DIRECTION-OF-ARRIVAL ESTIMATION OF WIDEBAND SOURCES USING ARBITRARY SHAPED MULTIDIMENSIONAL ARRAYS

Yeo-Sun Yoon[1] *Lance M. Kaplan*[2] *James H. McClellan*[1]

[1]Center for Signal and Image Processing
School of Electrical and Computer Engineering
Georgia Institute of Technology
Atlanta, GA 30332-0250

[2]CTSPS
Dept. of Engineering
Clark Atlanta University
Atlanta, GA 30314

ABSTRACT

We generalize a new incoherent wideband direction-of-arrival (DOA) estimation algorithm that provides higher resolution than traditional incoherent techniques. The new method is able to better adjust its beam response when multiple sources are present than incoherent MUSIC. The original algorithm was designed to work strictly for an uniform linear array. In this paper, we demonstrate how to generalize the algorithm to work over arbitrary 1-D or 2-D arrays. We demonstrate the higher resolution of the new algorithm against incoherent MUSIC for 1-D and 2-D arrays using simulations of two source signals.

1. INTRODUCTION

Sensor arrays are prevalent in many applications such as radar, wireless communications, and sonar. By using multiple sensors, it is possible to separate multiple signals based upon their different direction of arrivals (DOAs). Beamforming and DOA estimation will allow for increased capacity in wireless communication application. Tracking applications require the ability to localize the source of the signals. The DOA estimation problem has been studied intensively and many good methods are available [1]. However, most of these methods are designed to work exclusively for narrowband signals [2]. For signals whose bandwidth is significant relative to their center frequency, a few wideband DOA estimation techniques have been proposed (see [3] and the references therein). Most of the wideband methods decompose the signals into several narrowband frequency components through a series of filterbanks, estimate the spatial correlation matrix over each frequency component and use the structure of correlation matrices to arrive at a DOA estimate. There are two fundamental classes of methods to derive a DOA from the correlation matrices. First, the 'incoherent' methods process each frequency component independently and form a weighted average of DOA estimates over all frequency bins[4]. On the other hand, the 'coherent' methods form a global correlation matrix by averaging transformed versions of the individual correlation matrices for each frequency bin.

This paper generalizes the wideband DOA estimation method introduced in [5] for uniform linear arrays. The generalized method works for arbitrary 1-D and 2-D arrays. Furthermore, the paper modifies the approach in [5] to improve the robustness of the algorithm against noisy data through the use of a projection matrix.

This paper is organized as follows. Section 2 reviews the DOA estimator introduced in [5]. The generalization of this technique for arbitrary arrays is provided in Section 3. Section 4 introduces the projection matrix for better performance. We compare the performance of the new method against incoherent MUSIC (IMUSIC) [4] over simulated data in Section 5. Finally, Section 6 provides concluding remarks.

Prepared through collaborative participation in the Advanced Sensors Consortium sponsored by the U. S. Army Research Laboratory under the Collaborative Technology Alliance Program, Cooperative Agreement DAAD19-01-2-0008. The U. S. Government is authorized to reproduce and distribute reprints for Government purposes notwithstanding any copyright notation thereon.

2. WIDEBAND DOA ESTIMATION

This section describes the approach first introduced in [5]. The first step in the wideband DOA estimation is to decompose the wideband sources into several narrowband frequency components through a filterbank. The discrete Fourier transform (DFT) is a natural and common choice for the filterbank. After obtaining the frequency components, the wideband methods form a spatial correlation matrix of the sensor outputs for each frequency bin.

The wideband method in [5] assumes that the number of signal sources P is known. It uses K frequency bins by performing an eigen decomposition of the correlation matrices for each bin. Then each bin is divided into a signal and noise subspace. Let $\mathbf{F}_i$ be the signal subspace matrix of frequency ω_i and $\mathbf{W}_i$ be the noise subspace of frequency ω_i for $i = 1, \ldots, K$. Given that there are P sources,

$$\mathbf{F}_i = \mathbf{A}_i\mathbf{T}_i = [\ \mathbf{a}_i(\vec{\alpha}_1) \quad \cdots \quad \mathbf{a}_i(\vec{\alpha}_P)\ ]\mathbf{T}_i, \tag{1}$$

$$\mathbf{a}_i^H(\vec{\alpha}_p)\mathbf{W}_i = \mathbf{0}, \tag{2}$$

where $\mathbf{a}_i(\vec{\alpha}_p)$ is the array manifold (or steering) vector for the narrowband source signal corresponding to frequency ω_i and direction $\vec{\alpha}_p$. For a linear array, $\vec{\alpha}_p = \sin\theta/c$ where θ is the azimuth angle and c is the speed of propagation. For a uniform linear array of M sensors with displacement d between sensors

$$\mathbf{a}_i(\theta_j) = [\ e^{-j\omega_i\tau_j}\ e^{-j2\omega_i\tau_j}\ \ldots\ e^{-jM\omega_i\tau_j}\ ]^T$$

where $\tau_j = d\sin\theta_j/c$. For an uniform linear array, a simple relation exists between the array manifolds corresponding two different frequency bins ω_x and ω_z such that $\omega_x > \omega_z \geq 0$. The relationship is

$$\mathbf{a}_x(\theta_x) = \mathbf{\Phi}(\omega_y, \theta_y)\mathbf{a}_z(\theta_z) \tag{3}$$

where

$$\mathbf{\Phi}(\omega_y, \theta_y) = \mathrm{diag}\{e^{-j\omega_y\tau_y}, \ldots, e^{-jM\omega_y\tau_y}\}, \tag{4}$$

and

$$\omega_y = \omega_x - \omega_z.$$

The DOA corresponding to array manifold x is related to the DOA of array manifold z via

$$\sin\theta_x \quad = \quad \frac{\omega_y}{\omega_x}\sin\theta_y + \frac{\omega_z}{\omega_x}\sin\theta_z.$$

If θ_y happens to be equivalent to θ_z the DOA of the transformed manifold x is unchanged by the tranformation matrix $\mathbf{\Phi}$. This implies that if ones hypothesizes a DOA θ_y and applies (3) to transform an array manifold corresponding to frequency bin z to an array manifold corresponding to frequency bin x, then the DOA of the array manifold remains unchanged if $\theta_y = \theta_z$. This fact forms the central idea behind the wideband method in [5]. The DOA is estimated by transforming the signal subspace corresponding to frequency bin ω_1 via (3) for a hypothesized DOA θ and comparing the resulting subspace to the noise subspace for the given frequency bin. In other words, the estimator forms the matrix

$$\mathbf{D}(\theta) = \mathbf{F}_1^H \left[\mathbf{\Phi}_2^H(\theta)\mathbf{W}_2 \;\ldots\; \mathbf{\Phi}_K^H(\theta)\mathbf{W}_K\right] \tag{5}$$

where

$$\mathbf{\Phi}_k(\theta) = \mathbf{\Phi}(\omega_k - \omega_1, \theta).$$

If θ matches the DOA of one of the source signals, then $\mathbf{D}$ becomes rank deficient because the DOA of that source is preserved by the transformation in (3). Namely, if θ in (5) is same as the l-th DOA,

$$\mathbf{D}(\theta_l) = \mathbf{T}_1^H \left[\begin{bmatrix} \mathbf{a}_2^H(\hat{\theta}_{2,1}) \\ \vdots \\ \mathbf{a}_2^H(\theta_l) \\ \vdots \\ \mathbf{a}_2^H(\hat{\theta}_{2,P}) \end{bmatrix} \mathbf{W}_2 \quad \cdots \quad \begin{bmatrix} \mathbf{a}_K^H(\hat{\theta}_{K,1}) \\ \vdots \\ \mathbf{a}_K^H(\theta_l) \\ \vdots \\ \mathbf{a}_K^H(\hat{\theta}_{K,P}) \end{bmatrix} \mathbf{W}_K \right]. \tag{6}$$

By (2), the l-th row of the large matrix in (6) is zero and matrix $\mathbf{D}$ is singular.

The DOA estimator in [5] simply tests the singularity of matrix $\mathbf{D}$ for different hypothesized angles θ by taking the condition number or the reciprocal of the smallest eigenvalue of $\mathbf{D}$. Then, the DOAs are simply the peaks of the curve representing the singularity test versus the hypothesized DOA.

For a given frequency bin, it is possible that the transformation in (3) happens to transform the DOA of one source into the DOA for another source when θ_y does not match any of the source DOAs. Fortunately, the transformed DOA changes when another frequency bin is considered. To counter all possible scenarios where DOAs are transformed to DOAs of other sources, it is shown in [5] that

$$K \geq \lceil \max\{\frac{M}{M-P}, P+1\}\rceil$$

so that $\mathbf{D}$ only becomes rank deficient if the hypothesized DOA matches a source DOA.

For the case where one source is present, i.e., $P = 1$, then the matrix $\mathbf{D}$ becomes a vector. Then, for any hypothesized DOA, the algorithm simply calculates the norm of $\mathbf{D}$. It can be shown that this operation is equivalent to IMUSIC.

3. GENERALIZATION OF THE WIDEBAND DOA ESITMATOR

This section generalizes the DOA estimator to arbitrary sensor arrays. For arbitrary arrays, including multi-dimensional arrays, the DOA is now represented by the slowness vector

$$\vec{\alpha} = \frac{1}{c}(\sin\theta\sin\phi, \cos\theta\sin\phi, \cos\phi). \tag{7}$$

where θ and ϕ are the azimuth and elevation arrival angles, respectively. Note that the magnitude of the slowness vector is $1/c$. The n-th element of the array manifold vector of an arbitrary sensor array is

$$\exp\{j\omega(\vec{\alpha}\cdot\vec{x}_n)\},$$

where $\vec{x}_n$ is the 3-D location of the n-th sensor, i.e.,

$$\vec{x}_n = (x_n, y_n, z_n).$$

We redefine the transformation matrix in (4) as

$$\mathbf{\Phi}_i(\vec{\alpha}) = \mathrm{diag}\{\mathbf{a}(\omega_k - \omega_1, \vec{\alpha})\}. \tag{8}$$

The generalized DOA estimator is

$$\hat{\vec{\alpha}} = \arg\max_{\vec{\alpha}} \kappa\{\mathbf{D}(\vec{\alpha})\}.$$

where κ denotes the condition number.

We show below that the new estimator works under certain conditions for the array geometry. The expression for the $\mathbf{D}$ matrix in (5) is now

$$\mathbf{D}(\overrightarrow{\alpha}) = \mathbf{T}^H \left[\begin{bmatrix} \mathbf{b}_{2,1}^H \\ \vdots \\ \mathbf{b}_{2,P}^H \end{bmatrix} \mathbf{W}_2 \quad \cdots \quad \begin{bmatrix} \mathbf{b}_{K,1}^H \\ \vdots \\ \mathbf{b}_{K,P}^H \end{bmatrix} \mathbf{W}_K \right] \tag{9}$$

where

$$\mathbf{b}_{k,p} = \mathbf{\Phi}_k(\vec{\alpha})\mathbf{a}(\omega_1, \vec{\alpha}_p) \tag{10}$$

is the transformed array manifold. In the previous section, we showed that for uniform linear arrays, $\mathbf{b}_{k,p} = \mathbf{a}(\omega_k, \vec{\alpha}_p)$ and $\mathbf{D}$ loses rank if and only if $\vec{\alpha} = \vec{\alpha}_p$. For the new algorithm to be useful, we must show that the same property holds. For $\mathbf{D}$ to be rank deficient if and only if $\overrightarrow{\alpha}$ corresponds to the DOA of one sources, the following two conditions must hold for the transformed manifolds:

1. $\mathbf{b}_{k,p} = \mathbf{a}(\omega_k, \vec{\alpha}_p)$ for $k = 1, \ldots, K$ if and only if $\vec{\alpha}$ forming the transformation matrix $\mathbf{\Phi}$ in (8) matches the DOA for the p-th source, i.e. $\vec{\alpha}_p$.
2. The $\mathbf{b}_{k,p}$'s span the P-dimensional subspace.

If the $\mathbf{b}_{k,p}$'s are array manifolds of frequency k, the second condition is always satisfied. We show below that these conditions are satisfied for arbitrary 1-D and 2-D arrays.

Let

$$\mathbf{b} = \mathrm{diag}\{\mathbf{a}(\omega_1, \vec{\alpha}_1)\}\mathbf{a}(\omega_2, \vec{\alpha}_2).$$

The phase of the n-th element of $\mathbf{b}$ is

$$\omega_1(\vec{\alpha}_1\cdot\vec{x}_n) + \omega_2(\vec{\alpha}_2\cdot\vec{x}_n) = \omega_3(\vec{\beta}\cdot\vec{x}_n)$$

where

$$\omega_3 = \omega_1 + \omega_2$$

and

$$\vec{\beta} = \frac{\omega_1\vec{\alpha}_1 + \omega_2\vec{\alpha}_2}{\omega_3} = \gamma\vec{\alpha}_1 + (1-\gamma)\vec{\alpha}_2 \quad (11)$$

where $0 < \gamma < 1$. If the magnitude of $\vec{\beta}$ is $1/c$, we can say that it is n-th element of the array manifold vector of frequency ω_3. By triangle inequality,

$$\left|\vec{\beta}\right| \le \gamma|\vec{\alpha}_1| + (1-\gamma)|\vec{\alpha}_2| = \frac{1}{c}. \quad (12)$$

The equality holds in (12) if and only if $\vec{\alpha}_1$ is proportional to $\vec{\alpha}_2$, which means $\vec{\alpha}_1 = \vec{\alpha}_2$ because the magnitude of all the slowness vectors is $1/c$. This ensures that the first condition holds given that enough frequency bins are used to form the $\mathbf{D}$ matrix in (9). For the second condition to hold, $\vec{\beta}$ must lead to a valid array manifold. In general, the magnitude of $\vec{\beta}$ is not $1/c$. For one or two dimensional arrays, where one or two of the elements in $\vec{x}_i$ is zero, even if $\vec{\beta}$ is not a valid slowness vector, if there is a valid slowness vector $\vec{\alpha}_3$ such that

$$\vec{\beta}\cdot\vec{x}_i = \vec{\alpha}_3\cdot\vec{x}_i, \quad (13)$$

then the $\mathbf{b}$ vector can lead to a valid array manifold $\mathbf{b}$. Let the slowness vector and $\vec{\beta}$ be

$$\vec{\alpha}_i = (\alpha_{i,x}, \alpha_{i,y}, \alpha_{i,z}),$$
$$\vec{\beta} = (\beta_x, \beta_y, \beta_z).$$

Then,

$$\vec{\alpha}_i\cdot\vec{x}_j = \alpha_{i,x}x_j + \alpha_{i,y}y_j + \alpha_{i,z}z_j$$

The vectors of the phase terms of $\mathbf{b}$ are

$$\vec{\alpha}_i\cdot\vec{\mathbf{x}} = \alpha_{i,x}\mathbf{x} + \alpha_{i,y}\mathbf{y} + \alpha_{i,z}\mathbf{z},$$
$$\vec{\beta}\cdot\vec{\mathbf{x}} = \beta_x\mathbf{x} + \beta_y\mathbf{y} + \beta_z\mathbf{z},$$

where

$$\mathbf{x} = [\, x_1 \;\; \cdots \;\; x_M]^T,$$
$$\mathbf{y} = [\, y_1 \;\; \cdots \;\; y_M]^T,$$
$$\mathbf{z} = [\, z_1 \;\; \cdots \;\; z_M]^T,$$

Without loss of generality, for a 1-D array $\mathbf{y} = \vec{0}$ and $\mathbf{z} = \vec{0}$. Likewise, for the 2-D array $\mathbf{z} = \vec{0}$ and the vectors $\mathbf{x}$ and $\mathbf{y}$ are linearly independent. For both cases, we show that (13) holds, and $\vec{\beta}$ leads to a valid manifold vector.

One-dimensional (Linear) Array

For a linear array, $\mathbf{y}$ and $\mathbf{z}$ are zero vectors. Therefore, one must only show that $\alpha_{3,x} = \beta_x$. As a result, (13) is equivalent to

$$\gamma\sin\theta_1 + (1-\gamma)\sin\theta_2 = \sin\theta_3 \quad (14)$$

where the constant term $(1/c)$ cancels out. Because $0 < \gamma < 1$, there always exists a θ_3 that satisfies (14) for any θ_1 and θ_2. Therefore,

$$\mathbf{b} = \mathbf{a}(\omega_3, \theta_3).$$

This is consistent with Section 2.

Two-dimensional Array

For the case of a 2-D array, we have two vectors to consider, $\mathbf{x}$ and $\mathbf{y}$. The phase term is

$$(\gamma\vec{\alpha}_1 + (1-\gamma)\vec{\alpha}_2)\cdot\vec{\mathbf{x}}$$
$$= (\gamma\alpha_{1,x} + (1-\gamma)\alpha_{2,x})\mathbf{x} + (\gamma\alpha_{1,y} + (1-\gamma)\alpha_{2,y})\mathbf{y}.$$

If we change the variables as

$$u_i = \sin\theta_i,$$
$$v_i = \sin\phi_i.$$

then,

$$\alpha_{i,x} = v_i u_i,$$
$$\alpha_{i,y} = v_i\sqrt{1-u_i^2}.$$

Finally, (13) is equivalent to

$$\gamma v_1 u_1 + (1-\gamma)v_2 u_2 = v_3 u_3,$$
$$\gamma v_1\sqrt{1-u_1^2} + (1-\gamma)v_2\sqrt{1-u_2^2} = v_3\sqrt{1-u_3^2}.$$

Because $0 \le \gamma, |u_1|, |v_1|, |u_2|, |v_2| \le 1$, it can be shown that there exist unique values for u_3 and v_3 such that $0 \le |u_3|, |v_3| \le 1$. Therefore, once again, the $\mathbf{b}$ vector is an array manifold given by

$$\mathbf{b} = \mathbf{a}(\omega_3, \arcsin(u_3), \arcsin(v_3)).$$

One can conclude that arbitrary 2-D arrays satisfy the two conditions.

4. PROJECTION MATRIX

In the previous section, we showed that the estimator should not lead to spurious peaks when employing arbitrary 1-D and 2-D arrays. The performance of the estimator can be improved by adding the projection matrix $\mathbf{P}$ such as

$$\mathbf{P}_i(\vec{\alpha}) = \mathbf{I} - \frac{\mathbf{a}(\omega_i,\vec{\alpha})\mathbf{a}^H(\omega_i,\vec{\alpha})}{\mathbf{a}^H(\omega_i,\vec{\alpha})\mathbf{a}(\omega_i,\vec{\alpha})}$$

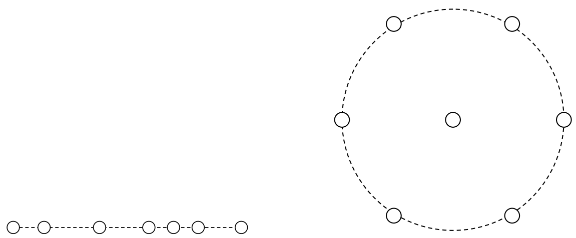
to the $\mathbf{D}$ matrix as

$$\mathbf{D}'(\vec{\alpha}) =$$
$$\mathbf{F}_1^H[\;\mathbf{\Phi}_2^H(\vec{\alpha})\mathbf{P}_2^H(\vec{\alpha})\mathbf{W}_2 \;\;\cdots\;\; \mathbf{\Phi}_K^H(\vec{\alpha})\mathbf{P}_K^H(\vec{\alpha})\mathbf{W}_K\;]$$

By inserting the projection matrices, the norm of the errors in the signal subspace and the noise subspace which are induced from the estimated correlation matrices can be reduced. This provides higher resolution.

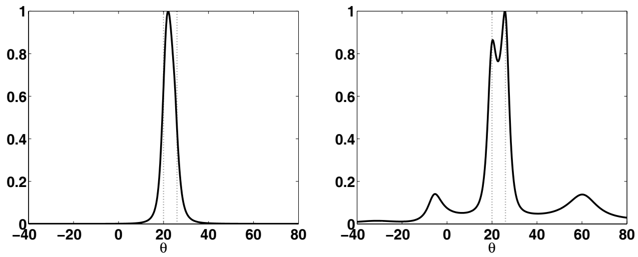
5. SIMULATIONS

To evaluate the new DOA estimator, we tested a random linear array and a 2-D array using simulated data representing two sources. Each source signal is a random sum of sinusoids. Both arrays have 7 sensors. The sensors in the 2-D array form a circle with one sensor at its center. The sensor spacing in the linear array is nonuniform. Figure 1 illustrates both arrays. In order to avoid aliasing, the sensor spacing in the linear array is less than $\lambda_m/2$, where λ_m



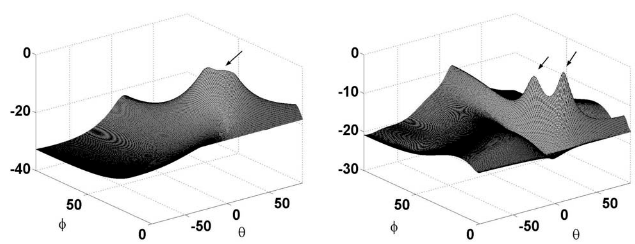
(a) Random linear (b) 2D circular

Fig. 1. Array structures.



(a) IMUSIC (b) New

Fig. 2. Results of DOA estimation for random linear array



(a) IMUSIC (b) New

Fig. 3. Results of DOA estimation for 2-D circular array

is the shortest wavelength of the wideband sources. For the 2-D array, the the distance between sensors is equal to $\lambda_m/2$.

The results are compared with that of incoherent MUSIC (IMUSIC) [4]

$$\vec{\alpha} = \arg\max_{\vec{\alpha}} \sum_{k=1}^{K} \frac{1}{\mathbf{a}^H(\omega_k, \vec{\alpha})\mathbf{W}_k \mathbf{W}_k^H \mathbf{a}(\omega_k, \vec{\alpha})}. \tag{15}$$

Note that if $P = 1$, the two methods would be equivalent. In both 1D and 2D simulations, 9 frequency bins are used and the SNR is 7 dB.

Figure 2 shows the example results for the random 1-D array. The left panel represents the output of IMUSIC, and the right panel represents the reciprocal of the smallest eigenvalue of $\mathbf{D}$. Two sources are located at 20° and 26°. Clearly, the new method can resolve the two sources. In addition, the new method exhibits little bias in this case.

Figure 3 shows the output of the estimators for the 2-D circular array where the two targets exist at $(20^\circ, 30^\circ)$ and $(30^\circ, 15^\circ)$. As in the 1-D case, only the new estimator resolves both signals. Interestingly, the new method does exhibit higher sidelobes, which we plan to study further.

6. CONCLUSIONS

This paper generalizes the new wideband DOA estimator introduced in [5] to work on arbitrary 1-D and 2-D arrays. Simulated results indicate that the new estimator, which uses both the signal and the noise subspaces, can resolve two closely spaced targets with higher resolution than IMUSIC, which uses only the noise subspaces. We anticipate that the new method can be used for multiple target tracking using acoustic sensors. The acoustic sensor of a target is composed of the sum of narrowband harmonics. The new method should be able to better detect multiple DOAs of targets demonstrating small angular separation. In future work, we will be able to test the new algorithm on real acoustic data. In addition, we are studying how to modify the algorithm to treat the case where the number of sources P is unknown.

[1] The views and conclusions contained in this document are those of the authors and should not be interpreted as representing the official policies, either expressed or implied, of the Army Research Laboratory or the U. S. Government.

7. REFERENCES

[1] D. Johnson and D. Dudgeon, *Array Signal Processing: Concepts and techniques*, Prentice Hall, Englewood Cliffs, NJ, 1993.

[2] M. Zatman, "How narrow is narrowband?," *IEE Proc.-Radar, Sonar Navig.*, vol. 145, no. 2, pp. 85–91, Apr. 1998.

[3] E.D. Di Claudio and R. Parisi, "WAVES: Weighted average of signal subspaces for robust wideband direction finding," *IEEE Trans. Signal Processing*, vol. 49, no. 10, pp. 2179–2190, Oct. 2001.

[4] M.Wax and T. Kailath, "Spatio-temporal spectral analysis by eigenstructure methods," *IEEE Trans. Acoust., Speech, Signal Processing*, vol. ASSP-32, no. 4, pp. 817–827, Aug. 1984.

[5] Y.-S. Yoon, L. Kaplan, and J. McClellan, "New signal subspace direction-of-arrival estimator for wideband sources," in *Proc. IEEE Acoustics, Speech, and Signal Processsing (ICASSP'03)*, Hong Kong, Apr. 2003.