{k=1}^{M} x_k^{ij}}{M}; \quad i = 1, 2, \ldots, N; j = i+1, \ldots, N$$

where

$$x_k^{ij} = \begin{cases} 1 & \text{if in the } k^{th} \text{ bit observation, receiver } i \\ & \text{and receiver } j \text{ make the same decision} \\ 0 & \text{if in the } k^{th} \text{ bit observation, receiver } i \\ & \text{and receiver } j \text{ make different decisions} \end{cases}$$

Based on N such pairwise statistics $\alpha_{12}, \alpha_{13}, \ldots, \alpha_{1N}$, α_{23}, the authors in [2] derive estimates of $P_e(1), P_e(2), \ldots\ , P_e(N)$ by solving N non-linear equations in N unknown $P_e(i)$ values. The set of N equations was derived by equating the N statistics, $\alpha_{12}, \alpha_{13}, \ldots, \alpha_{1N}, \alpha_{23}$ to their respective unbiased values as follows:

$$\begin{aligned}
1 - P_e(1) - P_e(2) + 2P_e(1)P_e(2) &= \alpha_{12} && (1.1)\\
1 - P_e(1) - P_e(3) + 2P_e(1)P_e(3) &= \alpha_{13} && (1.2)\\
&\vdots\\
1 - P_e(1) - P_e(N) + 2P_e(1)P_e(N) &= \alpha_{1N} && (1.N-1)\\
1 - P_e(2) - P_e(3) + 2P_e(2)P_e(3) &= \alpha_{23} && (1.N)
\end{aligned}$$

A closed-form solution to the above set of equations was presented. Furthermore, in [3] it was shown that the estimates derived in [2] are the maximum likelihood estimates (MLEs) of the receiver BERs.

Though the estimation algorithm presented in [2] renders MLEs of receiver BERs and was shown to be computationally inexpensive, it does not utilize all available pairwise information. As is easily verified, a total of $N(N-1)/2$ pairwise statistics, α_{ij}, are available. By the data processing theorem, more accurate estimates are expected to result if we are able to intelligently utilize all $N(N-1)/2$ pairwise statistics. Motivated by this concept, we wish to solve the following set of $N(N-1)/2$ equations in N unknowns to yield estimates of $P_e(1), P_e(2), \ldots, P_e(N)$.

$$\begin{aligned}
1 - P_e(1) - P_e(2) + 2P_e(1)P_e(2) &= \alpha_{12}\\
1 - P_e(1) - P_e(3) + 2P_e(1)P_e(3) &= \alpha_{13}\\
&\vdots\\
1 - P_e(1) - P_e(N) + 2P_e(1)P_e(N) &= \alpha_{1N}\\
1 - P_e(2) - P_e(3) + 2P_e(2)P_e(3) &= \alpha_{23}\\
1 - P_e(2) - P_e(4) + 2P_e(2)P_e(4) &= \alpha_{24}\\
&\vdots\\
1 - P_e(N-1) - P_e(N) + 2P_e(N-1)P_e(N) &= \alpha_{N-1N}
\end{aligned}$$

Let $\mathbf{P}$ be the column vector of receiver BERs defined as $\mathbf{P} = [P_e(1), P_e(2), \cdots, P_e(N)]^T$ and let $\overline{\mathbf{P}} = \mathbf{1}_N - \mathbf{P}$ where $\mathbf{1}_N$ denotes an N long column vector of ones. It is easy to see from above relationships that $\overline{\mathbf{P}}$ is an eigenvector of the matrix $\overline{\mathbf{P}}\overline{\mathbf{P}}^T$, corresponding to the eigenvalue $\|\overline{\mathbf{P}}\|^2$. Moreover, it can be shown that $\|\overline{\mathbf{P}}\|^2$ is the only non-zero eigenvalue of the matrix $\overline{\mathbf{P}}\overline{\mathbf{P}}^T$.

Let $\mathbf{L}$ be $N \times N$ real symmetric matrix with eigenvalues λ_i, *i=1,2,...,N* such that $|\lambda_1| > |\lambda_2| \geq |\lambda_N|$. Then given $\mathbf{v}_0 \,\epsilon\, \mathcal{R}^N$, the power method produces a sequence of vectors as follows [4]:

for i:=0,1,2... do
$\mathbf{v}_i = \mathbf{L}\mathbf{v}_i$
$\mathbf{v}_{i+1} = \mathbf{v}_i/\|\mathbf{v}_i\|^2$
end

The sequence of vectors $\mathbf{v}_i$ converges to the dominant eigenvector of $\mathbf{L}$ if the initial vector $\mathbf{v}_0$ has a component in the direction of the dominant eigenvector and the rate of convergence is dictated by $|\lambda_2|/|\lambda_1|$.

Based on the power method, $\mathbf{P}$ can be estimated as the dominant eigenvector of the matrix $\overline{\mathbf{P}}\overline{\mathbf{P}}^T$. However, the matrix $\overline{\mathbf{P}}\overline{\mathbf{P}}^T$ is unavailable to us. To be able to use a power method based procedure, we approximate the matrix $\overline{\mathbf{P}}\overline{\mathbf{P}}^T$ using the information available to us.

Consider another matrix $\mathbf{A}$ constructed from the $N(N-1)/2$ statistics α_{ij} as follows:

$$\begin{bmatrix} 0 & \alpha_{12} & \alpha_{13} & \cdots & \alpha_{1N} \\ \alpha_{12} & 0 & \alpha_{23} & \cdots & \alpha_{2N} \\ \alpha_{13} & \alpha_{23} & 0 & \cdots & \alpha_{3N} \\ \vdots & & & \ddots & \\ \alpha_{1N} & \alpha_{2N} & \cdots & \cdots & 0 \end{bmatrix}$$

Given infinite observations and hence, error free estimates α_{ij}, the matrix $\mathbf{A}$ can be decomposed as

$$\mathbf{A} = \mathbf{P}\mathbf{P}^T + \overline{\mathbf{P}}\overline{\mathbf{P}}^T - diag(\mathbf{P}\mathbf{P}^T + \overline{\mathbf{P}}\overline{\mathbf{P}}^T) \qquad (2)$$

where $diag(\cdot)$ denotes a diagonal matrix with the diagonal equal to the diagonal of the argument matrix.

We use Eq. (2) to approximate $\overline{\mathbf{P}}\overline{\mathbf{P}}^T$ for finite observations and derive the power method based iterations to estimate $\mathbf{P}$ as follows:

Initialization:
$\overline{\mathbf{P}}_0 = \mathbf{1}_N$
$\mathbf{P}_0 = \mathbf{1}_N - \overline{\mathbf{P}}_0$
Iterations:
for i = 0,1,2... do
$\overline{\mathbf{P}}_{i+1} = [\,\mathbf{A} + diag(\mathbf{P}_i\mathbf{P}_i^T + \overline{\mathbf{P}}_i\overline{\mathbf{P}}_i^T) - \mathbf{P}_i\mathbf{P}_i^T]\frac{\overline{\mathbf{P}}_i}{\|\overline{\mathbf{P}}_i\|^2}$
$\mathbf{P}_{i+1} = 1 - \overline{\mathbf{P}}_{i+1}$
end

It is easily verified that the desired solution is a stable point of the algorithm *i.e.* if the iterations converge to the desired solution (error-free value of $\mathbf{P}$), the error in the subsequent iterations is zero.

4. LOCAL STABILITY ANALYSIS OF THE PROPOSED ALGORITHM

In the previous section, we presented the iterative procedure to blindly estimate the BER of a set of networked receivers. In this section, we analyze the local stability of the proposed algorithm. To be able to do so, we assume that error-free values of α_{ij} are available and that at a particular iteration, the error in the estimate of $\mathbf{P}^*$ is given by $\mathbf{\Delta} = [\Delta_1, \Delta_2, \ldots, \Delta_N]^T$. Also, let $\mathbf{P}^*$ denote the error-free value of $\mathbf{P}^*$. Based on the above definitions, it can be shown that if $|\Delta_i| \ll \overline{\mathbf{P}^*}_i/2$, then in the next iteration, the error in $\mathbf{P}$, the estimate of $\mathbf{P}^*$, is given by $(\mathbf{M}\mathbf{\Delta} + \varepsilon)$ where

$$\mathbf{M} = \frac{1}{\|\overline{\mathbf{P}^*}\|^2}\Big[4diag(\mathbf{P}^*\overline{\mathbf{P}^*}^T) - 2diag(\mathbf{1}_N\overline{\mathbf{P}^*}^T) - (\mathbf{P}^{*T}\overline{\mathbf{P}^*})\mathbf{I}_N + \mathbf{P}^*\mathbf{P}^{*T} + \overline{\mathbf{P}^*}\overline{\mathbf{P}^*}^T - \mathbf{P}^*\mathbf{1}^T\Big] \quad (3)$$

and ε involves terms which are at least quadratic in $\mathbf{\Delta}$ and hence can be ignored. Here $\mathbf{I}_N$ denotes the identity matrix of size $N\times N$.

To be able to show that the algorithm is locally stable, we need to prove that all eigenvalues of $\mathbf{M}$ are less than one in magnitude. We prove this result in two steps: we first show that the eigenvalues of $\mathbf{M}$ are real, and we then show that the eigenvalues of the matrix $\mathbf{M} + \mathbf{I}_N$ lie between zero and two.

4.1. To show that the eigenvalues of $\mathbf{M}$ are real:

Let $\mathbf{S} = [4diag(\mathbf{P}^*\overline{\mathbf{P}^*}^T) - 2diag(\mathbf{1}_N\overline{\mathbf{P}^*}^T) - (\mathbf{P}^{*T}\overline{\mathbf{P}^*})\mathbf{I}_N + \mathbf{P}^*\mathbf{P}^{*T} + \overline{\mathbf{P}^*}\overline{\mathbf{P}^*}^T]$. We wish to show that the eigenvalues of $(\mathbf{S} - \mathbf{P}^*\mathbf{1}^T)$ are real. If λ is an eigenvalue of $(\mathbf{S} - \mathbf{P}^*\mathbf{1}^T)$, then we must have $det(\mathbf{S} - \lambda\mathbf{I}_N - \mathbf{P}^*\mathbf{1}^T) = 0$. It can be shown [5] that

$$det(\mathbf{S} - \lambda\mathbf{I}_N - \mathbf{P}^*\mathbf{1}^T) = det(\mathbf{S} - \lambda\mathbf{I}_N)\Big\{1 + \mathbf{1}_N^T(\mathbf{S} - \lambda\mathbf{I}_N)^{-1}\mathbf{P}\Big\}$$

The above equation implies that either $det(\mathbf{S} - \lambda\mathbf{I}_N) = 0$, in which case, λ is an eigenvalue of a real symmetric matrix and hence, is real or $(1 + \mathbf{1}_N^T(\mathbf{S} - \lambda\mathbf{I}_N)^{-1}\mathbf{P}) = 0$. Using the fact that if λ is an eigenvalue of a matrix, then so is the complex conjugate of λ, it can be shown that $(1 + \mathbf{1}_N^T(\mathbf{S} - \lambda\mathbf{I}_N)^{-1}\mathbf{P}) = 0$ implies that the imaginary part of λ is strictly zero for the above equality to hold. Therefore, the eigenvalues of $\|\overline{\mathbf{P}^*}\|^2\mathbf{M}$ and hence, those of $\mathbf{M}$ are real.

4.2. To show that if the eigenvalues of $\mathbf{M}$ are real, then the eigenvalues of $\mathbf{M} + \mathbf{I}_N$ lie between zero and two:

Let

$$\begin{aligned}
\mathbf{a} &= (\overline{\mathbf{P}^*} - \mathbf{P}^*)/\|\overline{\mathbf{P}^*}\|^2 \\
\mathbf{b} &= \overline{\mathbf{P}^*}^T/\|\overline{\mathbf{P}^*}\|^2 \\
\mathbf{D} &= \frac{1}{\|\overline{\mathbf{P}^*}\|^2}\Big[4diag(\mathbf{P}^*\overline{\mathbf{P}^*}^T) - 2diag(\mathbf{1}_N\overline{\mathbf{P}^*}^T) \\
&\quad -(\mathbf{P}^{*T}\overline{\mathbf{P}^*})\mathbf{I}_N\Big]
\end{aligned}$$

Then, the characteristic equation of the matrix $(\mathbf{M} + \mathbf{I}_N)$ can be written as [5]

$$\left(1 + \sum_{i=1}^{N}\frac{a_i b_i}{d_{ii} - \lambda}\right)\prod_{i=1}^{N}(d_{ii} - \lambda) = 0$$

where λ is an eigenvalue of the matrix in consideration, a_i, b_i are the i^{th} elements of the vectors $\mathbf{a}$, $\mathbf{b}$, respectively and d_{ii} is the i^{th} diagonal element of the diagonal matrix $\mathbf{D}$. It can be shown that the above equation is not satisfied for any value of λ less than zero. Furthermore, it can be shown that $\|\mathbf{M} + \mathbf{I}_N\|_\infty < 2$, which implies that all the eigenvalues of $(\mathbf{M} + \mathbf{I}_N)$ are less than 2 in magnitude.

The above two results together imply that the eigenvalues of $\mathbf{M}$ are bounded between -1 and 1 and hence, the proposed algorithm is locally stable.

5. PERFORMANCE COMPARISON OF THE PROPOSED ITERATIVE ALGORITHM

We now compare the performance of the proposed iterative algorithm to that of the algorithm proposed in [2]. In all the simulations presented in the paper, the proposed iterative algorithm was run for 20 iterations.

In Fig. 2 we compare the performance of the two algorithms for a network of 5 receivers having randomly chosen BERs of [0.0185 0.0269 0.0335 0.0612 0.0740]. Fig. 2 shows the comparison of bias in the estimate of the BER of the best receiver and the BER of the empirical fusion receiver for the two algorithms. The advantage of harnessing extra information is obvious from the plots in the figure. As is seen from the first subplot, the proposed power method based algorithm has a lower bias for small number of observations (less than 500 bits). It was observed via extensive simulations that the difference in bias resulting from the two algorithms is most prominent for the minimum BER receiver which in turn effects the BER of empirical fusion most. For other receivers in the network, though the proposed approach resulted in lower bias, the difference in performance reduces as the BER of the receiver increases.

Also plotted in the figure is the BER of empirical fusion receiver based on the BER estimates from the two algorithms. As can be seen in the figure, the proposed algorithm results in a much better performance for small number of observations and matches the performance of the optimal fusion receiver for modest number of observations.

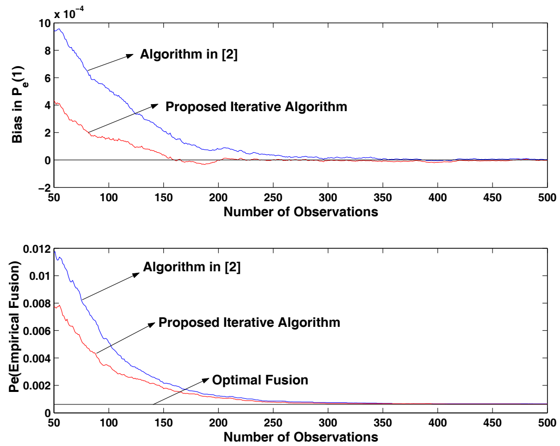

Fig. 2. Performance of the proposed algorithm compared with that of the algorithm in [2] based on (1) bias in estimate of BER of best receiver and (2) BER of empirical fusion. The proposed algorithm outperforms the one in [2].

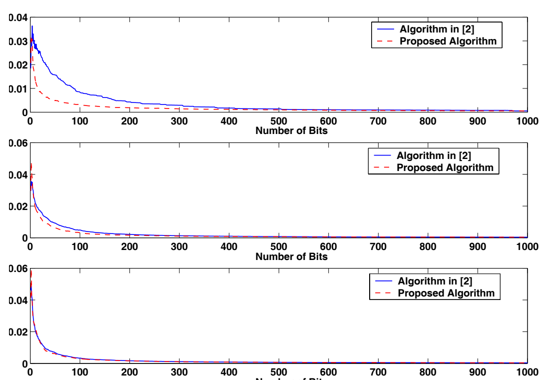

Fig. 3. Comparison of variance in the estimates rendered by the algorithms in consideration for a network of three receivers with BER values [0.0543 0.1866 0.2970].

Intuitively, it seems appealing that the variance in the estimates rendered by the proposed iterative procedure would be less than the variance in the estimates using the algorithm in [2]. However, a more meaningful analysis of the variance was elusive. In the absence of a theoretical result, we use simulations to compare the variances of the two estimates. In Fig. 3, we plot the variance in the estimates for the two algorithms for a bank of just three receivers with BER values [0.0543 0.1866 0.2970]. Interestingly, the proposed approach results in lower variance in its estimates even when the two approaches use the same set of information (both algorithms in this case use α_{12}, α_{13}, α_{23}). This result implies that for small number of observations, the new approach gives more consistent results compared to algorithm in [2], regardless of the number of receivers connected in the network.

Finally, we compare the performance of the two algorithms as the number of receivers in the network is increased and the number of bit observations is held constant at just 100 bits. In Fig. (4) we

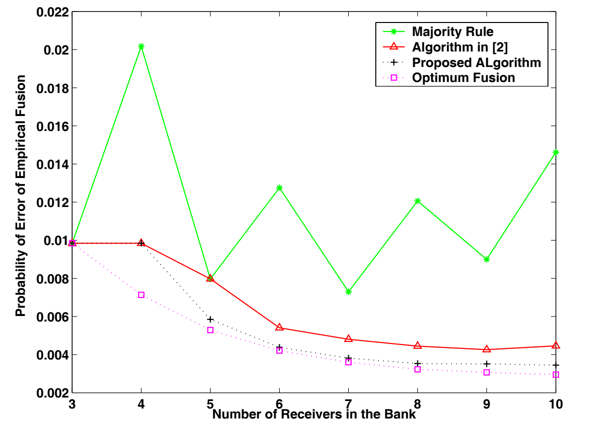

Fig. 4. Performance of proposed algorithm as the number of receivers in the bank are increased.

chose an increasingly large network of receivers with the number of receivers varying from 3 to 10 and each new receiver picked sequentially from the following set of BER values: [0.0200 0.0500 0.1300 0.1600 0.1700 0.2000 0.2500 0.2900 0.3400 0.3700]. In the figure, we compare the performance of (1) the majority rule receiver, (2) the empirical fusion receiver based on algorithm in [2], (3) the empirical fusion receiver based on the proposed iterative algorithm and (4) the optimal fusion receiver.

It can be seen from the figure that the proposed iterative algorithm consistently outperforms the majority rule receiver and the algorithm proposed in [2] as the number of participating receivers increases. Also, it should be noted that in this case, the proposed algorithm almost matches the performance of the optimal fusion receiver for as few as 100 bit observations.

6. CONCLUSION

In this work, we present a power method based iterative procedure to blindly estimate the BERs of N networked receivers. The proposed algorithm uses the complete set of information available for estimation and is computationally simple. It was shown that the desired solution is a stable point of the algorithm and the algorithm is locally stable. The proposed algorithm was shown to outperform the best receiver in the network and the standard majority rule receiver and was also shown to match the performance of the optimal fusion receiver for nearly all practical operating scenarios.

7. REFERENCES

[1] Z. Chair and P.K. Varshney, "*Optimal Data Fusion in Multiple Sensor Detection Systems,*" *IEEE Trans. Aerospace Elect. Syst.*, vol. AES-22 No.1, pp. 98-101, Jan 1986.

[2] A. Dixit and G. G. Orsak, "*Optimal Empirical Detector Fusion,*" *in Proc. 2000 IEEE Signal Processing Workshop*, Hunt, Texas, October 15-18, 2000.

[3] A. Dixit and G. G. Orsak, "*Asymptotically Optimal Blind Fusion of Bit Decisions,*" *in Proc. 2002 IEEE Signal Processing Workshop*, Callaway Gardens, Georgia, October 13-16, 2002.

[4] G. H. Golub and C. F. V. Loan, *Matrix Computations,* Johns Hopkins University Press, Baltimore 1996.

[5] F. A. Graybill, *Introduction to Matrices with Applications in Statistics,* Wadsworth: Belmont, CA 1969.