DISTRIBUTED EDGE SENSOR DETECTION WITH ONE- AND TWO-LEVEL DECISIONS

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ABSTRACT

A statistical approach to distributed edge sensor detection is proposed for wireless sensor networks. Edge sensor detection is a technique to decide whether a target sensor is an edge sensor based on data from its neighboring sensors. It is desirable that the computational complexity be low and the amount of data transferred among sensors be small. With some reasonable assumptions and the maximal likelihood technique, we propose both one-level and two-level decision methods to fulfill the above two constraints.

1. INTRODUCTION

With an emerging need in environmental monitoring, military surveillance and security protection, research on wireless sensor networks has received a great amount of interest recently. In these applications, attributions of the monitored event over a certain area should be collected and analyzed. The edge information is one of these important attributions. Although there are several well-developed edge detection algorithms in image processing, it is difficult to apply them directly to the edge sensor detection problem due to the randomness of sensor locations within an event.

Nowak and Mitra [1] proposed an edge approximation method for sensor networks using recursive dyadic partition (RDP). Chintalapudi and Govindan [2] proposed three approaches based on a localized edge detection technique for edge sensor detection; namely, the statistical approach, the filter-based approach and the classifier-based approach. The main difference between [1] and [2] is that a hierarchical network architecture was assumed and utilized in [1] whereas there was no hierarchy among sensors in [2]. Another major difference between them is that the real boundary was approximated in [1] while only edge sensors were detected in [2]. Also, [2] outperforms [1] in terms of a lower communication cost, which is critical in wireless sensor networks.

We follow the localized edge detection framework proposed in [2]. It was claimed in [2] that the performance of the classifier-based approach is better than that of the other two approaches, yet the statistical approach is more robust to a higher sensor error condition. To enhance the poor performance of the statistical approach, global and local decision rules based on the maximal likelihood ratio test are developed to detect edge sensors in this work. To be more specific, a distributed detection scheme [3], [4], [5] is used to solve the edge sensor detection problem, and new one-level and two-level decision rules are obtained. In the proposed schemes, the readings are statistically described so that the optimal processing is feasible in local sensors and the fusion center. It will be shown that the proposed one-level and two-level decision methods achieve good performance, which is close to the optimal solution.

2. SYSTEM AND OBSERVATION MODELS

Each sensor is assumed to be randomly deployed over the area of interest. Due to this unpredictability of each sensor's location, it is difficult to find a thin edge over the sensor network. To simplify the problem, we define an edge region as the region whose distance from the actual edge is within tolerance range r. Therefore, the width of the edge region is equal to $2r$. The sensors inside this edge region are called edge sensors. To decide whether a target sensor is an edge sensor, the measured data or local decision results of its neighboring sensors, whose distance from the target sensor is less than r, are collected and transmitted to the fusion center for the global decision making. We consider the white noise in wireless communication channels. Assumed there are N neighboring sensors around a target sensor and the measurement of the ith neighboring sensor at a particular time instance can be modeled as

$$y_i = m_i + n_i, \tag{1}$$

where $P(m_i = A) = P_A$, $P(m_i = 0) = 1 - P_A$ and n_i is a Gaussian random variable with zero mean and variance σ^2.

Furthermore, it is assumed that the PDF of the measurement of each neighbor sensor is i.i.d.. Thus, the PDF of y_i is equal to

$$P(y_i) = P_A \cdot N(A,\sigma^2) + (1 - P_A) \cdot N(0,\sigma^2), \tag{2}$$

where $N(A,\sigma^2)$ is the PDF of a Gaussian distribution with the mean and variance equal to A and σ^2, respectively.

3. PROPOSED DECISION METHODS

Based on the system model given in Section 2, we proceed to derive the optimal fusion rule for edge sensor detection. The problem of detecting edge sensors can be modeled as a binary hypotheses problem as follows.

H_{F0} : The target sensor is not an edge sensor.
H_{F1} : The target sensor is an edge sensor.

It was proved in [4] that the maximal likelihood ratio test can provide the optimal decision result for such a problem when the threshold is appropriately chosen, and we will use it to derive optimal edge sensor detection rule. Before starting the derivation, we first explain the difference between edge and non-edge (or regular) sensors as shown in Fig. 1. We see that a regular sensor

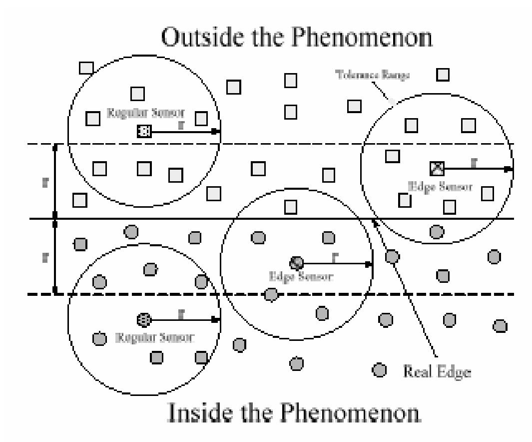

Figure 1. Illustration of edge and non-edge sensors.

has its neighboring sensors either all inside the phenomenon or all outside the phenomenon, whereas an edge sensor does not. Thus, we can use this characteristic as a criterion to distinguish between these two cases.

Approach I: One-Level Decision Method

In this case, the data measured by each sensor are transmitted to neighboring sensors within distance r directly without making any local decision. Thus, there is no information loss in this process. The assumption of independently identical distributions (i.i.d.) is made for measurements in the following derivation. Based on the observation model in Section 2 and the characteristic of an edge sensor, the likelihood functions can be shown as:

$$P(\underline{y} \mid H_{F0}) = (P_A)^N \cdot \prod_{i=1}^{N} N_i(A,\sigma^2) + (1-P_A)^N \cdot \prod_{i=1}^{N} N_i(0,\sigma^2) \tag{4}$$

$$P(\underline{y} \mid H_{F1}) = \prod_{i=1}^{N} [P_A \cdot N_i(A,\sigma^2) + (1-P_A) \cdot N_i(0,\sigma^2)] - P(\underline{y} \mid H_{F0}) \tag{5}$$

Then, the log likelihood ratio, $\ln(\Lambda(\underline{y}))$, can be computed and the log likelihood ratio test can be obtained as follows.

$$\ln(\Lambda(\underline{y})) = \ln\left\{ \frac{\prod_{i=1}^{N} [1 + \frac{P_A}{1-P_A} \cdot \Lambda(y_i)]}{1 + (\frac{P_A}{1-P_A})^N \cdot \prod_{i=1}^{N} \Lambda(y_i)} - 1 \right\} \begin{matrix} H_{F1} \\ > \\ < \\ H_{F0} \end{matrix} \ln\left(\frac{P(H_{F0})}{P(H_{F1})} \right), \tag{6}$$

where $\Lambda(y_i) = \frac{N_i(A,\sigma^2)}{N_i(0,\sigma^2)}$.

However, since $P(H_{F0})$ and $P(H_{F1})$ are unknown, it is not a trivial task to determine the threshold. We will show how the threshold value affects the system performance in Section 4.

Approach II: Two-Level Decision Method

In this case, the data measured by each sensor are first processed by each sensor to get a local decision. Then, the local decision is transmitted to neighboring sensors for further information fusion to make the global decision. Although some information is lost in the second data fusion step due to the first local decision step, it reduces the communication cost between sensors and the fusion center, which is the target sensor. In most applications, the available bandwidth is limited so that it would be costly to transfer raw data directly between sensors. Here, we consider the case where each local sensor makes a binary hypothesis test. Consequently, we have two levels of hypotheses as given below.

Local Hypotheses (the 1st level decision):

H_{L0} : The sensor is outside the phenomenon.
H_{L1} : The sensor is inside the phenomenon.

Global Hypotheses (the 2nd level decision):

H_{F0} : The target sensor is not an edge sensor.
H_{F1} : The target sensor is an edge sensor.

Although a similar derivation was conducted in [5], it dealt with the identical local and global hypotheses. In this work, hypotheses in these two levels are different so that the derivation becomes more complicated. Let C_{ij} represent the cost of deciding H_{Fi}, given that H_{Fj} is present. The Bayesian cost function after algebraic manipulations becomes

$$R_B = C + C_F \cdot P_F - C_D \cdot P_D, \tag{7}$$

where

$$C = C_{01} \cdot [1 - P(H_{F0})] + C_{00} \cdot P(H_{F0}),$$

$$C_F = (C_{10} - C_{00}) \cdot P(H_{F0}), C_D = (C_{01} - C_{11}) \cdot [1 - P(H_{F0})],$$

$$P_F = P(H_{F1} \mid H_{F0}), \; P_D = P(H_{F1} \mid H_{F1}).$$

We would like to find a pair of optimal global and local decision rules so that (7) can be minimized. Here, we adopt the person-by-person optimization to reduce the complexity of the optimization problem. The inequality below was proved to be an optimal global decision rule in [5].

$$\frac{P(\underline{x} = \underline{x}^* \mid H_{F1})}{P(\underline{x} = \underline{x}^* \mid H_{F0})} \begin{matrix} P(u=1 \mid \underline{x} = \underline{x}^*) = 1 \\ > \\ < \\ P(u=1 \mid \underline{x} = \underline{x}^*) = 0 \end{matrix} \frac{C_F}{C_D}, \tag{8}$$

where $\underline{x} = (x_1, x_2, \cdots, x_N)$, x_i is the local decision result of the ith neighbor sensor and u is the global decision result of the fusion center. It is obvious that the optimal solution for the global decision rule is a likelihood ratio test. Consequently, we can determine the value of u if the likelihood ratio is known for all possible $\underline{x}^*$. In other words, we can make a global decision when the local decision result of each neighbor sensor is known.

To compute the likelihood ratio, we must know the local decision rule first. Next, we will derive the local decision rule. First, we can rewrite (7) as

$$\begin{aligned} R_B = C &+ \sum_{\underline{x}} P(u=1 \mid \underline{x}_\mu^0)[C_F P(\underline{x}_\mu^0 \mid H_{F0}) - C_D P(\underline{x}_\mu^0 \mid H_{F1})] \\ &+ \sum_{\underline{x}} P(u=1 \mid \underline{x}_\mu^1)[C_F P(\underline{x}_\mu^1 \mid H_{F0}) - C_D P(\underline{x}_\mu^1 \mid H_{F1})], \end{aligned} \tag{9}$$

where $\underline{x}_\mu^k = (x_1, x_2, \cdots, x_\mu = k, \cdots, x_N)$. Then, $P(\underline{x}_\mu^k \mid H_{F0})$ and $P(\underline{x}_\mu^k \mid H_{F1})$ can be found via

$$P(\underline{x}_\mu^k \mid H_{F0}) = \int_{y_\mu} P(x_\mu = k \mid y_\mu) G(y_\mu, \underline{x}^\mu) dy_\mu, \tag{10}$$

$$P(\underline{x}_\mu^k \mid H_{F1}) = \int_{y_\mu} P(x_\mu = k \mid y_\mu)[L(y_\mu, \underline{x}^\mu) - G(y_\mu, \underline{x}^\mu)] dy_\mu, \tag{11}$$

where $\underline{x}^\mu = (x_1, x_2, \cdots, x_{\mu-1}, x_{\mu+1}, \cdots, x_N)$,

$$\begin{aligned} G(y_\mu, \underline{x}^\mu) = &[P(H_{L0})]^N P(y_\mu \mid H_{L0}) P(\underline{x}^\mu \mid H_{L0}) \\ &+ [P(H_{L1})]^N P(y_\mu \mid H_{L1}) P(\underline{x}^\mu \mid H_{L1}), \end{aligned}$$

$$\begin{aligned} L(y_\mu, \underline{x}^\mu) = &[P(H_{L0}) P(y_\mu \mid H_{L0}) \\ &+ P(H_{L1}) P(y_\mu \mid H_{L1})] \cdot B^\mu, \end{aligned}$$

$$B^{\mu} = \prod_{i \neq \mu} [P(H_{L0})P(x_i \mid y_i)P(y_i \mid H_{L0}) + P(H_{L1})P(x_i \mid y_i)P(y_i \mid H_{L1})].$$

After substituting (10) and (11) back into (9), the Bayesian cost function can be rewritten as

$$R_B = C + C_{\mu} + \sum_{\underline{x}^{\mu}} A_{\underline{x}^{\mu}} [\int_{y_{\mu}} P(x_{\mu} = 1 \mid y_{\mu}) \times \{C_F \cdot G(y_{\mu}, \underline{x}^{\mu}) - C_D \cdot L(y_{\mu}, \underline{x}^{\mu}) + C_D \cdot G(y_{\mu}, \underline{x}^{\mu})\} dy_{\mu}], \quad (12)$$

where

$$C_{\mu} = \sum_{\underline{x}^{\mu}} P(u = 1 \mid \underline{x}_{\mu}^{0})[C_F P(\underline{x}^{\mu} \mid H_{F0}) - C_D P(\underline{x}^{\mu} \mid H_{F1})],$$

$$A_{\underline{x}^{\mu}} = P(u = 1 \mid \underline{x}_{\mu}^{1}) - P(u = 1 \mid \underline{x}_{\mu}^{0}).$$

By observing (12), it is easy to find that the optimal local decision rule is given by

$$\frac{P(y_{\mu} \mid H_{L1})}{P(y_{\mu} \mid H_{L0})} \underset{x_{\mu}=0}{\overset{x_{\mu}=1}{\gtrless}} T_L, \quad (13)$$

where

$$T_L = \frac{(C_F + C_D)A_0 \cdot \prod_{i \neq \mu} P(x_i \mid y_i)P(y_i \mid H_{L0}) - C_D P(H_{L0})B^{\mu}}{P(H_{L1})B^{\mu} - (C_F + C_D)A_1 \cdot \prod_{i \neq \mu} P(x_i \mid y_i)P(y_i \mid H_{L1})},$$

$$A_0 = [P(H_{L0})]^N \text{ and } A_1 = [P(H_{L1})]^N.$$

It is obvious from (13) that the optimal local decision rule is also a likelihood ratio test. We can easily compute the probability of detection, P_D, and the probability of false alarm, P_{FA}. Then, the optimal global decision rule can be written as

$$\ln(\Lambda(\underline{x}_m)) = \ln\left(\frac{P(\underline{x}_m \mid H_{F1})}{P(\underline{x}_m \mid H_{F0})}\right) = \ln\left\{\frac{[1 + \frac{P(H_{L0})P_{FA}}{P(H_{L1})P_D}]^m \cdot [1 + \frac{P(H_{L0})(1 - P_{FA})}{P(H_{L1})(1 - P_D)}]^{N-m}}{1 + [\frac{P(H_{L0})P_{FA}}{P(H_{L1})P_D}]^m \cdot [\frac{P(H_{L0})(1 - P_{FA})}{P(H_{L1})(1 - P_D)}]^{N-m}} - 1\right\} \underset{H_{F0}}{\overset{H_{F1}}{\gtrless}} T_F, \quad (14)$$

where $\underline{x}_m$ is the vector $\underline{x}$ with m 1's.

However, the formula (14) is too complicated for a sensor to fuse data, and further simplification is needed.

To do so, we observe the trend of the log likelihood ratio. If the ratio is a monotonically increasing function of m, then K-out-of-N rule can be adopted. If it is not, we have to find another way to simplify it. Fig. 2 shows the trend of log likelihood ratio. The value of SNR means the ratio between the received signal power and the noise power of a local sensor and N is the number of neighboring sensors inside the tolerance range. Obviously, it is not a monotonically increasing function, and the K-out-of-N rule is not appropriate. From this figure, we find that the value of the log likelihood ratio is always larger than the given threshold in some continuous interval of m. Thus, we propose the following global decision rule:

Decide H_{F0}: $m < m_1$ or $m > m_2$,

Decide H_{F1}: $m_1 < m < m_2$,

where $m_1 < m_2$.

With this optimal global decision rule, the fusion center can perform the complicated data fusion processing easily and fast.

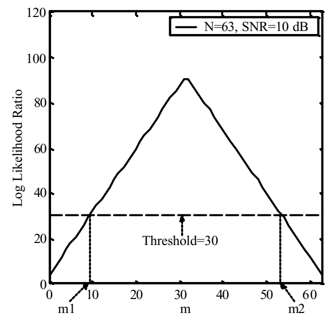

Figure 2. The trend of log likelihood ratio

4. SIMULATION RESULTS

We conducted computer simulations with the following settings. The locations of sensors are assumed uniformly distributed. The phenomenon region is of a circular shape as shown in Figs 3 and 5. The total number of sensors is 2000, the SNR value is 10dB and the tolerance range, r, is set to 1.

4.1. One-Level Decision Method

Figs. 3 and 4 provide simulation results for the one-level decision method, where edge sensors are detected using the log likelihood ratio test given by (6). The a priori probability, $P_A = P(H_{L1})$, is assigned as 0.5 for each sensor. Since we do not know the ratio of $P(H_{F0})$ and $P(H_{F1})$ in advance, the threshold required in (5) is difficult to decide. The effect on the performance of edge sensor detection under different values of threshold is shown in Fig. 4.

Fig. 3 shows one edge sensor detection result. The solid line is the actual edge location while the two dashed lines indicate the tolerance limits. The detection and the false alarm rates under different thresholds are given in Fig. 4. From this figure, both rates are decreasing with an increasing threshold value. However, the decreasing rate of the false alarm probability is faster than that of the detection probability when the threshold is smaller than 15. Also note that the detection rate can be maintained around 80% when the false alarm rate is lower than 20%.

4.2. Two-Level Decision Method

Figs. 5 and 6 show the simulation results with the two-level decision method that has a lower communication cost at the cost of poorer performance. The global decision rule in (14) is used and the a priori probability $P_A = P(H_{L1})$ is assigned as 0.5.

The value of threshold is again a very important factor in this case. Thus, we also observe the impact on the detection and false alarm performance from different threshold values. Since some information is lost in the local decision process, the performance for this case should be worse than the one-level decision method that has a higher communication cost.

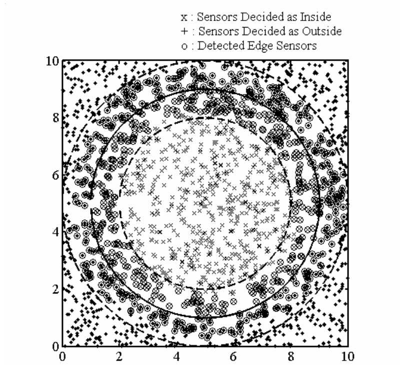

Figure 3. Example of the edge sensor detection result when threshold=20

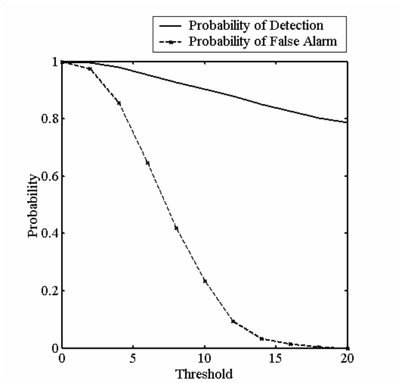

Figure 4. The detection and false alarm rates for the one-level decision method.

In other words, the performance of edge sensor detection using the one-level decision method can serve as a performance bound for the two-level decision method. Fig. 5 shows the edge sensor detection result. Again, we have the phenomenon region of a circular shape, and the tolerance range is 1. Fig. 6 illustrates the detection and the false alarm rates with different threshold values. From this figure, we see that the decreasing rate of the false alarm probability is faster than that of the detection probability while the threshold is smaller than 25. Nevertheless, the probability of detection decreases to around 0.6 when the probability of false alarm is approximately zero.

Finally, we compare the performance of the one-level and the two-level methods using the Receiver Operating Characteristics (ROC) curves as shown in Fig. 7. For the same false alarm rate, the one-level method has a higher detection rate than the two-level method.

5. REFERENCES

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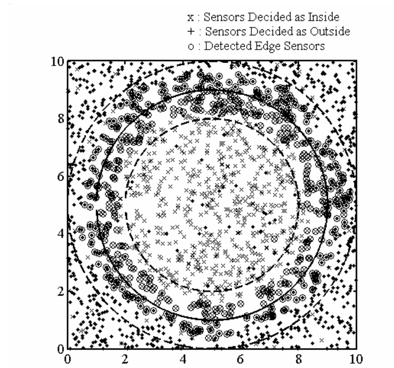

Figure 5. Example of the edge sensor detection result when threshold=30

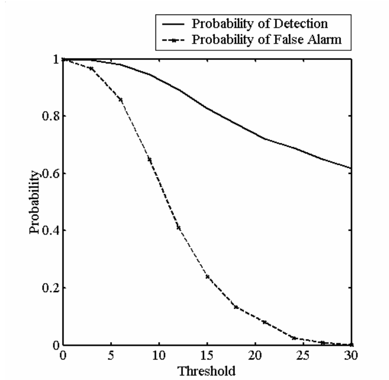

Figure 6. The detection and false alarm rates for the two-level decision method.

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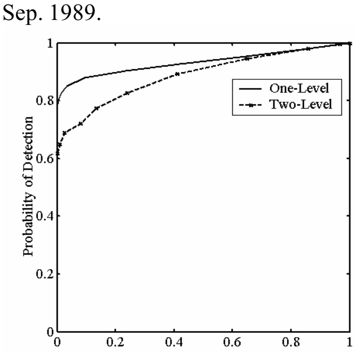

Figure 7 ROC (Receiver Operating Characteristics) Curves