PARALLEL BEAMFORMER DESIGN UNDER RESPONSE EQUALIZATION CONSTRAINTS

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ABSTRACT

In some applications, it may be desirable to design a certain number of fixed beamformers that have different look directions but the same response to a certain pre-determined signal, referred to as the equalization signal. In this article we present a method to accomplish this task, and show that it is optimal for a natural directivity criterion.
We then demonstrate the effectiveness of this method with an application to an audio conference phone where the equalization signal is the direct-path loudspeaker coupling signal.

1. INTRODUCTION

We consider the problem of designing several beamformers on the same microphone array, each beamformer having its own look direction but all of them being constrained to have the same response to a particular signal, referred to as the equalization signal.
One approach to this problem might be to first choose the value of the response to the equalization signal and then use the classical Linearly Constrained Minimum Variance (LCMV) beamformer design method (see [2]) to design each beamformer.
In the frequency domain, the LCMV approach can be formulated as follows. Let $W_i(\nu)$, $1 \le i \le M$, be the weight vector of the i^{th} beamformer at frequency ν, $S_i(\nu)$ the look direction steering vector for the i^{th} beamformer (the length of $S_i(\nu)$ being the number of microphones *N*) and $S(\nu)$ the equalization steering vector. If *g* denotes the chosen equalized response value, then the LCMV optimization problem for the i^{th} beamformer can be written as follows:

$$\operatorname*{Min}_{W_i(\nu)}\left(W_i^H(\nu)R_i(\nu)W_i(\nu)\right) \text{ subject to } C_i^H(\nu)W_i(\nu) = G(\nu),$$

where ^{H} denotes the Hermitian transposition.

In this formulation, $R_i(\nu)$ is the noise correlation matrix for the i^{th} beamformer, $C_i(\nu) = [S_i(\nu) \quad S(\nu)]$ is the constraint matrix (size $N \times 2$) and $G(\nu) = [1 \quad g]^T$ the gain vector (length 2), common to all beamformers.
Assuming that the equalization steering vector is linearly independent of the span of the look direction steering vectors, this optimization problem has a unique solution

$$W_i(\nu) = R_i^{-1}(\nu)C_i(\nu)\left[C_i^H(\nu)R_i^{-1}(\nu)C_i(\nu)\right]^{-1}G(\nu). \qquad (1)$$

The main drawback of this method is that the directivity of the resulting beamformers depends heavily on the arbitrary choice of the equalization value *g* (in both magnitude and phase). A sub-optimal value may impose unnecessary stress on the solution and result in significant loss of directivity.
To illustrate this fact, we take the example of a free-space elliptical microphone array with axes 20cm and 10cm and with 4 microphones positioned at azimuth 0, 90, 180 and 170 degrees. We consider two beamformers with look directions 0 and 90 degrees in the far field and an equalization source also in the far field, in the vertical direction from the center of the array (see Figure 1).
We first design two Minimum-Variance Distortionless Response (MVDR) beamformers (see [2]) without any specific constraint on their response to the equalization signal. We then design the equalized beamformers with (1) using two different choices for the equalized response value: first the response value of one MVDR beamformer to the equalization signal; then the average of the response values of the two MVDR beamformers.
Figures 2 and 3 show polar beampattern plots at different frequencies for the two resulting beamformers, with thick lines for the equalized LCMV beamformers and thin lines for the MVDR beamformers. It appears that the directivity can indeed be greatly affected by the value chosen for the target equalization response value.
In general, this might seem a natural approach to solving our problem: first design the individual beamformers with no special constraint with respect to the equalization signal, then take the average response over all beamformers as the value of *g* in (1).

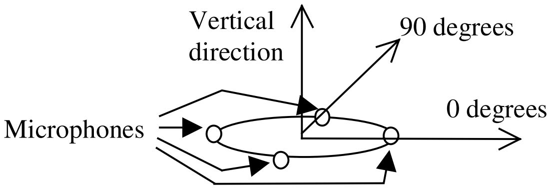

Figure 1: elliptical microphone array with vertical direction of equalization signal

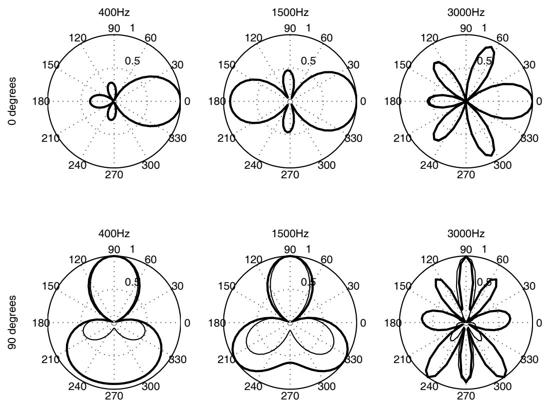

Figure 2: polar plots for MVDR and equalized beamformers (equalized response chosen as the response of one MVDR beamformer)

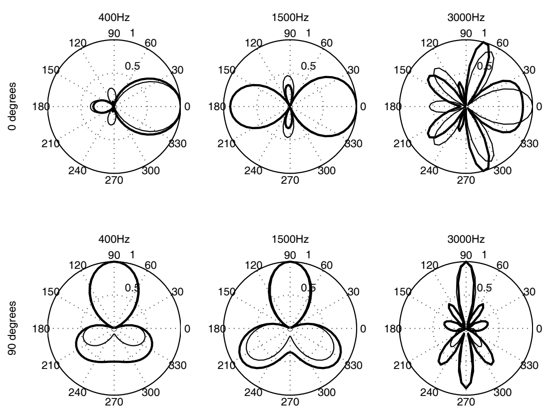

Figure 3: polar plots for MVDR and equalized beamformers (equalized response chosen as the average response of the MVDR beamformers)

This, however, is not optimal in the sense that there is no guarantee that the average response is the best possible value for the equalized response. In the next section we present another method which is direct, and optimizes the directivity of the resulting beamformers.

2. PARALLEL BEAMFORMER DESIGN METHOD

In order to specify an equalization constraint without forcing a specific value for the equalized response, we define a "parallel" optimization problem on the whole set of beamformers.

If we denote $\hat{W}(\nu)$ the vector of length $M.N$ formed by concatenating all weight vectors for the M beamformers, that is,

$$\hat{W}(\nu)=\begin{bmatrix} W_1(\nu) \\ W_2(\nu) \\ \dots \\ W_M(\nu) \end{bmatrix},$$

then the distortionless constraints for all beamformers can be written as

$$\hat{C}_d^H(\nu)\hat{W}(\nu)=\hat{G}_d(\nu),$$

where $\hat{C}_d(\nu)$ is the combined distortionless constraint matrix (size $M.N \times M$) defined as

$$\hat{C}_d(\nu)=\begin{bmatrix} S_1(\nu) & 0 & \dots & 0 \\ 0 & S_2(\nu) & 0 & \dots \\ \dots & 0 & \dots & 0 \\ 0 & \dots & 0 & S_M(\nu) \end{bmatrix}$$

and $\hat{G}_d(\nu)$ is the combined distortionless response vector (length M) given by

$$\hat{G}_d(\nu)=\begin{bmatrix} 1 \\ 1 \\ \dots \\ 1 \end{bmatrix}.$$

The equalization constraints can be included as $M-1$ linear constraints on the coefficients of the combined beamformer weights:

$$\begin{cases} S^H(\nu)(W_1(\nu)-W_2(\nu))=0 \\ S^H(\nu)(W_2(\nu)-W_3(\nu))=0 \\ \dots \\ S^H(\nu)(W_{M-1}(\nu)-W_M(\nu))=0 \end{cases}.$$

All constraints can then be combined as follows:

$$\hat{C}^H(\nu)\hat{W}(\nu)=\hat{G}(\nu),$$

where $\hat{C}(\nu)$ is the combined constraint matrix of size $M.N\times(2M-1)$ given by

$$\hat{C}(\nu)=\left[\hat{C}_d(\nu)\quad \begin{bmatrix} S(\nu) & 0 & \dots & & 0 \\ -S(\nu) & S(\nu) & \dots & & \dots \\ & \dots & \dots & & 0 \\ & & 0 & -S(\nu) & S(\nu) \\ 0 & & \dots & 0 & -S(\nu) \end{bmatrix}\right]$$

and $\hat{G}(\nu)$ is the combined gain vector of length $2M-1$ given by

$$\hat{G}(\nu)=\begin{bmatrix}1\\ \ldots\\ 1\\ 0\\ \ldots\\ 0\end{bmatrix}\begin{matrix}\left.\vphantom{\begin{matrix}1\\ \ldots\\ 1\end{matrix}}\right\} M\\ \left.\vphantom{\begin{matrix}0\\ \ldots\\ 0\end{matrix}}\right\} M-1\end{matrix}.$$

A natural overall directivity performance estimator can also be defined as the combined variance

$$\hat{W}^{H}(\nu)\hat{R}(\nu)\hat{W}(\nu),$$

where $\hat{R}(\nu)$ is the $(M.N\times M.N)$ combined noise correlation matrix

$$\hat{R}(\nu)=\begin{bmatrix}R_1(\nu) & 0 & \ldots & 0\\ 0 & R_2(\nu) & & \ldots\\ \ldots & & \ldots & 0\\ 0 & \ldots & 0 & R_M(\nu)\end{bmatrix}.$$

The combined LCMV optimization problem can then be written as follows:

$$\underset{\hat{W}(\nu)}{\mathrm{Min}}\left(\hat{W}^{H}(\nu)\hat{R}(\nu)\hat{W}(\nu)\right)\text{ subject to }\hat{C}^{H}(\nu)\hat{W}(\nu)=\hat{G}(\nu).$$

It is easy to see that since the equalization steering vector $S(\nu)$ and the individual look direction steering vectors $S_i(\nu)$ were assumed to be linearly independent, so are the linear constraints of the combined optimization problem (that is, the columns of the combined constraint matrix $\hat{C}(\nu)$). Therefore the combined LCMV problem is well-defined and its solution is given by

$$\hat{W}(\nu)=\hat{R}^{-1}(\nu)\hat{C}(\nu)\left[\hat{C}^{H}(\nu)\hat{R}^{-1}(\nu)\hat{C}(\nu)\right]^{-1}\hat{G}(\nu). \qquad (2)$$

Note that given the block structure of $\hat{R}(\nu)$, the combined variance is the sum of the individual variances:

$$\hat{W}^{H}(\nu)\hat{R}(\nu)\hat{W}(\nu)=\sum_{i=1}^{M}W_i^{H}(\nu)R_i(\nu)W_i(\nu).$$

If we apply this method to the example in the introduction, we obtain the beampattern plots shown in Figure 4. Note that in that particular case, the “average response” was indeed quite a good guess for the equalized response, although not optimal for the combined minimum-variance criterion.

In closing this section, let us note that additional linear constraints on the individual beamformers can be included in the parallel design method. These constraints are simply appended as column vectors to the combined noise correlation matrix with the corresponding gain appended to the combined gain vector.

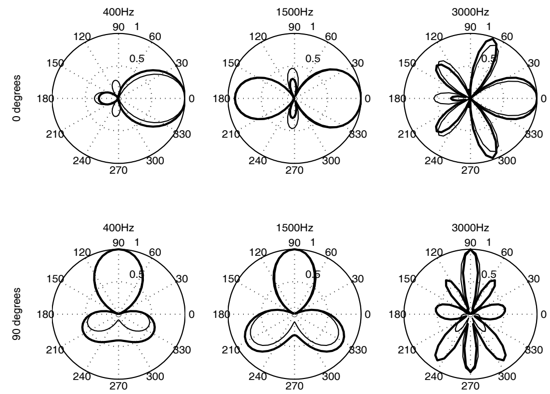

Figure 4: beampattern plots for optimally equalized beamformers

3. APPLICATION TO ACOUSTIC COUPLING EQUALIZATION

The method presented in this article was developed to solve a specific problem encountered during the development of an audio conference unit based on the microphone array technology (see [3]).

In audio systems based on beamforming and performing full-duplex communication, one known question (see [4]) is whether to perform acoustic echo cancellation (AEC) before the beamformer (that is, multi-channel AEC on the microphone signals) or after the beamformer, as shown in Figure 5. When AEC is performed after the beamformer, the various beams that cover the spatial span of the system may present a different response to the loudspeaker coupling signal. This is particularly true if the microphone array is asymmetric with respect to the loudspeaker. In order to reduce the impact of beam switching on the AEC (see [3] for details), it may be desirable to design the beams in such a way that they present the same (or relatively close) response to the coupling signal. The components of the acoustic echo that pertain to the acoustic environment (reflections, etc) cannot be predicted, but the direct-path coupling signal may be possible to predict through simulation or measurement. Since it accounts for most of the energy of the acoustic echo signal, equalizing the responses to the direct-path coupling signal may be enough to smooth the transitions when beam switching occurs (see [3]).

This problem can be formulated as an equalization problem as above, with the measured or simulated direct-path coupling signal as the equalization signal. One can apply the parallel design method to equalize the beamformers’ coupling responses.

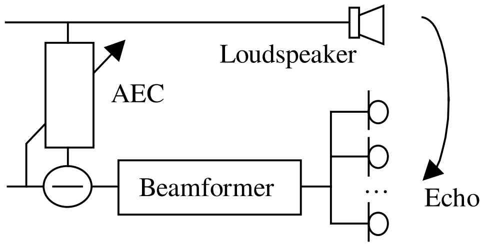

Figure 5: AEC after beamformer

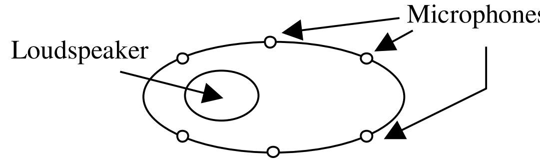

Figure 6: microphone array with loudspeaker

We show results obtained with a microphone array with 6 microphones in an elliptic enclosure with the loudspeaker located towards one end of the enclosure (Figure 6). We used 12 beamformers pointed every 30 degrees around the circle. Figure 7 shows the direct-path loudspeaker coupling response across the telephony frequency band for the individual MVDR beamformers (thin lines) and the equalized beamformers (thick lines). Figure 8 shows beampattern plots at 2000Hz for these beamformers. Figure 9 shows the direct-path coupling responses where random perturbations ($\pm 2\,\text{dB}$ in magnitude and ± 10 degrees in phase) were added to the coupling signal at the microphones to simulate variations that occur in a practical system due to loudspeaker-induced structural vibrations, acoustic leakage, or manufacturing variability. All beamformers were regularized to increase their robustness (see [1]). This is compatible with the parallel design method: one can add a regularization factor to the diagonal of the combined noise correlation matrix $\hat{R}(\nu)$.

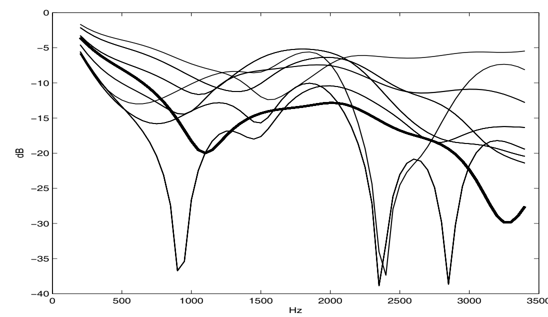

Figure 7: Nominal coupling response of MVDR and optimally equalized beamformers

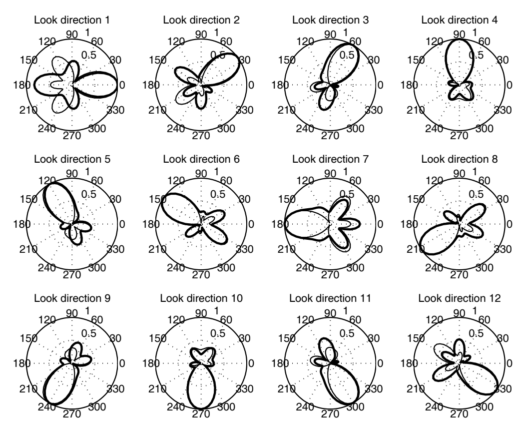

Figure 8: polar plots for MVDR and optimally equalized beamformers (2000 Hz)

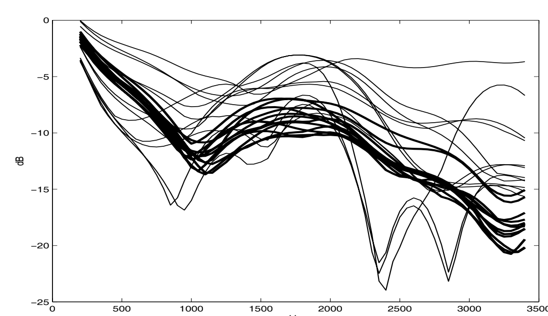

Figure 9: Perturbed coupling response of MVDR and optimally equalized beamformers

4. SUMMARY

We have presented a method to design an arbitrary number of fixed beamformers to have the same, a priori undetermined, response to an equalization signal in a way that is optimal for a natural criterion that consists of the sum of the individual "mimimum-variance" quantities.
This method is compatible with other linear constraints as well as with classical regularization techniques. It is the object of a pending patent filed by Mitel Networks.

5. REFERENCES

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[2] B.D. Van Veen and K.M. Buckley, "Beamforming: a versatile approach to spatial filtering", *IEEE Acoustics, Speech and Signal Processing Magazine*, pp. 4-24, April 1998.

[3] M. Tetelbaum and F. Beaucoup, "Application of microphone array technology to a commercial conference phone design (Mitel Networks 53xx)", submitted for publication.

[4] W. Kellermann, "Strategies for combining acoustic echo cancellation and adaptive beamforming microphone arrays", *Proc. IEEE ICASSP'97*, pp 219-222, Apr. 1997.