DECISION FUSION WITH CENSORED SENSORS

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ABSTRACT

Motivated by the sensor censoring idea, we consider in this paper a canonical decentralized detection problem with each sensor employing an on/off signaling scheme. Novel to the current work is the integration of the fading transmission channels in the fusion algorithm design. The on/off signaling, in addition to its low communication overhead which is crucial to bandwidth limited applications, also enables the decision fusion being carried out without the knowledge of channel phase. Resorting to incoherent detection schemes, we develop optimal fusion rules for the following two scenarios: 1) when the fading channel envelopes are available at the fusion center; and 2) when only the fading statistic is available. Under the low signal-to-noise ratio regime, we further reduce the optimal fusion rule into simple nonlinearities that are both easy to implement and are not subject to prior knowledge constraint.

1. INTRODUCTION

Distributed detection involving multiple sensors observing a common phenomenon has been pursued for years using classical inference tools. The recent emergence of wireless sensor networks (WSN), however, has added some new dimensions into this classical problem. For example, transmission of local sensor decisions to the fusion center becomes an integral part of the system. Previous work mostly assumes that local sensor decisions are accessible at the fusion center which may be justifiable from a pure information theoretic point of view (i.e., rate v.s. capacity). This needs to be revisited, however, in the context of WSN as practical constraints, including power, bandwidth, cost, and delay, may render this assumption unrealistic.

Attempt has been made to integrate the transmission and processing (fusion) in order to achieve better performance with limited resources. Examples include channel aware decision fusion algorithms [1,2] and channel optimized sensor signaling (quantization) scheme [3] that coherently combine the information transmission and decision making into a unified process. In [1,2], under the canonical parallel sensor fusion paradigm, optimal and suboptimal fusion algorithms are developed that take into account the presence of fading channels. The results, however, relies on some assumptions that are not always easy to satisfy in practices. For example, the phase information of the flat fading channels is assumed known at the fusion center. This may not be reasonable for resource constrained WSN applications.

Motivated by the sensor censoring idea, first proposed by Rago *et al* [4], we propose the on/off signaling scheme for local sensor output. In addition to the reduced communication rate, this on/off signaling allows the fusion center to employ fusion rules that do not rely on the channel phase information, much in the same way as incoherent detection can be devised for on/off signaling in digital communications [5]. With censored sensors, only sensors with informative observation, measured by its local likelihood ratio (LR) value, send their LR to the fusion center. Using the canonical parallel fusion structure with binary hypothesis and conditional independent sensor observations, it was shown in [4] that the optimal 'no-send' region, in both the Bayesian and Neyman-Pearson (NP) sense, was a single interval in the LR domain. This is illustrated in Fig. 1(a) where $[t_1, t_2]$ correspond to the 'no-send' region. Furthermore, in the case of sufficiently small prior probability of the target-present hypothesis and severe communication constraint, the optimal (in the sense of minimizing error probability) lower threshold of the 'no-send' region was shown to be 0, i.e., $t_1 = 0$ (see Fig. 1(b)). Similar result was also established later in [6] using the NP criterion. An intuitive explanation is that when a target is less likely to be present, the extreme communication constraint prohibits sending low LR value that happens much more often.

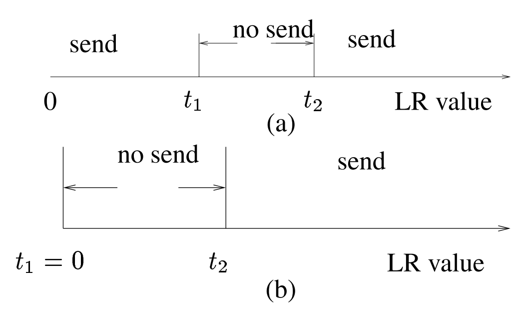

Fig. 1. (a) Sensor censoring region; (b) Special case when t_1=0.

For the case with $t_1 = 0$, the sensor censoring is effectively a LR test (LRT)-based transmission scheme: whenever the local LR exceeds t_2, the sensor transmits the LR; otherwise the sensor remains silent. In this paper, we take the above censoring sensor to its extreme case – if the local LR exceeds t_2, the sensor sends only a single bit indicating that the LR falls into the 'send' region, instead of sending the LR value in its entirety. This is again, motivated by the resource constraint in WSN applications – the energy consumption is proportional to the number of bits transmitted. Further, such extreme sensor censoring greatly simplifies our ensuing derivation and provides insights into some simple intuitive test statistic. For example, since only alarmed sensors send signals to the fusion center, one intuitive detection scheme is to employ an energy detector. Indeed, we show that this simple scheme is the optimal detector in the low signal-to-noise ratio (SNR) regime.

We remark here that our emphasis in this work is the development of fusion algorithms with on/off signaling for a fading environment. Another important issue, the local sensor signaling (i.e., the selection of the threshold t_2), is not addressed in this paper. We assume, instead, that local sensors employ the extreme sensor censoring scheme with known t_2. Therefore, the local sensor performance indices (probabilities of false alarm and detection) can be calculated directly. The organization of the paper is as follows. In the next section, we introduce the system model and derive the optimal LR based fusion rule with the knowledge of channel fading envelope. Two suboptimal fusion statistics are also provided. In Section 3, we derive, under a Rayleigh fading channel model, an optimal fusion rule assuming only the knowledge of the channel statistics. Numerical simulation is provided in Section 4 followed by conclusion in Section 5.

2. SYSTEM MODEL AND OPTIMAL FUSION RULE WITH FADING CHANNEL ENVELOPE

The binary sensor censoring system with a parallel fusion structure is depicted in Fig. 2. The K sensors collect observations and calculate their respective LR values. For each sensor, if its LR value exceeds a pre-calculated threshold, it transmits a binary signal (say, $u_k = 1$) to a fusion center. Otherwise, if the LR falls below the threshold, $u_k = 0$, i.e., the sensor remains silent during this transmission period. We assume the observations are independent across sensors conditioned on any hypothesis. The probabilities of false alarm and detection of the k^{th} local sensor node are denoted by P_{fk} and P_{dk}, respectively. These performance indices are assumed known at the fusion center in deriving the optimal LR based fusion rule. The local sensor outputs, u_k for $k = 1, \cdots, K$, are transmitted over parallel channels that are assumed to undergo independent fading and we denote by h_k and ϕ_k the fading envelope and phase, respectively. We further assume a slow fading channel whereby the channel remains constant during the transmission of one decision.

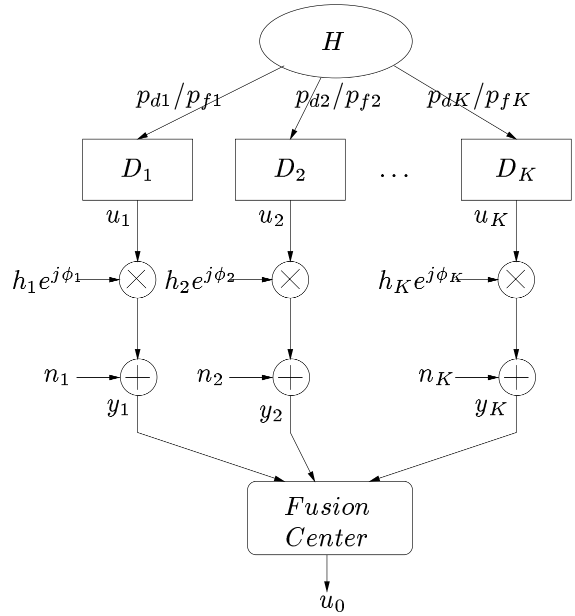

Fig. 2. Parallel fusion model in the presence of fading and noisy channel between the local sensors and the fusion center

The above model results in the channel output for the k_{th} sensor, y_k, being

$$y_k = \begin{cases} n_k & \text{The } k^{th} \text{ sensor decides } H_0. \\ h_k e^{j\phi_k} + n_k & \text{The } k^{th} \text{ sensor decides } H_1. \end{cases}$$

where n_k is a zero mean complex Gaussian noise whose real and imaginary parts have equal variance σ^2. Further, n_k's are assumed to be independent across sensors.

If both h_k and ϕ_k are known, an optimum LR based decision fusion rule can be easily derived which is essentially the same as that in [2, 7]. Notice that we are replacing $\{+1, -1\}$ with $\{1, 0\}$ with censoring, hence with phase information, the equivalence between the two schemes (save some scaling factors) is reminiscent to the rotational invariance principle in digital modulation. Thus we concentrate now on the incoherent case, i.e., we develop fusion statistics based on the output envelope, or equivalently, output power.

Denoted by z_k the signal power for the k^{th} channel output, i.e., $z_k = |y_k|^2$, it is easy to get

$$\begin{aligned} p(z_k|u_k = 0, h_k) &= \frac{1}{2\sigma^2} e^{-\frac{z_k}{2\sigma^2}} \\ p(z_k|u_k = 1, h_k) &= \frac{1}{2\sigma^2} I_0 \left(\sqrt{\frac{h_k^2 z_k}{\sigma^4}} \right) e^{-\frac{h_k^2 + z_k}{2\sigma^2}} \end{aligned}$$

where $I_0(.)$ is zeroth order modified Bessel function of the first kind. Using z_k instead of y_k in the fusion rule design and assuming knowledge of the fading channel envelope and the local sensor performance indices, the log LR (LLR) can be derived in a straightforward manner as

$$\Lambda = \log \frac{f(z_1, \cdots, z_K|H_1)}{f(z_1, \cdots, z_K|H_0)} = \sum_k \log \frac{P_{dk} I_0 \left(\sqrt{\frac{h_k^2 z_k}{\sigma^4}}\right) e^{-\frac{h_k^2}{2\sigma^2}} + (1 - P_{dk})}{P_{fk} I_0 \left(\sqrt{\frac{h_k^2 z_k}{\sigma^4}}\right) e^{-\frac{h_k^2}{2\sigma^2}} + (1 - P_{fk})} \quad (1)$$

We consider next the low SNR approximation for Λ.

Proposition 1 *As the channel noise variance $\sigma^2 \to \infty$, i.e., SNR $\to 0$, Λ in equation (1), assuming identical sensor performance, reduces to the following statistic:*

$$\Lambda_{IMRC} = \frac{1}{K} \sum_k h_k \sqrt{z_k} \quad (2)$$

To show this, we use the fact that as $\sigma^2 \to \infty$,

$$I_o \left(\frac{h_k}{\sigma^2}\sqrt{z_k}\right) \to e^{\frac{h_k}{\sigma^2}\sqrt{z_k}}.$$

Plug this into equation (1) and use, for small x, $e^{-x} \approx 1-x$, we have

$$\Lambda \overset{\sigma^2 \to \infty}{=} \sum_k \log \left[1 + \frac{(P_{dk} - P_{fk}) \left(\frac{h_k}{\sigma^2}\sqrt{z_k} - \frac{h_k^2}{2\sigma^2}\right)}{1 + P_{fk} \left(\frac{h_k}{\sigma^2}\sqrt{z_k} - \frac{h_k^2}{2\sigma^2}\right)}\right]$$

Use $\log(1+x) \approx x$ for $x \to 0$, we have

$$\Lambda \approx \sum_k (P_{dk} - P_{fk}) \left(\frac{h_k}{\sigma^2}\sqrt{z_k} - \frac{h_k^2}{2\sigma^2}\right)$$

which is equivalent to equation (2) under the assumption of identical sensors.

This is very similar to the maximum ratio combining (MRC) fusion statistic presented in [1] except instead of phase coherent combining, we use envelope ($\sqrt{z_k}$) here, hence the name incoherent MRC (IMRC). One can further simplify this statistic, somewhat heuristically, by applying equal weight to the envelope, i.e.,

$$\Lambda_{IEGC} = \frac{1}{K} \sum_k \sqrt{z_k} \quad (3)$$

which, for similar reason, is termed incoherent equal gain combiner (IEGC). This IEGC fusion rule has a simple form and does not require the channel gain information.

3. CHANNEL STATISTICS BASED FUSION RULE

The optimal fusion rule developed in the previous section requires knowledge of the channel fading envelope. Due to the limited resources (energy and/or bandwidth), this information may not be available at the fusion center. Without this envelope information but with knowledge of channel phase, a new LR based fusion rule that requires only the knowledge of the fading channel statistics has been developed in [2]. In the absence of both channel envelope and phase information, we derive in this part the channel statistic-based LRT using the channel output power z_k.

Without loss of generality, we assume that the Rayleigh fading channel has unit power, i.e., $E[h_k^2] = 1$. Thus,

$$p(h_k) = 2h_k e^{-h_k^2}$$

We then calculate the conditional probability density function (pdf) $p(z_k|u_k)$

$$p(z_k|u_k) = \int_0^\infty p(z_k|h_k, u_k) p(h_k) dh_k$$

These can be computed straightforwardly as

$$p(z_k|u_k = 0) = \frac{1}{2\sigma^2} e^{-\frac{z_k}{2\sigma^2}}$$

$$p(z_k|u_k = 1) = \frac{1}{1 + 2\sigma^2} e^{-\frac{z_k}{1+2\sigma^2}}$$

Plug the two conditional pdf into the LLR, we have

$$\Lambda' = \sum_k \log \frac{P_{dk} \frac{1}{1+2\sigma^2} e^{-\frac{z_k}{1+2\sigma^2}} + (1 - P_{dk}) \frac{1}{2\sigma^2} e^{-\frac{z_k}{2\sigma^2}}}{P_{fk} \frac{1}{1+2\sigma^2} e^{-\frac{z_k}{1+2\sigma^2}} + (1 - P_{fk}) \frac{1}{2\sigma^2} e^{-\frac{z_k}{2\sigma^2}}} \quad (4)$$

Again, consider low SNR regime and we have

Proposition 2 *At low channel SNR, i.e., $\sigma^2 \to \infty$, the LLR in equation (4) reduces to a form equivalent to*

$$\sum_k \frac{1}{K} (P_{dk} - P_{fk}) \frac{z_k}{2\sigma^2(1 + 2\sigma^2)} \quad (5)$$

The proof is straightforward by applying first order Taylor series expansion for e^{-x} and $\log(1+x)$ for $x \to 0$.

With identical local sensors, the above low SNR approximation of LLR is equivalent to the energy detector (ED)

$$\Lambda_{ED} = \frac{1}{K} \sum_k z_k$$

4. PERFORMANCE EVALUATION

Fig. 3 shows the detection probability as a function of channel SNR for various fusion statistics, including the optimal LRT assuming the knowledge of the channel phase information [1]. The system false alarm rate is fixed at $P_{f0} = 0.01$. In this example, the total number of sensor is 8 with sensor level $P_{fk} = 0.05$ and $P_{dk} = 0.5$. From the NP lemma, it is clear that the LR based fusion rule provides the best detection performance. Among the three LRT, the performance degrades as the prior information utilized in each LRT decreases. Thus coherent LRT performs better than incoherent LRT using channel fading envelope, which in turn is better than incoherent LRT using only the fading statistics. Further, as SNR decreases, both incoherent LRT approach to their corresponding low SNR approximations, the IMRC and ED statistics. Somewhat surprisingly, the heuristic IEGC fusion statistic seems to perform better than IMRC and ED at moderate SNR values.

To further compare the performance among the three suboptimal statistics, one can use some single letter performance measures, such as the so-called deflection measure [8]. Notice that all three statistics bear the form of sum of independent and identically distributed random variables hence with a limiting distribution as Gaussian. Deflection measure for Gaussian hypotheses testing amounts to the detection SNR, thus provides a natural performance characterization. The deflection measure is defined as

$$D(\Lambda) = \frac{[E(\Lambda|H_1) - E(\Lambda|H_0)]^2}{Var(\Lambda|H_0)}$$

Fig. 4 shows the deflection measure for IMRC, IEGC, and ED statistics. As easily seen in the figure, IEGC appears to perform as well as or better than the other two at moderate SNR values. This is consistent with the result in Fig. 3.

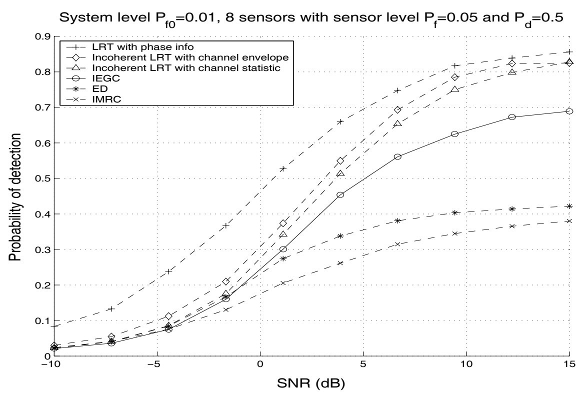

Fig. 3. Probability of detection as a function of channel SNR for Rayleigh fading channels

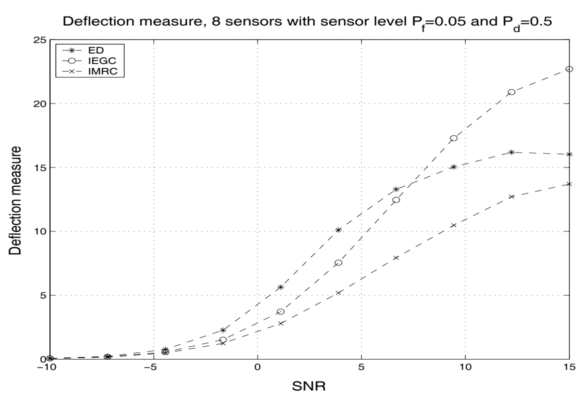

Fig. 4. The deflection measure for the three suboptimal statistics.

5. CONCLUSION

Fusion of the decisions with on/off signaling and transmitted over fading channels in WSN is studied in this paper. The on/off signaling, in addition to lower communication overhead, allows the development of fusion statistics without knowledge of channel phase information. Both cases, one assuming the knowledge of channel fading envelope and the other the fading statistics only, are treated and optimal LRT are derived under each scenarios. Suboptimal detection statistics, derived as low SNR approximations of the optimal LRT, are obtained and provide some theoretical justification to some intuitive test statistics, such as the energy detector.

In the current work, we address only the decision fusion rule design assuming a fixed sensor censoring threshold. Optimal censoring schemes that jointly consider the transmission channel will be investigated in our future work.

6. REFERENCES

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