LOW RANK ADAPTIVE SIGNAL PROCESSING FOR RADAR APPLICATIONS

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ABSTRACT

This paper addresses the problem of radar target detection in heterogeneous clutter environments. We present the performance of the low rank normalized matched filter (LRNMF) test and the low rank normalized adaptive matched filter (LRNAMF) in a background of disturbance consisting of clutter having a covariance matrix with known structure and unknown scaling plus background white Gaussian noise with unknown variance. It is shown that the LRNMF test retains invariance with respect to the unknown scaling as well as the background noise level and is approximately CFAR. Analytical expressions for calculating the false alarm and detection probabilities are presented. Performance analysis of the LRNAMF test using simulated data from the KASSPER program is presented. Issues of sample support for subspace estimation, constant false alarm rate (CFAR) and detection performance are presented.

1. INTRODUCTION

This paper addresses the problem of radar signal detection in disturbance composed of clutter, having a covariance matrix with known structure but unknown level and background white noise. The technique developed in this paper ensures invariance with respect to the unknown level and the background noise power. Typically a radar receiver front end consists of an array of J antenna elements processing N pulses in a coherent processing interval.

Previous efforts [1, 2] derived the normalized matched filter (NMF) test for the problem of detecting a rank one signal in additive clutter modeled as a spherically invariant random process [3]. The NMF test is given by

$$\Lambda_{NMF} = \frac{|\mathbf{e}^H \mathbf{R}_c^{-1} \mathbf{x}|^2}{[\mathbf{e}^H \mathbf{R}_c^{-1} \mathbf{e}][\mathbf{x}^H \mathbf{R}_c^{-1} \mathbf{x}]} \underset{H_0}{\overset{H_1}{\gtrless}} \lambda_{NMF}. \tag{1}$$

where $\mathbf{e}$ is the known spatio-temporal signal steering vector, and $\mathbf{R}_c$ is the known clutter covariance matrix. A detailed overview of various efforts concerning the development of the NMF test is provided in [4, 5]

This paper seeks to extend previous work by including the effect of additive white Gaussian noise. Specifically, we consider the binary hypothesis testing problem given by

$$\begin{array}{rl} H_0: & \mathbf{x} = \mathbf{d} = \mathbf{c} + \mathbf{n} \\ H_1: & \mathbf{x} = a\mathbf{e} + \mathbf{d} = a\mathbf{e} + \mathbf{c} + \mathbf{n} \end{array} \tag{2}$$

where $\mathbf{c}$ denotes the Gaussian clutter vector having a covariance matrix $s\mathbf{R}_c$ with known structure and unknown level s, $\mathbf{n}$ denotes the additive white Gaussian noise vector having covariance matrix $\sigma^2\mathbf{I}$, where $\mathbf{I}$ is the $JN \times JN$ identity matrix and σ^2 is the unknown noise power, $\mathbf{e}$ denotes the $JN \times 1$ spatio-temporal steering vector and a is the unknown complex amplitude of the target. Consequently, the disturbance covariance matrix is given by $\mathbf{R}_d = s\mathbf{R}_c + \sigma^2\mathbf{I}$. Invariance of the NMF fails for this problem [4, 5].

However, in many practical airborne radar applications $\mathbf{R}_c$ has rank r which is much less than the spatio-temporal product $M = JN$. For example, the clutter rank in the airborne linear phased array radar problem, is given by the Brennan rule [6]

$$r = J + \beta(N-1) \tag{3}$$

where β is the slope of the clutter ridge. A nominal value of $\beta = 1$, yields a clutter rank $r \approx J + (N-1) << M$ especially for large J and N. This fact is advantageously used ensure approximate invariance to the unknown clutter power and noise level. In addition, the low rank approximation enables reduction of training data support compared to full dimension STAP processing. For $s\lambda_i >> \sigma^2$, it follows from [7] that the inverse covariance matrix can be approximated as

$$\mathbf{R}_d^{-1} \approx \frac{1}{\sigma^2}(\mathbf{I} - \mathbf{P}) \tag{4}$$

where $\mathbf{P} = \sum_{i=1}^{r} \mathbf{u}_i \mathbf{u}_i^H$ is a rank r projection matrix formed from the eigenvectors corresponding to the dominant eigenvalues of $\mathbf{R}_d$. For $\mathbf{R}_c$ with known structure, the dominant modes are readily determined and are unaffected by s.

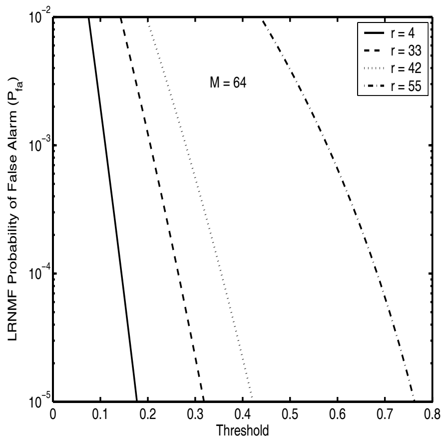

Fig. 1. P_{fa} versus threshold

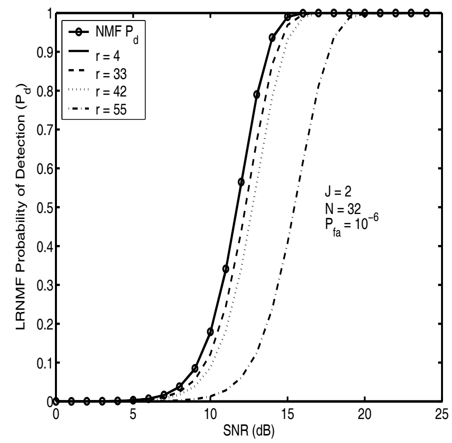

Fig. 2. P_d versus Signal-to-noise-ratio

2. LOW RANK NMF TEST

We now use the form of $\mathbf{R}_d^{-1}$ given by (4) to express the LRNMF test as

$$\Lambda_{lr} = \frac{|\mathbf{e}^H(\mathbf{I}-\mathbf{P})\mathbf{x}|^2}{[\mathbf{e}^H(\mathbf{I}-\mathbf{P})\mathbf{e}][\mathbf{x}^H(\mathbf{I}-\mathbf{P})\mathbf{x}]} \underset{H_0}{\overset{H_1}{\gtrless}} \lambda_{lr}. \quad (5)$$

It has been shown in [4,5] that the LRNMF test performance is invariant to s and σ^2. Furthermore, analytical expressions for the detection and false alarm probability of the LRNMF test were derived in [4,5] and are given by

$$P_{fa} = P(\Lambda_{lr} > \lambda_{lr}|H_0) = (1-\lambda_{lr})^{M-r-1} \quad (6)$$

$$\begin{aligned} P_d &= P(\Lambda_{lr} > \lambda_{lr}|H_1) \\ &= 1 - E \\ E &= (1-\lambda_{lr})^{M-r-1} \sum_{k=1}^{M-r-1} \frac{\Gamma(M-r)}{\Gamma(k+1)\Gamma(M-r-k)} F \\ F &= \left(\frac{\lambda_{lr}}{1-\lambda_{lr}}\right)^k [1 - gammainc(A^2(1-\lambda_{lr}), k)] \end{aligned} \quad (7)$$

where $\Gamma(.)$ is the Eulero-Gamma function, $A^2 = \frac{|a|^2\mathbf{e}_1^H\mathbf{e}_1}{\sigma^2}$ is the signal-to-noise-ratio in the sub-dominant interference space, and

$$gammainc(\theta,\ M) = \frac{1}{\Gamma(M)} \int_0^{\theta} z^{M-1} exp(-z) dz. \quad (8)$$

Figure 1 shows a plot of the false alarm probability versus threshold for the LRNMF test with the clutter rank r as a parameter. We observe a significant increase in the threshold with increasing clutter rank for a given P_{fa} value. Fig. 2 presents a comparison of the performance for the LRNMF with $r = 4$ and $r = 55$ with the full rank NMF for the case where the disturbance consists of white noise alone with a full rank covariance matrix and no clutter. Relevant test parameters are reported in the plot. These curves reveal important features of the low-rank approximation to the covariance matrix. Curve 1 corresponding to $r = 0$ (no clutter) upper bounds the performance of the low-rank approximation. Furthermore, for the clutter rank $r = 4$ performance of the LRNMF test attains its upper bound, i.e., the full rank($M \times M$ covariance matrix) NMF test performance in background white noise with unknown power level. As the clutter rank increases, performance of the LRNMF degrades. The performance degradation (approximately 4 dB loss) with increasing rank (from $r = 4$ to $r = 55$) can be accounted for due to the fact that the threshold incurs an increase with increasing clutter rank. Furthermore, A^2 decreases with increasing r. Since P_d is a monotonic function of A^2, performance is degraded as r increases.

3. LOW RANK NORMALIZED ADAPTIVE MATCHED FILTER

In this section we present the performance analysis of an adaptive version of the LRNMF test of (5). The disturbance covariance matrix is seldom known in practice and thus must be estimated using representative training data. Specifically, we consider the LRNMF test of (5) with $\mathbf{P}$ replaced by its estimate $\hat{\mathbf{P}}$ formed from a singular value decomposition (SVD) of a data matrix $\mathbf{Z}$ whose columns $\mathbf{z}_i$, $i = 1, 2, \dots K$ contain representative training data. The resulting test is called the low rank normalized adaptive matched filter (LRNAMF). It can be readily demonstrated using arguments similar to those employed for the LRNMF test that the LRNAMF offers invariance to the unknown clutter power as well as the background noise power for large clutter-to-noise-ratio (CNR), i.e., $s\lambda_i >> \sigma^2$. In radar applications this condition is satisfied in most instances. For example, the MCARM [8] and KASSPER [9] data sets offer

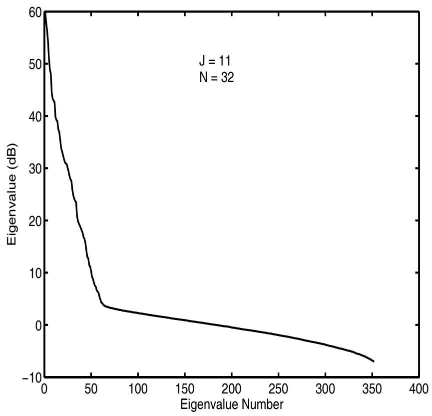

Fig. 3. Eigenspectrum of KASSPER Data

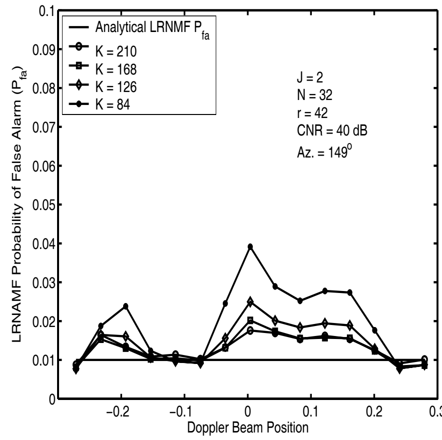

Fig. 4. P_{fa} versus Normalized Doppler Beam Position

an average CNR of 40 dB.

Typically r is unknown in practice. Consequently, a key issue in this context is the determination of r from the training data. Details of estimating r from the training data are contained in [5] and the references therein.

Data from the L-band data set of the KASSPER program [9] is used for carrying out performance analysis of the LRNAMF. The L-band data set consists of a datacube of 1000 range bins corresponding to the returns from a single coherent processing interval (CPI) from 11 channels and 32 pulses resulting in a spatio-temporal product of 352. Relevant system parameters for the L-band data set are available in [9]. Since analytical expressions for P_d and P_{fa} for the LRNAMF are mathematically intractable, we resort to performance evaluation using Monte Carlo simulation.

Fig. 3 shows the eigenspectrum of the KASSPER data. This plot is obtained from a SVD of the KASSPER datacube. Relevant test parameters are reported in the plot. We observe that the eigenspectrum exhibits a significant roll off (nearly 60 dB) after approximately 50 eigenvalues. The rank of the clutter subspace is determined using the procedure outlined in [10]. The procedure of [10] yields a clutter rank of 42 for this example, which is in agreement with the Brennan rule (3).

Fig. 4 shows the variation of the LRNAMF P_{fa} as a function of normalized Doppler beam position for a fixed steering angle. Relevant test parameters are reported in the plot. The curve which reflects a constant false alarm probability as a function of Doppler corresponds to the LRNMF and is obtained using (6). A modest CFAR loss is incurred with respect to the normalized Doppler beam position (particularly in the vicinity of zero Doppler). Our simulations also reveal that the threshold is insensitive to the unknown clutter power level and noise variance as long as the clutter-to-noise-ratio (CNR) is high.

Fig. 5 shows a plot of the LRNAMF P_d as a function of SINR. Relevant test parameters are reported in the plot. The analytical curve corresponding to the LRNMF represents the upper bound on performance of the LRNAMF. With $K = 2r$ training data samples for estimating $\hat{\mathbf{P}}$, we observe a 4dB decrease in performance with respect to the analytical curve. The performance loss is reduced with increasing K. Specifically, for 3 dB detection (P_d) performance $K = 5r$ training data vectors are needed. The work of [11] noted that 3 dB performance in terms of signal-to-noise-ratio requires the use of $K = 2M$ target free training data vectors. In [12] it was shown that 3 dB performance for P_d using sample matrix inversion calls for $K = 5M$ training data vectors. This is due to the fact that although P_d is a monotonic function of signal-to-noise-ratio, it is a highly nonlinear function. Consequently, the training data support needed for 3 dB signal-to-noise-ratio performance is quite different from the 3 dB performance for P_d. The work of [7, 13] derives an expression for the PDF of the output signal-to-noise-ratio of the principal components inverse technique and shows that the training data support for 3 dB performance is $K = 2r$. A similar result is also noted in [14] while considering the low rank problem in a maximum likelihood estimation framework. Fig. 5 reports for the first time the corresponding training data support for 3 dB performance in terms of P_d for low rank adaptive processing methods. This result is very similar in spirit to that reported in [12] for the sample matrix inversion technique.

4. CONCLUSIONS

This paper discusses the invariance properties of the LRNMF and LRNAMF tests in a background of clutter plus background white noise, both having unknown power levels. Analytical expressions for the detection and false alarm probabilities are presented and illustrated with numerical examples in the form of plots of P_d vs SNR which reveal a mod-

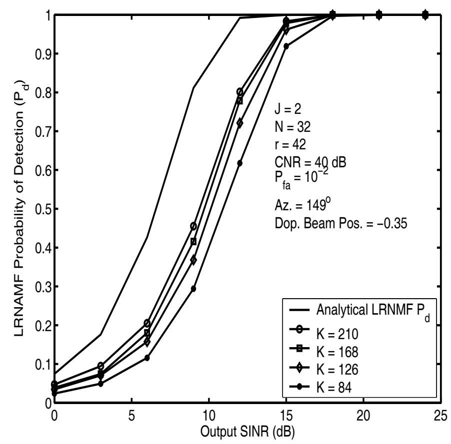

Fig. 5. P_d versus SINR

est degradation with increasing clutter rank. Performance of the LRNAMF, is studied using the KASSPER radar data. We observe a 4 dB degradation in performance due to the finite sample support used in estimating the clutter subspace. Furthermore, we note a loss of CFAR for the LRNAMF due to the threshold dependence on the Doppler beam position. An important feature of the LRNAMF is the ability to reduce the training data support for subspace estimation. Finally, we note that accurate determination of the rank of the clutter subspace significantly impacts detection performance. Space limitations require us to be rather brief in this paper. However, a more comprehensive treatment can be found in [5].

5. ACKNOWLEDGMENT

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