AN EFFICIENT DIGITAL BEAMSTEERING SYSTEM FOR DIFFERENCE FREQUENCY IN PARAMETRIC ARRAY

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ABSTRACT

For a digital beamsteering system, the smallest time delay available is equal to the sampling period of the digital signal processing (DSP) board. As most of the time the sampling frequency is not high enough, the smallest steering angle available is large, which is undesirable. This limitation also occurs when performing beamsteering in a parametric array [2] digitally. Although partial delay or frequency domain algorithms can be used to improve the steering angle, most of the algorithms are either computational intensive or introduce error during the process. In this paper, an algorithm is proposed to beamsteer the difference frequency in parametric array. The proposed system can be used to steer the difference frequency to a small angle, without the need to increase the sampling frequency or implement partial delay.

1. INTRODUCTION

It is always desirable to have a steerable audio system. However, if beamsteering in parametric array is to perform in digital domain, the available steering angle is limited by the sampling frequency of the DSP processor. In this paper, an algorithm is proposed to rectify this problem by applying separate delays to the carrier and sideband frequency. It extends our previous work on beamforming in parametric array [4].

This paper is organized in the following sections. In the next section, the theory on the difference frequency generated by using bi-frequency Gaussian source will be presented. This is followed by Section 3, which proposes a digital beamsteering system for the difference frequency in parametric array. Section 4 shows some simulation results for different steering angles, and Section 5 concludes this paper.

2. THEORY

For a source with Gaussian amplitude shading [4] [5], the complex pressure amplitude, $q_1(r,z)$ can be written as:

$$q_1(r,0) = p_0 \exp\left[-(r/a)^2\right], \tag{1}$$

where r is the radial distance in cylindrical coordinate, z is the coordinate along the axis of the beam, p_0 is the peak source pressure, and a is the effective source radius.

The solution of a finite-amplitude sound beam generated by the Gaussian source, which includes the effects of diffraction, absorption, and nonlinearity can be written as:

$$q_1(r,z) = \frac{p_0 e^{-\alpha_1 z}}{1 - j\,z/z_0} \exp\left[-\frac{(r/a)^2}{1 - j\,z/z_0}\right], \tag{2}$$

where α_1 is the thermoviscous attenuation coefficient, $z_0 = \frac{1}{2}ka^2$ and k is the wave number. Noted that Equation (2) is in closed form and is linear. The far-field directivity can be easily derived as:

$$D_1(k,\theta) = \exp\left[-\frac{1}{4}(ka)^2 \tan^2\theta\right], \tag{3}$$

where θ is the angle with respect to the axis of the beam.

By using bi-frequency Gaussian source, the difference frequency's far-field directivity is given by the product of the primary beam directivities [5], i.e.:

$$D_-(\theta) = D_{1a}(\theta) D_{1b}(\theta), \tag{4}$$

where $D_{1a}(\theta)$ and $D_{1b}(\theta)$ is the far-field directivity for frequency ω_a and ω_b, respectively.

Consider a group of M weighted primary sources, which are equally spaced (uniform linear array). Due to the fact that the solution of a primary source by using quasilinear theory is linear, the far-field difference frequency's directivity can be roughly estimated by the product of four terms [4]:

$$D_-(\theta) = D_1(k_a,\theta) H(k_a,\theta) D_1(k_b,\theta) H(k_b,\theta), \tag{5}$$

where $H(k,\theta)$ is the far-field array response. If delay-and-sum beamforming technique is used [1] [6], the far-field array response can be written as:

$$H(k,\theta) = \frac{1}{M}\sum_{m=0}^{M-1} w_m e^{-j\omega m\tau_0} e^{j\omega\frac{md}{c}\sin\theta}, \tag{6}$$

where w_m are the weightings for frequency ω for $m=0,1,2,\cdots,M-1$ and τ_0 is the time delay, which can be calculated as:

$$\tau_0 = \frac{d}{c}\sin\theta_{0,} \tag{7}$$

d is the inter-element spacing, c is the speed of sound and θ_0 is the desired steering angle for the delay-and-sum beamformer. Hence, Equation (5) can be rewritten as:

$$\begin{aligned} D_-(\theta) = \frac{1}{M^2} D_1(k_a,\theta) \sum_{m=0}^{M-1} w_{am} e^{-j\omega_a m\tau_{a0}} e^{j\omega_a \frac{md}{c}\sin\theta} \\ \times D_1(k_b,\theta) \sum_{n=0}^{M-1} w_{bn} e^{-j\omega_b n\tau_{b0}} e^{j\omega_b \frac{nd}{c}\sin\theta}. \end{aligned} \tag{8}$$

3. PROPOSED DIGITAL BEAMSTEERING SYSTEM FOR PARAMETRIC ARRAY

As the frequency involved in a parametric array is very high, a small error in the time delay (τ_0) will cause a relatively large error in the phase shift ($\omega\tau_0$). This will affect the accuracy of the difference frequency's steering angle. One way to solve the precision problem is to apply the beamsteering algorithm in digital domain as shown in Figure 1.

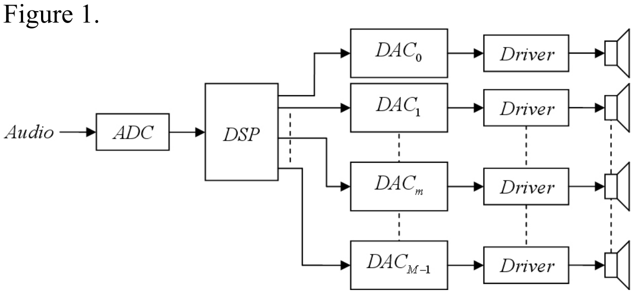

Figure 1 Hardware configuration of the proposed digital beamsteering system in parametric array.

Although there are many advantages when implementing the beamsteering system in digital domain, the delay for the delay-and-sum beamformer is dependent on the sampling frequency, F_S of the DSP. For a $160kHz$ sampling frequency DSP board, the sampling period, T is $6.25\mu s$. The smallest value for the time delay, τ_0 is equal to the sampling period, T. Hence, by using Equation (7), the smallest steering angle step for $c=344ms^{-1}$ and $d=4.9mm$ is:

$$\theta_0 = \sin^{-1}\frac{6.25\mu\times 344}{4.9m} = 26.03°. \tag{9}$$

This means it is impossible to steer the difference frequency to an angle less than $26.03°$. Similarly, if a digital beamsteering system is required to steer to $1°$, it would require a $\sim 4MHz$ sampling frequency DSP board, which is very costly to build. Although it is possible to improve the steering angle by using partial delay or performing beamsteering in frequency domain, it requires high computational power due to filtering or discrete Fourier transformation.

Figure 2 shows the block diagram of the proposed algorithm for beamsteering. Different delays and weighting functions are added to the carrier and sideband frequencies. For lower sideband modulation (LSB), the output of the DSP to m^{th} DAC is given as:

$$\begin{aligned} \varphi_{LSB,m}(nT) = 0.5\{ w_{am}\cos[\omega_c(nT - m\tau_{a0})] \\ + w_{bm}\cos[(\omega_c - \omega_-)(nT - m\tau_{b0})]\}, \end{aligned} \tag{10}$$

where ω_c and ω_- are the angular frequencies of the carrier frequency and difference frequency, respectively.

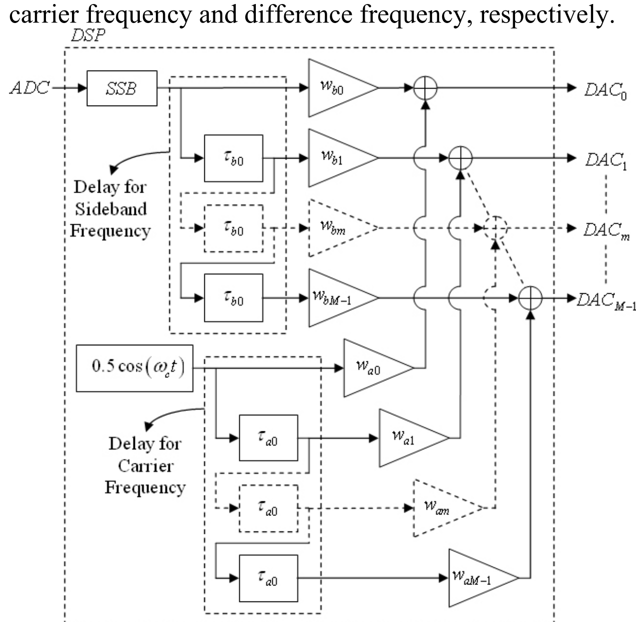

Figure 2 Block diagram of the proposed algorithm.

3.1. Time Delays

As the carrier frequency is independent from the input, the carrier frequency with the exact delay ($\tau_{a0} = \tau_0$) can be computed offline or modified when necessary by the following formula:

$$\varphi_{carrier,m}(nT) = 0.5 w_{am}\cos[\omega_c(nT - m\tau_0)]. \tag{11}$$

It is also preferred that, the sampling frequency, F_S is chosen to be a multiple integers of carrier frequency, f_c:

$$F_S = \alpha f_c, \tag{12}$$

where α is a positive integer greater than 2 (Nyquist rate [3]). Hence, in this case, only one cycle of the carrier frequency with α sample points need to be stored in the look-up table.

On the other hand, as the sideband frequency is calculated based on the input (real-time), the delay has to be rounded to the nearest sampling period, i.e.:

$$\tau_{b0} = \beta T, \qquad (13)$$

where β is an integer.

3.2. Weighting Functions

The objectives of the carrier frequency's weighting function, w_{am} are to control the difference frequency's beamwidth and to attenuate the carrier frequency's sidelobe. As for the sideband frequency's weighting function, w_{bm} is to generate a flat directivity response over a range of angles (similar to lowpass filter), such that the amplitude gain of the difference frequency is the same for different steering angles. The weighting function, w_{bm} also attenuates the sideband frequency's sidelobe, such that the difference frequency generated has lower sidelobe directivity.

The procedures for designing the weighting functions w_{am} and w_{bm} are stated as follows:

Step 1: Determine the beamwidth required for the difference frequency, θ_-.

Step 2: Calculate the required sidelobe attenuation (dB) for $H(k_a,\theta)$, R:

$$R = 20\log\left\{T_n\left[\frac{\cos\left(\dfrac{\pi}{2(M-1)}\right)}{\cos\left(\dfrac{k_a d\sin(\theta_-/2)}{2}\right)}\right]\right\}, \qquad (14)$$

where $T_n(x) = \cos\left[(M-1)\cos^{-1}x\right]$ is the Chebyshev polynomial.

Step 3: Specify the number of element, M used in the array, and the amount of sidelobe attenuation, R to design a Chebyshev weighting function, w_{am}.

Step 4: Plot the carrier frequency's directivity, $H(k_a,\theta)$ by using the weighting function, w_{am} in step 3, and delay, $\tau_{a0} = T/2$. Locate the first null location of the main lobe, $\theta_{Null,T/2}$, which is further away from the broadside as shown by an example (dashed line) in Figure 3.

Step 5: The weighting function, w_{bm} can be designed by using the coefficients of the symmetric FIR filter in DSP as shown in Figure 4. The passband edge, ω_{pass} (in π) for designing weighting function, w_{bm} is:

$$\omega_{pass} = 2 f_{\max}\frac{d}{c}\sin\theta_{Null,T/2}, \qquad (15)$$

where $f_{\max}$ is the maximum sideband frequency. The stopband edge, ω_{stop} (in π) is:

$$\omega_{stop} = \omega_{pass} + \omega_{trans}, \qquad (16)$$

where ω_{trans} is the transition band and the recommended value is 0.05π.

The FIR filter coefficients can be calculated by using Parks-McClellan optimal equiripple FIR filter design method. The M coefficients can be calculated by entering the following MATLAB command:

```
remez(M-1, [0 ωpass ωstop 1], [1 1 0 0]);
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The calculated coefficients can be used for the sideband frequency's weighting function, w_{bm}.

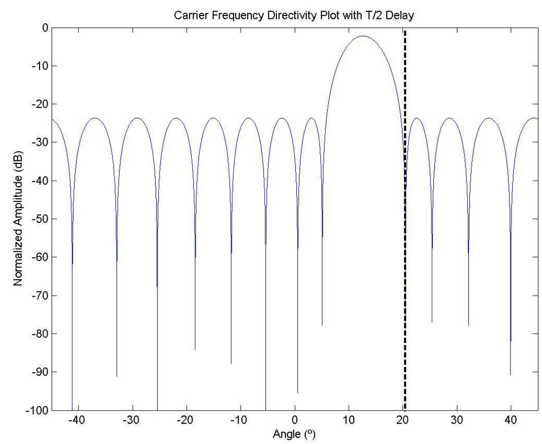

Figure 3 Carrier frequency's directivity with delay, $\tau_{a0} = T/2$.

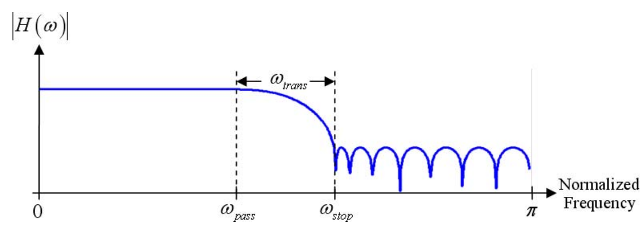

Figure 4 Lowpass filter's frequency response.

4. SIMULATION RESULTS

For the simulations, the carrier frequency of the lower sideband modulation is set at $40kHz$. The sampling frequency of the DSP board, F_S is $160kHz$. The input of the beamsteering system accepts frequency from $500Hz$ to $19{,}500Hz$, with $500Hz$ increment. A total of $M = 32$ elements is formed with inter element spacing, $d = 4.9mm$. The effective source radius is set at $a = 3.85mm$. The speed of sound, c is $344ms^{-1}$ and the difference frequency's beamwidth, θ_- is $10°$.

As the weighting function is symmetrical, only first half of the weighting function (truncated) is shown in Table 1 and Table 2 for w_{am} and w_{bm}, respectively.

Table 1 Weighting function for carrier frequency, w_{am} with $\theta_{-} = 10°$.

$m = 0,\cdots,3$	0.2843	0.1906	0.2484	0.3124
$m = 4,\cdots,7$	0.3817	0.4549	0.5303	0.6062
$m = 8,\cdots,11$	0.6806	0.7515	0.8168	0.8746
$m = 12,\cdots,15$	0.9231	0.9609	0.9868	1.0000

Table 2 Weighting function for sideband frequency, w_{bm} with $\theta_{-} = 10°$.

$m = 0,\cdots,3$	-0.0246	-0.1607	0.0078	0.0495
$m = 4,\cdots,7$	0.0563	-0.0168	-0.0850	-0.0530
$m = 8,\cdots,11$	0.0646	0.1297	0.0290	-0.1642
$m = 12,\cdots,15$	-0.2103	0.0718	0.5885	1.0000

The difference frequency's directivity simulations across the audible frequency are shown in Figure 5 to Figure 7 for $10°$, $20°$ and $30°$ beamsteering.

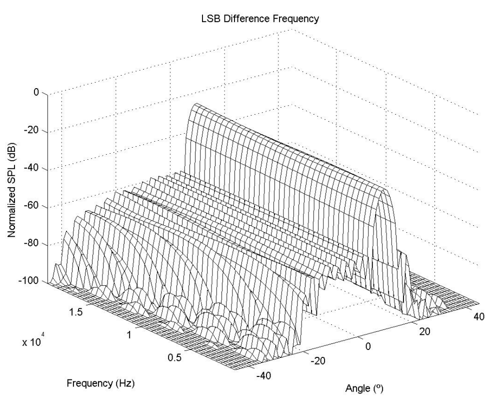

Figure 5 Difference frequency directivity for $10°$ beamsteering ($\tau_{a0} = 2.47\mu s$, $\tau_{b0} = 0 \times T$).

5. CONCLUSION

There are many advantages for the proposed beamsteering system. Firstly, the amplitude gain across the audible frequency is the same. Secondly, the inter-null beamwidth is constant throughout the audible frequency and the beamwidth can be controlled by the weighting function, w_{am}. Thirdly, the design procedure is simple and the difference frequency can be steered accurately. Most importantly, the proposed beamsteering system is able to steer to any angle (assume no aliasing), without the need to increase the sampling frequency or implement partial delay.

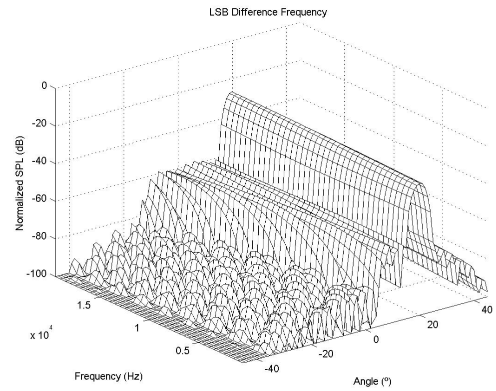

Figure 6 Difference frequency directivity for $20°$ beamsteering ($\tau_{a0} = 4.87\mu s$, $\tau_{b0} = 1 \times T$).

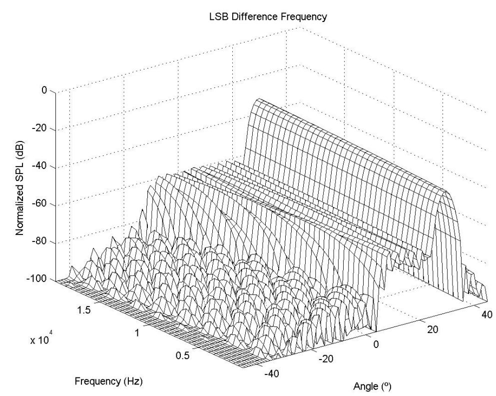

Figure 7 Difference frequency directivity for $30°$ beamsteering ($\tau_{a0} = 7.12\mu s$, $\tau_{b0} = 1 \times T$).

6. REFERENCES

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