DOA ESTIMATION FOR BROADBAND CHIRP SIGNALS

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Abstract- We derive two new methods, based on the ambiguity function, for estimating the direction-of-arrival (DOA) of broadband chirp signals. These new methods make use of the time-frequency structure of the chirp signal and can be applied to any aperture array and any chirp rate. As long as the signals are separable in the ambiguity function plane, the number of sources can be greater than the number of sensors. These methods can be applied to single chirp and multiple chirps with either the same or different chirp rates. Simulation results show that the DOA estimation performance can be improved, especially for the low SNR and spatially close sources' cases.

I. INTRODUCTION

The conventional broadband DOA estimation methods usually use Fourier transform to decompose the broadband signal into narrow band signals. Subsequently, the narrowband DOA estimation method is used in each frequency bin or in the focused/ interpolated data. When these methods are applied to signals with distinctive time-frequency structure, such as chirps, the SNR in each frequency bin may be very low and this usually results in poor DOA estimation performance.

Recently, there are some works dealing with the DOA estimation for chirp signals [1]-[5]. The DOA estimation results are improved considerably, especially for the low SNR case. But the method proposed in [1] can be only used for narrowband chirp signals. The method in [2] uses an iterative algorithm to estimate the broadband chirp DOA, but the convergence is not guaranteed. The methods presented in [3] and [4] assume that the instantaneous signal frequencies do not change during the time needed for a wave to travel across the array aperture. This assumption is not always true, especially for larger aperture array and high chirp rate signals. The method proposed in [5] requires that the chirp rates of multiple signals are different.

In this paper, we introduced two broadband DOA estimation methods for chirp signals based on the ambiguity function [6]. The advantages of our proposed methods are (1) they can be used for any aperture size and chirp rate, (2) the number of sources can be greater than the number of sensors, (3) no special resolution limitation, (4) the signal frequency can be higher than the normal design frequency, (5) the multiple chirps can have different or the same chirp rate and (6) the methods can be applied to low SNR cases. The proposed methods require that the chirp rates are known and the signals must be separable in the ambiguity function plane. In active sonar, radar and communication applications, the chirp rate is usually known. If the signal's chirp rate is not known a priori, the chirp rate estimation methods in [7], [8] can be applied first.

II. SIGNAL MODEL AND AMBIGUITY FUNCTION

Assume an array of M sensors receiving L wideband chirp signals emanating from the unknown directions $\{\theta_l\}_{l=1}^{L}$. The Mx1 vector of sensor outputs can be modeled as:

$$\mathbf{x}(t) = \mathbf{A}(\theta,t)\mathbf{s}(t) + \mathbf{n}(t) \tag{1}$$

where $\mathbf{A}(\theta,t) = [\mathbf{a}(\theta_1,t),\ldots,\mathbf{a}(\theta_L,t)]$ is the array manifold, $\mathbf{a}(\theta_i,t)$ is the Mx1 time-varying direction vector of the i^{th} source at the time t, $\theta = [\theta_1,\theta_2,...,\theta_L]^T$ is the Lx1 DOA vector, $\mathbf{s}(t) = [s_1(t), s_2(t),..., s_L(t)]^T$ is the Lx1 vector of the L source signals at the time t and $\mathbf{n}(t)$ is the Mx1 vector of white noise.

A chirp signal is described by:

$$s(t) = pe^{j2\pi(\alpha t + \frac{\beta}{2}t^2)} \tag{2}$$

where p is the amplitude, α is the lower frequency, ß is the chirp rate and $0 \le t \le T$. As the frequency changes with time, the array manifold is time dependent as well. Assuming a chirp source located at direction ?, the m[th] sensor output arising from this signal is:

$$x_m(t) = s(t-\tau_m) + n(t) = s(t)e^{-j2\pi(\alpha\tau_m - \frac{\beta}{2}\tau_m^2)}e^{-j2\pi\beta t\tau_m} + n(t) \tag{3}$$

where τ_m is the time delay relative to the first sensor and it depends on the source direction, n(t) is white Gaussian noise and is not correlated with the signal. From Equation (3), we note that the direction vector depends not only on space but also on time, which can be written as,

$$\mathbf{a}(\theta,t) = [1, e^{-j2\pi(\alpha\tau_1 - \frac{\beta}{2}\tau_1^2)}e^{-j2\pi\beta t\tau_1},..., e^{-j2\pi(\alpha\tau_{M-1} - \frac{\beta}{2}\tau_{M-1}^2)}e^{-j2\pi\beta t\tau_{M-1}}]^T \tag{4}$$

The ambiguity function of signal *x(t)* is defined as,

$$A_x(u,f) = \int x(t)x^*(t-u)e^{-j2\pi ft}dt \tag{5}$$

where u is time delay, f is the Doppler frequency, and * refers to the complex conjugate. To simplify the discussion, we consider the single chirp scenario at first. The solution for multiple chirps will be discussed later. Replacing the m[th] sensor output (3) into (5) and considering that the signal and the white noise are not correlated, we arrive at,

$$A_{x_m}(u,f) = A_s(u,f)e^{-j2\pi\beta u\tau_m} + E_n\delta(u,f) \tag{6}$$

where E_n is the noise energy,

$$\delta(u,f) = \begin{cases} 1, & u=0, f=0 \\ 0, & others \end{cases} \tag{7}$$

and $A_s(u,f)$ is the signal ambiguity function given by,

$$\begin{aligned} A_s(u,f) &= \int s(t)s^*(t-u)e^{-j2\pi ft}dt = \\ &= p^2(T-|u|)e^{-j\pi(f-\beta u)(T+u)}e^{j2\pi(\alpha u - \frac{\beta}{2}u^2)}\operatorname{sinc}[(T-|u|)(f-\beta u)] \end{aligned} \tag{8}$$

where $-T \le u \le T$. This shows that the signal energy is concentrated along the straight line: $f = \beta u$ and passes through the origin point u=0 and $f = 0$.

When the chirp rate is known, the two dimensional matrix (8) can be reduced to a one dimensional vector by using the values located only along the straight line $f = \beta u$:

$$A_{x_m}(u) = A_{x_m}(u, f)\Big|_{f=\beta u} = A_s(u)e^{-j2\pi\beta u\tau_m} + E_n\delta(u) \tag{9}$$

where

$$A_s(u) = A_s(u, f)\Big|_{f=\beta u} = p^2(T - |u|)e^{j2\pi(\alpha u - \frac{\beta}{2}u^2)} \tag{10}$$

The importance of using the ambiguity function is that the signal's absolute time t is converted into Doppler frequency, which is related to the time delay. The time delay is a known parameter and a relative variable that does not depend on the signal absolute time. Another observation is that the noise is also concentrated at the origin, reducing considerably its influence on the quality of the DOA estimates.

III. INCOHERENT CHIRP BASED BROADBAND DOA ESTIMATION (ICCBBDOA)

From equation (9), the direction vector is:

$$\mathbf{a}(\theta, u) = [1, e^{-j2\pi\beta u\tau_1}, ..., e^{-j2\pi\beta u\tau_{M-1}}]^T \tag{11}$$

Comparing Equations (4) and (11), the new direction vector depends on the source DOA and the known time delay u. Thus, the source DOA is the only unknown parameter in this expression instead of two unknowns as in Equation (4).

The obtained direction vector is time-delay dependent, therefore the delay-and-sum DOA estimation method can be applied to (9) for each u. By averaging the spectra, it obtains the incoherent chirp based broadband DOA estimation method (ICCBBDOA) where ßu is equivalent to the signal frequency in conventional spatial processing.

From the direction vector, when the time delay is close to zero, there is no spatial resolution. And just as in the spatial processing, the equivalent frequency is limited by the sensors' interspacing. Another observation is the very low SNR at the beginning and the end of the ambiguity function because of the short signal overlap. In conclusion, the time delay for ICCBBDOA method will be limited by:

$$0 < |u| < \min\left[T, c/2d\beta\right] \tag{12}$$

where c is the wave propagation speed, d is the inter-element distance and min[x,y] refers to the smaller between x and y. The noise influence is reduced as well.

As the ambiguity function is not related to the signal absolute time, the different parts of the signals can be used to compute the ambiguity functions. Therefore, the full rank covariance matrix can be obtained by averaging them just as in the incoherent DOA estimation approach. The correlation based DOA estimation methods, such as MVDR and MUSIC can be applied. However, this is computationally expensive and in the next section, we introduce a more efficient method.

IV. COHERENT CHIRP BASED BROADBAND DOA ESTIMATION (CCBBDOA)

By applying the focusing method in [3] to Equation (9), we can obtain the coherent broadband DOA estimation, albeit at an additional computational cost. Here we propose a new method for coherent broadband DOA estimation for chirp signal based on the result of the previous part.

From Equation (9), the product of the ambiguity function at $u + \Delta u$ of the m[th] sensor and its conjugate at u is given by,

$$\begin{aligned} P_{x_m}(u) &= A_{x_m}{}^*(u)A_{x_m}(u + \Delta u) \\ &= p^4(T - |u|)(T - |u + \Delta u|)e^{-j2\pi(\alpha\Delta u - \beta u\Delta u - \frac{\beta}{2}\Delta u^2)}e^{-j2\pi\beta\Delta u\tau_m} \end{aligned} \tag{13}$$

where $\Delta u = U$ is a positive constant, and $-T \le u \le T - U$. The product of the m[th] sensor $P_{x_m}(u)$ and the n[th] sensor $P_{x_n}{}^*(u)$ is,

$$P_{x_m}(u)P_{x_n}{}^*(u) = p^8(T - |u|)^2(T - |u + \Delta u|)^2 e^{-j2\pi\beta\Delta u(\tau_m - \tau_n)} \tag{14}$$

From this result, the direction vector becomes:

$$\mathbf{a}(\theta) = [1 .e^{-j2\pi\beta\Delta u\tau_1 c}, ..., e^{-j2\pi\beta\Delta u\tau_{M-1}}]^T \tag{15}$$

It depends only on the DOA if Δu is constant. Thus, the narrow band DOA estimation methods can be applied directly to $\mathbf{P_x}(u)$. The covariance matrix for the DOA estimate is calculated by,

$$\mathbf{R_P} = \frac{1}{2T - U + 1}\sum_{u=-T, u\neq 0}^{T-U}\mathbf{P_x}(u)\mathbf{P_x}^H(u) \tag{16}$$

where $\mathbf{P_x}(u)$ is a Mx1 vector at time delay u. Any narrowband DOA estimation method can therefore be applied here.

We refer to this method as the coherent chirp based broadband DOA estimation method (CCBBDOA). It provides a direct broadband DOA estimation for chirp signals without the focusing procedure. Because the selection of the one-dimensional ambiguity function from the two ambiguity function is equivalent to a 2 dimensional filtering, the obtained result is more robust to noise compared to the conventional incoherent broadband DOA estimation methods.

For CCBBDOA, the equivalent frequency is $\beta\Delta u$, the larger the Δu , the better the spatial resolution. However, the equivalent frequency is limited by the sensors' interspacing d. Thus Δu should satisfy,

$$0 < \Delta u \le c/2d\beta \tag{17}$$

In the derivation of (14), the noise is neglected as it is concentrated at only one point $u = 0$, and the DOA estimation is based on the average of certain points. The CCBBDOA method is hence more efficient than the ICCBBDOA in computation when used with MUSIC and MVDR.

V. MULTIPLE CHIRPS

For the case of two chirp signals with different parameters, the m[th] sensor output can be expressed as:

$$x_m(t) = pe^{j2\pi(\alpha_1(t-\tau_{m1}) + \frac{\beta_1}{2}(t-\tau_{m1})^2)} + qe^{j2\pi(\alpha_2(t-\tau_{m2}) + \frac{\beta_2}{2}(t-\tau_{m2})^2)} + n(t) \tag{18}$$

where p and q are amplitudes, α_1, α_2 are the lower frequencies, β_1, β_2 are the chirp rates, and τ_{m1} and τ_{m2} are the time delays corresponding to the different DOAs. The ambiguity function of $x_m(t)$ is:

$$\begin{aligned} A_{x_m}(u, f) = {} & p^2 e^{j2\pi((\alpha_1 - \beta_1\tau_{m1})u - \frac{\beta_1}{2}u^2 + (f - \beta_1 u)\frac{T-|u|}{2})} \operatorname{sin}c[(f - \beta_1 u)(T - |u|)] + q^2(T - |u|)e^{j2\pi((\alpha_2 - \beta_2\tau_{m2})u - \frac{\beta_2}{2}u^2 + (f - \beta_2 u)\frac{T-|u|}{2})} \operatorname{sin}c[(f - \beta_2 u)(T - |u|)] \\ & + pq(T - |u + \tau_{m1} + \tau_{m2}|)\Big\{ e^{j2\pi((\alpha_1 - \beta_1\tau_{m1})u - \frac{\beta_1}{2}u^2 + \gamma_m)} \big\{ H(f) * \operatorname{sin}c[(f - \beta_1 u + \phi_m)(T - |u + \tau_{m1} + \tau_{m2}|)]e^{j\pi(f - \beta_1 u + \phi_m)(T - |u + \tau_{m1} + \tau_{m2}|)} \big\} + \\ & e^{j2\pi((\alpha_2 - \beta_2\tau_{m2})u - \frac{\beta_2}{2}u^2 - \gamma_m)} \big\{ H(-f) * \operatorname{sin}c[(f - \beta_2 u - \phi_m)(T - |u + \tau_{m1} + \tau_{m2}|)]e^{j\pi(f - \beta_2 u - \phi_m)(T - |u + \tau_{m1} + \tau_{m2}|)} \big\} \Big\} + E_n\delta(u, f) \end{aligned} \tag{19}$$

where

$$\phi_m = \alpha_1 - \alpha_2 - \beta_1\tau_{m1} + \beta_2\tau_{m2} \quad (20)$$

$$\gamma_m = \alpha_1\tau_{m1} - \alpha_2\tau_{m2} - \frac{\beta_1}{2}\tau_{m1}^2 + \frac{\beta_2}{2}\tau_{m2}^2 \quad (21)$$

$$H(f) = F\left[e^{j\pi(\beta_1-\beta_2)t^2} rect\left(\frac{t-(T-|u|)/2}{T-|u|}\right)\right] \quad (22)$$

F refers to the Fourier transform and *rect(t)* is the rectangle function. In Equation (19), the first two terms are the auto ambiguity functions of each chirp and the 3rd and 4th terms are the cross ambiguity functions between the two chirps. As the energies of the two auto-ambiguity functions are concentrated along $f = \beta_1 u$ and $f = \beta_2 u$, the one dimensional ambiguity function for each chirp can be obtained as:

$$A_{x_m,s_1}(u) = p^2(T-|u|)e^{j2\pi(\alpha_1 u - \frac{\beta_1}{2}u^2)}e^{-j2\pi\beta_1 u\tau_{m1}} + E_n\delta(u) \quad (23)$$

and

$$A_{x_m,s_2}(u) = q^2(T-|u|)e^{j2\pi(\alpha_2 u - \frac{\beta_2}{2}u^2)}e^{-j2\pi\beta_2 u\tau_{m2}} + E_n\delta(u) \quad (24)$$

Thus the previously proposed methods for single chirp scenario can be applied to (23) and (24) separately. The energies from the cross terms may be partially overlapping with the auto terms, but they can be treated as noise. The spatial resolution is not a problem as they are separated in the ambiguity function. The same method can be extended naturally to more than two chirps.

If the multiple chirps have the same chirp rate, the auto ambiguity functions for different chirps are overlapped in the ambiguity function plane. In this case, only the ICCBBDOA based delay-and-sum method can be applied directly to the auto terms for multiple chirps' DOA estimation. The correlation based methods can not be applied because the two sources are coherent. Here we propose to make use of the cross terms for this scenario.

As the two cross ambiguity functions are separated in the ambiguity function plane and have the same chirp rate as the auto terms but with different frequency shifts. They are concentrated along the straight lines:

$$f = \beta u + \alpha_1 - \alpha_2 - \beta\tau_{m1} + \beta\tau_{m2} \quad (25)$$

and

$$f = \beta u - \alpha_1 + \alpha_2 + \beta\tau_{m1} - \beta\tau_{m2} \quad (26)$$

The one dimensional cross ambiguity functions along these two lines can be expressed as:

$$A_{x_m,c1}(u) = pq(T - |u + \tau_{m1} + \tau_{m2}|)e^{j2\pi(\alpha_1 u - \frac{\beta}{2}u^2 + \gamma_m)}e^{-j2\pi\beta\tau_{m1}u} \quad (27)$$

and

$$A_{x_m,c2}(u) = pq(T - |u + \tau_{m1} + \tau_{m2}|)e^{j2\pi(\alpha_2 u - \frac{\beta}{2}u^2 - \gamma_m)}e^{-j2\pi\beta\tau_{m2}u} \quad (28)$$

As γ_m is related to the DOA of the two sources, the cross terms of (27) and (28) can not be used for ICCBBDOA. However, the CCBBDOA approach detailed in Equations (14)-(16) can be applied to (27) and (28) separately.

But, in order to use the cross terms for DOA estimation, we need to first detect the cross terms. A simple way to determine the cross terms is to detect the peaks in the auto correlation function away from the center point and then the cross terms should pass through these peaks with the known chirp rate.

When there are more than two chirps, the same method can be applied. The proposed ICCBBDOA and CCBBDOA require that the chirps and their cross terms must be separable in the time-delay and Doppler frequency plane.

VI. SIMULATION

Assume a linear array of 8 sensors with inter-element spacing 0.5m, an echo of an active source emanating from -30? with SNR -8dB. The transmitted signal frequency is from 0.5kHz to 1.5kHz within a time duration of 0.2s. The spatial spectra obtained by conventional incoherent broadband delay-and-sum and our proposed ICCBDOA based delay-and-sum are given in Fig. 1. In ICCBBDOA, one ambiguity function for each sensor is used for the whole signal. Figure 2 shows the same results for the case of a lower SNR of -10dB.

We next simulated the case with the same array as before, but with two sources of a SNR of 0dB, located at -30? and -10?. Their frequencies are [0.5kHz, 1.5kHz] and [1.5kHz, 0.5kHz]. The signals' time duration is 0.2s. Fig.3 and Fig.4 compare the results of the conventional incoherent methods with our proposed CCBBDOA based methods.

Finally, we simulated the case of two sources having the same chirp rate. The frequencies of the two sources are [0.1kHz, 1.1kHz] and [0.5kHz 1.5kHz] and their time durations are 0.2s. The DOA estimation results obtained by incoherent broadband methods and our CCBBDOA method are given in Fig.5 and Fig.6.

From these results, the output SNR of the ICCBBDOA based delay-and-sum method is about 5 times that of the incoherent broadband delay-and-sum method. The last two experiments show that our proposed CCBBDOA methods have better resolution and estimation accuracy, regardless of whether the chirp rates are the same or different.

VII. CONCLUSION

In this work, we have proposed 2 new methods for the DOA estimation of broadband chirp signals. Using the ambiguity function of the sensor outputs, the spatial and time structure of the chirp signal are translated to the spatial and time-delay plane. As the time-delay is a known parameter, the two unknown parameters problem is converted to a single unknown parameter problem. This makes it possible to apply existing narrowband DOA estimation methods to the ambiguity function. These two methods consider the exact time-space structure of the chirp signal which can be applied in any aperture array and any chirp rate. As the signals are separated in ambiguity plane, there is no limitation in the number of sources and special resolution. Simulation results show that our proposed methods are more robust to noise and have higher resolution than the conventional incoherent DOA estimation methods.

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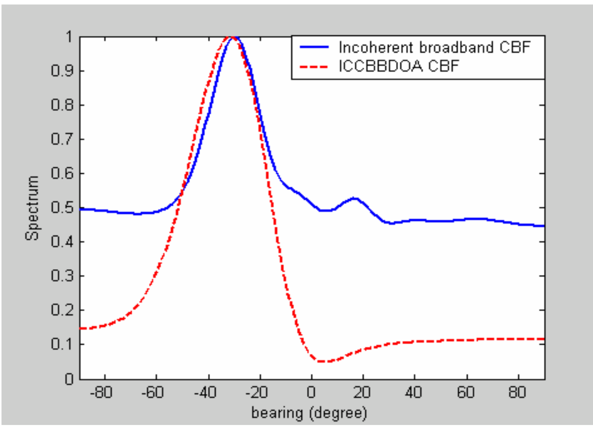

Fig.1 Comparison of incoherent CBF ICCBBDOA CBF (SNR = -6dB)

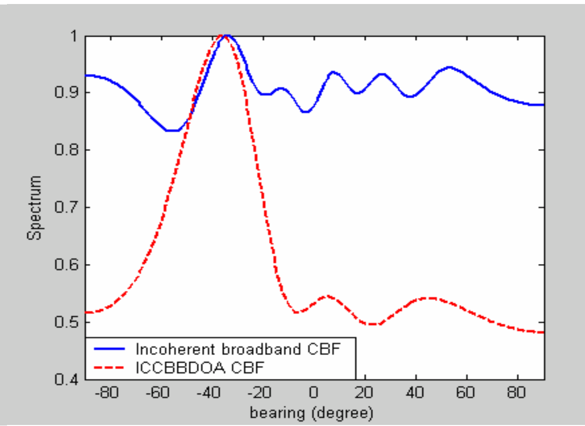

Fig.2 Comparison of incoherent CBF and ICCBBDOA CBF (SNR = -15dB)

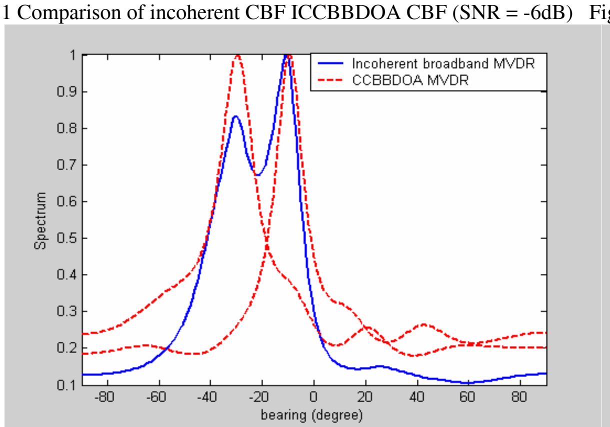

Fig.3 Comparison of incoherent MVDR and CCBBDOA MVDR

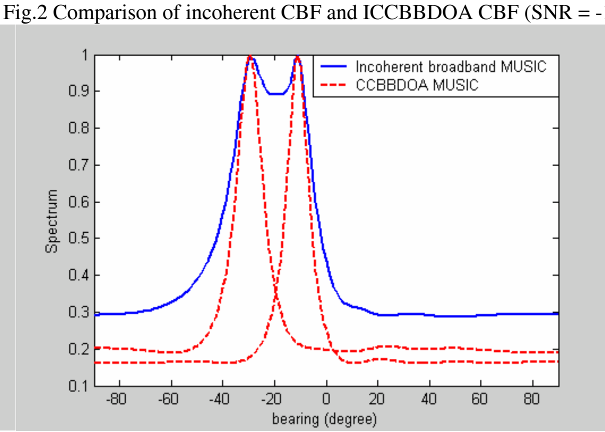

Fig.4 Comparison of incoherent MUSIC and CCBBDOA MUSIC

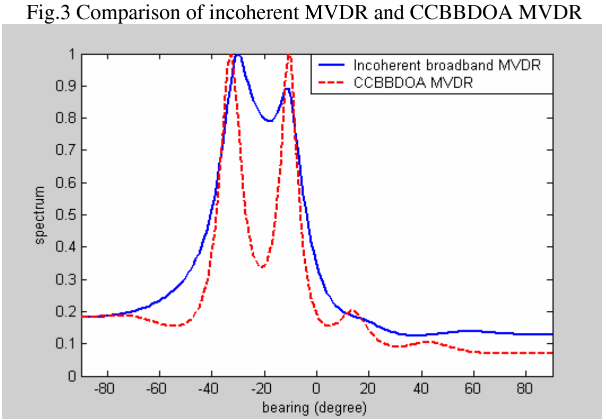

Fig.5 Comparison of incoherent MVDR and CCBBDOA MVDR

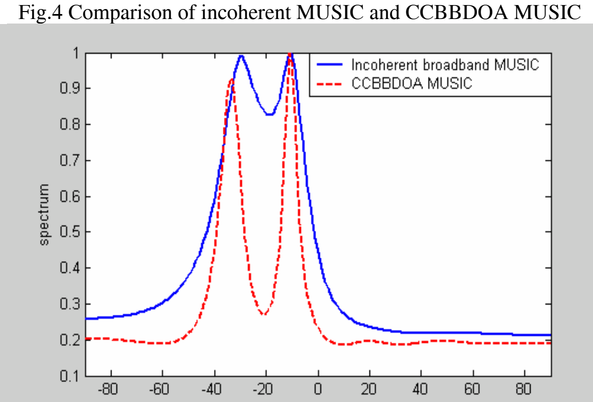

Fig.6 Comparison of incoherent MUSIC and CCBBDOA MUSIC