# A KALMAN FILTER BASED REGISTRATION APPROACH FOR ASYNCHRONOUS SENSORS IN MULTIPLE SENSOR FUSION APPLICATIONS

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## ABSTRACT

In this paper, a Kalman filter based registration approach is proposed for multiple asynchronous sensors. In the approach, a linear time-varying measurement model is formulated using a first order approximation and is shown to be *uniformly completely observable*. The sensor registration errors are estimated based on the application of a modified two-stage Kalman estimator. The proposed registration approach is computationally efficient and is capable of handling asynchronous sensor measurements. Simulation and real-life data are used to demonstrate the effectiveness of the proposed approach. Results are compared with the popular least squares (LS) method.

## 1. INTRODUCTION

Data fusion has been defined as a process of dealing with association, correlation and combination of data and information from single or multiple sources to achieve refined position and identity estimates. Sensor registration is a part of the level one processing of data fusion which includes association, filtering and identifications. It refers to the process of ensuring the requisite error free coordinate conversion of multiple sensor data, and is a prerequisite for a data fusion system to accurately estimate and correct systematic errors. In multisensor integration, data from each sensor are transformed into a common reference system for merging. Direct transformation of data usually leads to limited success due to the failure to register adequately by the individual sensors. The effect of sensor registration errors is to introduce biases into fusion, generating ghost targets for multisensor signal processing [1].

Various registration algorithms [1][2][3][4][5][6][7][8] have been developed recently. However, there are two problems that have not been addressed: system observability and asynchronous sensor measurements. System observability plays an important role in system analysis and parameter estimation. It decides if a system state can be recovered from the measurements. It also determines the optimality of the Kalman filtering. Most of the algorithms have assumed that the system under consideration is observable by default, an assumption that may not hold without a rigorous proof. Secondly, most registration algorithms assume that the sensors are synchronized, which may not be true in practical applications. In reality, sensors often operate asynchronously. The scanning process of different radars may not be synchronized and have different rate.

In this paper, a Kalman filter based registration approach is proposed for asynchronous sensor measurements. Constant range and azimuth errors are considered. The sensors are assumed to be asynchronous and have different update rates. The near constant velocity model is used to describe the target motion dynamics. A linear time-varying measurement model is formulated using a first order approximation. Observability analysis is carried out and it is shown that the linear time-varying measurement model is *uniformly completely observable*. The observability of the system ensures that sensor registration errors can be estimated from the sensor measurements. A modified sensor measurement model is formulated which contains both the target state and sensor registration errors. The two-stage Kalman estimator is applied for estimating the target state and the sensor registration errors. The estimation procedures can be described as follows. At each sensor, a bias-free estimate of the target state is first computed using a local Kalman filter when a sensor measurement arrives. The sensor registration errors are estimated by another Kalman filter cycle based on the measurement residual of the bias-free target state estimate. The *a posteriori* estimates of the bias-free states and the sensor registration errors along with their covariance matrices are propagated to the sensor site where the next measurement is received. The algorithm iterates between the sensors as new measurements arrive. The two-stage Kalman filter is equivalent to but computationally more efficient than the augmented Kalman estimator because it involves state vectors of smaller dimensions. The algorithm has a distributed structure and can be implemented in a parallel way. It is robust and suitable for real time applications. In this paper, simulated and real-life multiple radar data are used to evaluate the performance of the proposed approach. Comparisons are made with the popular LS registration method.

## 2. PROBLEM FORMULATION

Consider two sensors $A$ and $B$ in a common plane. Without loss of generality, we assume that sensor $A$ is located at the origin of the system coordinate and sensor $B$ at coordinates $(u, v)$. Assume that sensors $A$ and $B$ both measure the range and azimuth of a target.

The standard near constant velocity model is used for target motion

$$\underline{x}(k+1) = \Phi(k+1, k)\underline{x}(k) + \Gamma(k+1)\nu(k), \quad k = 1, 2, \ldots, \tag{1}$$

where $\underline{x}(k) = [x(k), v_x(k), y(k), v_y(k)]^T$ denotes the state vector at the $k$th time instance. In the state vector, $x(k)$ and $y(k)$ are the actual target coordinates in the system plane, and $v_x(k)$ and $v_y(k)$ denote the speed in $x$ and $y$ coordinate, respectively; $\nu(k)$ is the process noise; $\Phi(k+1, k)$ and $\Gamma(k+1)$ are the state and noise transition matrix, respectively [9].

Let $\{\bar{r}_A(k), \bar{\theta}_A(k)\}$ and $\{\bar{r}_B(k), \bar{\theta}_B(k)\}$ denote the polar coordinates of the target at the $k$th time instance relative to sensor

$A$ and $B$, respectively. We use $\{\Delta r_A, \Delta\theta_A\}$ and $\{\Delta r_B, \Delta\theta_B\}$ to denote the range and azimuth biases of sensor $A$ and $B$, respectively. Let $\{w_{rA}(k), w_{\theta A}(k)\}$, $\{w_{rB}(k), w_{\theta r B}(k)\}$ denote the range and azimuth measurement noise of sensor $A$ and $B$, respectively. They are assumed to be independent, identically distributed (*i.i.d*) Gaussian processes with zero-mean and variances $\sigma^2_{rA}$. $\sigma^2_{\theta A}$, $\sigma^2_{rB}$ and $\sigma^2_{\theta B}$, respectively. We use $\{x_A(k), y_A(k)\}$ and $\{x_B(k), y_B(k)\}$ to denote the Cartesian coordinates of the target reported by sensors $A$ and $B$, respectively, in the system plane. Using a first order approximation, the sensor measurement models can be written as

$$
\begin{aligned}
\underline{z}_A(k) &= L\underline{x}(k) + \Sigma_A(k)\underline{\delta}_A + \Sigma_A(k)\underline{w}_A(k) \\
\underline{z}_B(k) &= L\underline{x}(k) + \Sigma_B(k)\underline{\delta}_B + \Sigma_B(k)\underline{w}_B(k),
\end{aligned}
\tag{2}
$$

where

$$
\begin{aligned}
\underline{z}_A(k) &= [x_A(k), y_A(k)]^T \\
\underline{z}_B(k) &= [x_B(k), y_B(k)]^T \\
\underline{w}_A(k) &= [w_{rA}(k), w_{\theta A}(k)]^T \\
\underline{w}_B(k) &= [w_{rB}(k), w_{\theta B}(k)]^T \\
\underline{\delta}_A &= [\Delta r_A, \Delta\theta_A]^T \\
\underline{\delta}_B &= [\Delta r_B, \Delta\theta_B]^T.
\end{aligned}
\tag{3}
$$

In (2), $\Sigma_A(k)$ and $\Sigma_B(k)$ are given by

$$
\Sigma_A(k) = \begin{bmatrix} \sin\theta_A(k) & r_A(k)\cos\theta_A(k) \\ \cos\theta_A(k) & -r_A(k)\sin\theta_A(k) \end{bmatrix}
\tag{4}
$$

$$
\Sigma_B(k) = \begin{bmatrix} \sin\theta_B(k) & r_B(k)\cos\theta_B(k) \\ \cos\theta_B(k) & -r_B(k)\sin\theta_B(k) \end{bmatrix},
\tag{5}
$$

and matrix $L$ is defined as

$$
L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}.
\tag{6}
$$

Define $\underline{\eta}_A(k)$ and $\underline{\eta}_B(k)$ as the augmented state vector for sensor $A$ and $B$, respectively, which are formed by appending the sensor biases to the target state vector. The measurement model (2) can be written in the augmented form as

$$
\underline{z}_A(k) = H_A(k)\underline{\eta}_A + \Sigma_A(k)\underline{w}_A(k)
\tag{7}
$$

$$
\underline{z}_B(k) = H_B(k)\underline{\eta}_B + \Sigma_B(k)\underline{w}_B(k),
\tag{8}
$$

where $H_A(k) = [L \mid \Sigma_A(k)]$, $H_B(k) = [L \mid \Sigma_B(k)]$. The augmented state transition matrix can be written accordingly as

$$
\Phi_a(k+1,k) = \begin{bmatrix} \Phi(k+1,k) & \mathbf{0} \\ \mathbf{0} & I \end{bmatrix}.
\tag{9}
$$

### 2.1. Observability analysis

The observability of a system can be defined based on the properties of information matrix of the system. In the following, we discuss the system for sensor $A$ only. The same conclusion can be drawn for sensor $B$. The information matrix $\mathcal{J}(k+S,k)$ for system (1) and (7) is given by [10]

$$
\mathcal{J}(k+S,k) = \sum_{i=k}^{k+S} [\Phi_a(i,k)]^T H_A^T(i) R_A^{-1}(i) \Phi_a(i,k) H_A(i),
$$

where $R_A(i) = \Sigma_A(i) Q_{wA} \Sigma_A^T(i)$ and $Q_{wA} = \mathrm{diag}[\sigma^2_{Ar}, \sigma^2_{A\theta}]$.

A system is said to be *uniformly completely observable* if there exists a positive integer $S$ and positive $\xi_1$ and $\xi_2$ such that

$$
\xi_1 I \le \mathcal{J}(k+S,k) \le \xi_2 I,
\tag{10}
$$

for all $k > 0$. It can be shown that the information matrix can be decomposed into $\mathcal{J}(k+S,k) = U^T\Omega^T\Omega U$, where $\Omega$ is a block diagonal matrix with the $i$th sub-matrix given by $Q_{wA}^{-1/2}\Sigma_A^{-1}(k+i-1)$, and

$$
U = \begin{bmatrix} H_A(k) \\ H_A(k+1)\Phi_a(k+1,k) \\ \vdots \\ H_A(k+S-1)\Phi_a(S+k,k) \end{bmatrix}.
\tag{11}
$$

The matrix $U$ is called the observability matrix [11]. Let $S = 3$. It is shown in [12] that $U$ has a full column rank, i.e., its columns are linearly independent of each other, and the observability matrix $U$ satisfies the observability rank condition. It can also be verified that $\Sigma_A(k)$ and $\Omega$ are non-singular. Since $U$ has a full column rank, it is sufficient that $\Omega U$ is also of full column rank. According to a preposition by Song and Grizzle in [13], since $\Omega U$ has a full column rank for all $\theta_A(k)$ (a compact set), there must exist positive constant $\xi_1$ and $\xi_2$ such that (10) holds, i.e., the dynamical system (1) and (7) is *uniformly complete observable*.

## 3. SENSOR REGISTRATION

The Kalman filter is a natural candidate for estimating the sensor biases from the asynchronous sensor measurements. We first reformulate the measurement model to include both the sensor $A$ and sensor $B$ measurement model as

$$
\underline{z}(k) = L\underline{x}(k) + \Sigma(k)\underline{\delta} + \underline{w}(k),
\tag{12}
$$

where $\underline{\delta} = [\Delta r_A, \Delta\theta_A, \Delta r_B, \Delta\theta_B]^T$, $\underline{z}(k)$, $\Sigma(k)$ and $\underline{w}(k)$ are assigned different values depending on whether the measurement at $k$th time instance is from sensor $A$ or from sensor $B$. The index $k$ is a combination of indices for sensor $A$ and $B$ measurements ordered in time. When sensor $A$ and $B$ measurements occur at the same time, we place the sensor $A$ measurement in front of that of sensor $B$ for processing convenience. Let $k^{(A)}$ and $k^{(B)}$ denote the measurement indices for sensor $A$ and $B$, respectively. When the $k$th measurement is the $k^{(A)}$th measurement from sensor $A$, $\underline{w}(k) = \Sigma_A(k^{(A)})\underline{w}_A(k^{(A)})$ and

$$
\underline{z}(k) = \underline{z}_A(k^{(A)}), \quad \Sigma(k) = \begin{bmatrix} \Sigma_A(k^{(A)}) & \mathbf{0} \end{bmatrix},
\tag{13}
$$

where $\mathbf{0}$ denotes an all-zero matrix. If the $k$th measurement is the $k^{(B)}$th measurement from sensor $B$, $\underline{w}(k) = \Sigma_B(k^{(B)})\underline{w}_B(k^{(B)})$ and

$$
\underline{z}(k) = \underline{z}_B(k^{(B)}), \quad \Sigma(k) = \begin{bmatrix} \mathbf{0} & \Sigma_B(k^{(B)}) \end{bmatrix}.
\tag{14}
$$

When sensor $A$ and $B$ measurements occur at the same time, since the time difference between the two measurements is zero, the transition matrix $\Phi(k+1, k)$ reduces to an identity matrix. The target dynamical model becomes

$$\underline{x}(k+1) = \underline{x}(k). \tag{15}$$

The proposed registration approach is based on the application of the two-stage Kalman estimator to the reformulated measurement model (12). The two-stage Kalman estimator is due to Ignagni [14]. At each sensor site, a bias-free estimator computes the bias-free target state estimate when a new measurement arrives. Another Kalman filter cycle is used to estimate the sensor registration errors based on the measurement residual of the bias-free Kalman estimator. The *a posteriori* estimates of the bias-free target states and the sensor biases along with their corresponding covariance matrices are propagated to the sensor site where the next measurement is received. The algorithm iterates between the sensors and provides accurate sensor registration estimates as sensor measurements arrive.

## 4. PERFORMANCE EVALUATION

We first use computer simulations to study the performance of the proposed registration approach. Two sensors are simulated with sensor $A$ located at the system origin and sensor $B$ at $(u, v)$, where $u = 300$km and $v = 0$km. The standard near constant velocity model is used for the target motion dynamics. The standard deviation of the process noise $\sigma_\nu$ is set to $0.06$ times the mean velocity in one sampling interval.

The track is simulated to have a slope of 1 relative to the line connecting the two sensors. This track pattern is one of the most typical track patterns that would be encountered by a pair of sensors in sensor fusion applications. The sampling intervals are assumed to be $4s$ and $3s$ for sensor $A$ and $B$, respectively. The time delays at the beginning of the two sensor measurements are assumed to be zero.

The LS method [5] is used for comparison. The LS algorithm is a special case of the GLS algorithm [1] in which the measurement covariance is assumed to be an identity matrix. Since the LS method requires synchronized sensor measurements, we use the one-step fixed-lag smoothing approach proposed by Helmick *et al.* [15] to time-translate sensor $A$ measurements to the times of sensor $B$ measurements. For simplicity, a single near constant velocity model is used for the target dynamics. In the smoother, a one-step predictor is first used to predict to the subsequent sensor $B$ measurement time, which is corrected using the next sensor $A$ measurement to produce the smoothed estimates. The smoothed estimates are then passed to the LS method for sensor registration.

Figures 1 shows the variations of the standard deviation (STD) of the sensor registration error estimates for different measurement noise cases. Five measurement noise cases are simulated. The noise is assumed to be *i.i.d* Gaussian distributed with zero mean and standard deviations given in Table 1. The sensor registration errors are simulated as $\Delta r_A = 1$km, $\Delta\theta_A = 0.0105$rad, $\Delta r_B = -0.8$km and $\Delta\theta_B = 0.0087$rad. The numbers of sensor measurements are $K_1 = 100$ and $K_2 = 135$ for sensor $A$ and $B$, respectively. Each test is repeated 200 times to obtain the averaged results. In the figure, the lines with a diamond legend denote the LS results and lines with 'x' are results obtained by the Kalman filter based approach. The Kalman filter based approach demonstrates clear improvements over the LS method. The LS method performs poorly and fails when the measurement noise increases.

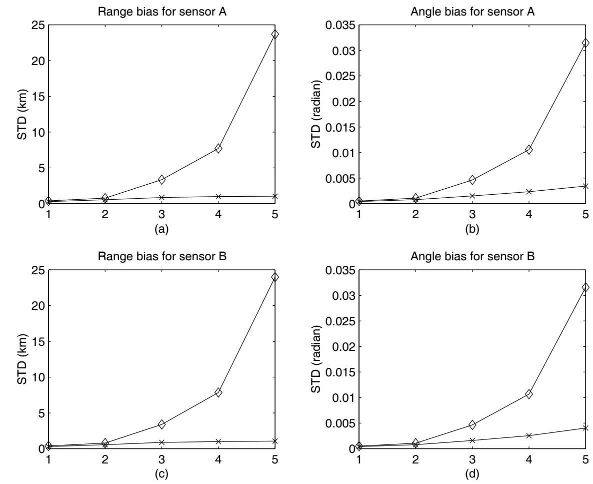


**Fig. 1**. Variation of the STDs of the sensor registration error estimates via different measurement noise.

**Table 1**. Standard Deviations of Noise in Range and Azimuth

| | case 1 | case 2 | case 3 | case 4 | case 5 |
|---|---|---|---|---|---|
| $\sigma_{rA}$ (km) | 0.05 | 0.1 | 0.5 | 1.0 | 1.5 |
| $\sigma_{\theta A}$ (rad) | 0.0056 | 0.0011 | 0.0028 | 0.0056 | 0.0111 |
| $\sigma_{rB}$ (km) | 0.05 | 0.1 | 0.5 | 1.0 | 1.5 |
| $\sigma_{\theta B}$ (rad) | 0.0056 | 0.0011 | 0.0028 | 0.0056 | 0.0111 |

We also apply the Kalman filter based registration approach to real-life multiple radar data collected from an air surveillance network. The radar network is composed of seventeen 24-by-24 foot phased array L-band long range radars located along the coastline of Canada. With a range of 200 nautical miles, these long range radars measure the range, azimuth as well as the elevation of the targets. Each radar also carries an IFF for target identification. Tracks of air targets arise from commercial airlines.

To remove the effects of the false targets (clutters), target ID's provided by the identification of friend and foe (IFF) beacon are used to extract the true aircraft tracks from radar returns for registration calculation. Each site in the system has two satellite dishes for communications, and the communications between the radars are via the Anik communications satellite. This radar network employs the stereographic projection [16] to map the elliptic earth on a plane to get the stereographic ground range from the slant range of a target. The target azimuth is measured relative to the true north at the radar location and is adjusted so that it is relative to the true north at the origin of the common coordinate system. Figure 2 shows the target track measurements collected by two radars of the air surveillance network. In the figure, the solid lines are tracks observed by radar A and the dots denote the tracks observed by radar B. The measurements have been converted to the common system coordinate with the location of radar $A$ as the origin. Radar $B$ is located at $(u, v) = (272.6406, 19.3287)$ nautical miles in the system plane. Five tracks are reported by radar $A$ and $B$. We use a segment of track 2 data for registration. The number of data points is 45. The standard deviation of the noise process of the target dynamics is chosen to be approximately equal to the maximum acceleration observed. This gives $\sigma_{\nu x} = 0.0023$ nautical miles/$s^2$ and $\sigma_{\nu y} = 0.0059$ nautical miles/$s^2$ in the $x$ and $y$

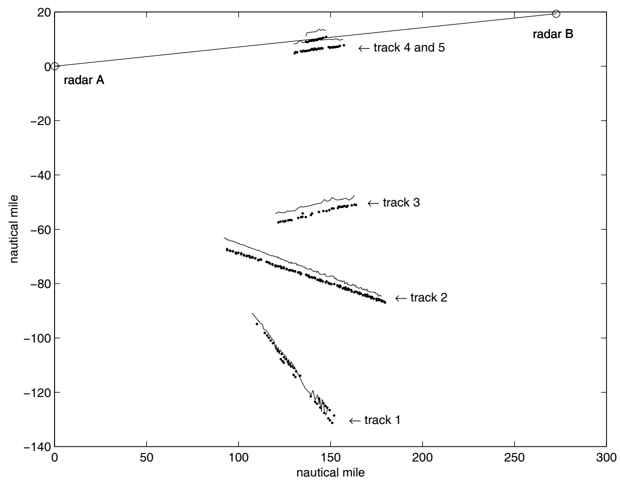


**Fig. 2**. Real target track measurements collected by two radars of the air surveillance network.

direction, respectively. The sensor registration errors are then estimated by applying the Kalman filter based registration approach and the LS method.

Since there is no ground truth about the sensor registration errors, i.e., the actual radar biases are unknown, we use the distance between the track measurements by the two sensors before and after the registration as an performance index to measure the quality of registration algorithm. The distances between the tracks that are used for estimating the sensor registration errors before registration are given by $3.1704$ nautical miles. After registration, the distance reduces to $0.2731$ and $0.5055$ nautical miles, for the Kalman filter based approach and the LS method, respectively. Both methods are able to successfully reduce the track distance after registration to a great extent. However, the Kalman filter based approach outperforms the LS method in that it produces a track distance that is only about half of that by the LS method. We also apply the sensor registration error estimates to the other tracks that are not used in the registration process to examine the generalization ability of the algorithms. In Table 2, we list the calculated track distances for different tracks before and after registration using the Kalman filter based approach and the LS method. It can be seen that both the Kalman based approach and the LS method can reduce the track distance for tracks that are not used in the registration process. However, the generalization errors by the Kalman filter based approach are smaller than those by the LS method. In particular, for track 1, a segment of which is used in the registration process, the generalization error by the Kalman filter based approach is only half of that by the LS method. This indicates that the Kalman based approach has a better generalization ability than the LS method.

**Table 2**. Track Distances Before and After Registration

| | before reg. (nautical miles) | LS method (nautical miles) | Kalman appro. (nautical miles) |
|---|---|---|---|
| T 1 | 3.7795 | 0.5548 | 0.2816 |
| T 2 | 3.3720 | 0.7618 | 0.5406 |
| T 3 | 4.4511 | 0.5666 | 0.3159 |
| T 4 | 3.0976 | 0.7014 | 0.5150 |
| T 5 | 3.0174 | 0.6491 | 0.5240 |

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