A NOVEL PARTIAL ADAPTIVE BROAD-BAND BEAMFORMER USING CONCENTRIC RING ARRAY

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ABSTRACT

The design of an adaptive concentric ring array for broad-band beamforming is a challenge due to the large number of adapting coefficients. In this paper, the concept of setting the weights among different rings to enhance the beamformer output is used to design a partial adaptive ring array for applications in non-stationary signal environment. We decompose the weights of the array into two components: the weights for each receiving element within a ring and the weights for each ring. The partial adaptive array is obtained by setting the weight for each receiving element based on a priori knowledge of the Direction Of Arrival (DOA) of the desired signal, and only the weights for each ring are adapted in real-time. The partial array design results in faster convergence rate, better performance and smaller amount of computation compared to a fully adaptive array. Experimental results demonstrate the advantage of the partial adaptive array design.

1. INTRODUCTION

The ambiguity set of DOA for a linear array is a cone wrapping around itself, which increases the background noise level in the beamformer output and pick up some directional interference coming through the ambiguity directions. Two dimensional(2-D) array is generally superior to linear array in that the amount of ambiguity is reduced to 2 DOAs. Thus 2-D arrays have been widely used in 3-D beamforming.

Among 2-D arrays, circular ring array has received considerable interest for its symmetric and compact structure. Stearns *et al.* [1] have proposed a method to achieve low sidelobe level for continuous concentric ring antennas by optimizing the weights among different ring's beampattern through the Fourier-Bessel series inversion. In [2], Vu explored sidelobe control in a circular ring array. His method, however, is only effective for broadside direction beamforming. Kumar *et al.* [3] proposed a design of low sidelobe circular ring array by element radius optimization, which is suitable for narrow-band beamforming only.

In [4], we generalize Stearns' method for broad-band beamforming in 3-D and deduce the discrete ring array. We further apply a compound ring array design to make the concentric ring array suitable for broad-band signal acquisition. The design results in a deterministic beamformer with desired sidelobe level. A drawback of the design is that it may not achieve enough attenuation for some strong directional interference. Thus the application of the deterministic beamformer in the presence of strong interference or under non-stationary signal environment is limited.

Adaptive array can adapt its weights by learning the characteristics of an environment in real-time, thus they are more suitable for applications in non-stationary environment. Although many existing adaptive array design methods are available, the difficulty of designing an adaptive broad-band concentric ring array lies in that it usually consists of a large number of array elements. Thus if the array operates in fully adaptive manner, the computation burden can be a serious issue for real-time applications. Moreover, the convergence rate can be slow, which makes the adaptive array ineffective in rapidly changing signal environment.

To increase the convergence rate and reduce computation, we propose a partial adaptive array by making use of the specific structure of the concentric ring array. In the proposed design, only a small portion of the array weighs are adapted while the others are kept fixed by using a priori knowledge of the desired signal's DOA, that is usually available to a beamformer. We implement the proposed design using a General Sidelobe Canceller (GSC) configuration. Results from the deterministic beamformer design can then be incorporated to reduce the sidelobe level in the GSC's quiescent beampattern [5]. Two adaptation algorithms for the proposed partial adaptive array will be given.

The rest of the paper is organized as follows. Section 2 reviews the adaptive broad-band ring array design and its implementation. The disadvantage of a large number of coefficients in an array is also described. The partially adaptive design is proposed in Section 3 and its implementation is described in detail. Section 4 compares the performance of the fully adaptive and partially adaptive design using a simulation example. Section 5 is the summary.

2. BROAD-BAND BEAMFORMING USING CONCENTRIC RING ARRAY

A concentric ring array is composed of M rings as shown in Fig.1. Each ring has $N_i, i = 1 \dots M$ receiving elements allocated at equal spacing along its circumference. The total number of elements is $K = \sum_{i=1}^{M} N_i$. For a narrow-band signal impinging on the array, the beamformer output at time instant l is:

$$z(l) = \sum_{i=1}^{M} \sum_{j=1}^{N_i} v_{ij}^* x_{ij}(l) = \mathbf{v}^H \mathbf{x}(l) \quad (1)$$

where $x_{ij}(l)$ is the received signal from the jth element on the ith ring, v_{ij} is the corresponding weight and $(^*)$ denotes conju-

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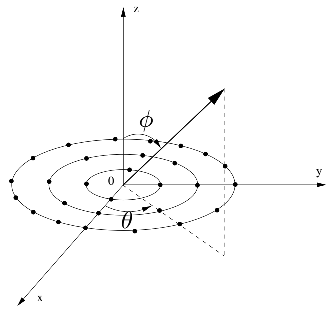

Fig. 1. Geometry of a concentric ring array.

gate. $\mathbf{x}(l)$ and $\mathbf{v}$ are the received data vector and weight vector respectively, both are of size $K \times 1$. The superscript $(^H)$ denotes the conjugate transpose.

A commonly used criteria to obtain $\mathbf{v}$ is the Linearly Constrained Minimum Variance (LCMV) criteria. A LCMV beamformer minimizes the output power, $E[|z(l)|^2] = \mathbf{v}^H \mathbf{R}_{xx} \mathbf{v}$ subject to a set of Q linear constraints in the form

$$\mathbf{C}^H \mathbf{v} = \mathbf{f}, \tag{2}$$

where $\mathbf{C}$ is a $K \times Q$ constraint matrix and $\mathbf{f}$ is a $Q \times 1$ vector of the constraint values. Here $\mathbf{R}_{xx}$ denotes the spatial correlation matrix of input data $\mathbf{x}(l)$. The solution is known to be [6]:

$$\mathbf{v}_{opt} = \mathbf{R}_{xx}^{-1} \mathbf{C} (\mathbf{C}^H \mathbf{R}_{xx}^{-1} \mathbf{C})^{-1} \mathbf{f}. \tag{3}$$

The GSC is an alternate implementation of the LCMV beamformer. GSC decomposes the LCMV weights into the constrained and unconstrained components. Fig. 2 shows the GSC structure of the concentric ring array. The upper branch is not adaptive and

$$\mathbf{v}_q = \mathbf{C}(\mathbf{C}^H \mathbf{C})^{-1} \mathbf{f}. \tag{4}$$

It is the LCMV solution when $\mathbf{R}_{xx} = \mathbf{I}$. The array response in the upper branch is termed as the array's quiescent response [5].

The lower branch performs unconstrained optimization. $\mathbf{B}_1$ is a $K \times (K - Q)$ blocking matrix satisfying $\mathbf{B}_1^H \mathbf{C} = \mathbf{0}$. $\mathbf{v}_a$ is a $(K - Q) \times 1$ weight vector corresponding to the unconstrained component of $\mathbf{v}$.

The adaptive array discussed above assumes a narrow-band signal. In broad-band beamforming, the received broad-band signal is usually decomposed into many narrow-band components using Fast Fourier Transform (FFT) and different components are processed by narrow-band beamformers. Denote the number of FFT components as L, there are a total of $(K-Q) \times L$ number of weights that need to be adapted. When K or L is large, the total computation required can be a huge burden and become the major obstacle for real-time applications.

Besides the computation burden, the large number of adaptive weights also result in slow convergence rate. When using adaptive algorithm such as Least Mean Squares (LMS) [6], the convergence rate of $\mathbf{v}_a$ is inversely proportional to the eigenvalue spread of $\tilde{\mathbf{x}}(l)$'s spatial correlation matrix, whose size is $(K-Q) \times (K-Q)$. For large K, the eigenvalue spread of the correlation matrix tends to be large, which results in slow convergence of $\mathbf{v}_a$. Other adaptive algorithm such as Recursive Least Squares (RLS) [6] offers faster convergence rate independent of the eigenvalue spread, but the computation involved is roughly $O((K - Q)^2)$, and this amount of complexity restricts its applications in large size

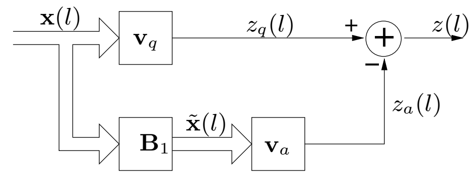

Fig. 2. Fully adaptive GSC structure of a concentric ring array.

fully adaptive array. Consequently, when the signal environment is changing rapidly, a broad-band concentric ring array operating in fully adaptive manner will have low quality output due to its slow convergence rate.

3. PARTIALLY ADAPTIVE CONCENTRIC RING ARRAY

A concentric ring array usually consists of a large number of elements. For some demanding applications, the total number is in the magnitude of several hundreds. To alleviate the computation burden and increase the convergence rate for the array, an effective way is to develop partial adaptive array to reduce the number of adaptive coefficients. The multi-ring structure of the array provides as a convenient basis for this purpose.

Note that each ring in the concentric ring array can be viewed as a stand alone array with the output:

$$y_i(l) = \mathbf{h}_i^H \mathbf{x}_i(l) \tag{5}$$

where $\mathbf{x_i}(l) = [x_{i1}(l), x_{i2}(l), \ldots, x_{iN_i}(l)]^T$ is a $N_i \times 1$ vector containing the data received by the elements on the ith ring and $\mathbf{h}_i$ denotes the corresponding weight vector. The superscript $(^T)$ denotes transpose.

The final output $z(l)$ of the concentric ring array can then be formed as a weighted sum of the output from each ring:

$$z(l) = \mathbf{w}^H \mathbf{y}(l) \tag{6}$$

where $\mathbf{y}(l) = [y_1(l), y_2(l), \ldots, y_M(l)]^T$ and $\mathbf{w} = [w_1, w_2, \ldots, w_M]^T$ is the weight vector that governs the contribution of the individual rings to the final output.

Putting (5) into (6) yields:

$$z(l) = \mathbf{u}^H \mathbf{x}(l) \tag{7}$$

where $\mathbf{x}(l) = [\mathbf{x}_1^T(l), \mathbf{x}_2^T(l), \ldots, \mathbf{x}_M^T(l)]^T$ is the $K \times 1$ array data vector, and

$$\mathbf{u} = [w_1 \mathbf{h}_1^T, w_2 \mathbf{h}_2^T, \ldots, w_M \mathbf{h}_M^T]^T \tag{8}$$

is a $K \times 1$ weight vector that combines the weights among the rings and within each ring.

Comparison of (7) and (1) shows that $\mathbf{u}$ is a partitioned form of $\mathbf{v}$ and there are a total $K + M$ number of weighting parameters in $\mathbf{u}$. Among those $K + M$ weights, we can set $\mathbf{h}_i$ to some fixed weights computed based on a priori knowledge of the desired signal's DOA, which is usually available. During the adaptation process, we only adapt $\mathbf{w}$ to track the changing characteristics of the signal environment, resulting in a partially adaptive array. $\mathbf{w}$ has the physical interpretation of creating nulls in the overall beampattern to remove interference by combining the beampattern from the individual rings.

Many standard beamforming techniques can be used to choose $\mathbf{h}_i$. For simplicity, we set $\mathbf{h}_i$ to be the delay-and-sum weights.

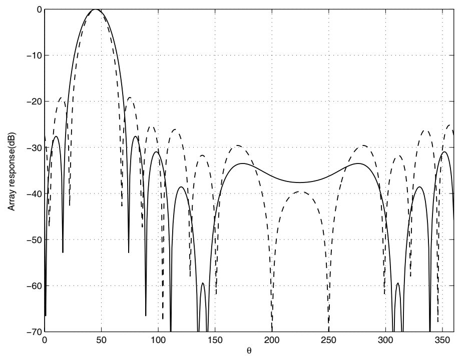

Fig. 3. Quiescent beampattern obtained from deterministic design from [4] (solid line) vs. quiescent beampattern obtained using (12)(dashed line).

Assuming the signal's DOA is (θ_0, ϕ_0), the delay-and-sum weight for the jth element in the ith ring can be calculated as [7]:

$$h_{ij} = \frac{1}{N_i} e^{jkR_i[\sin\phi_0 \, \cos(\theta_0 - v_j)]} \tag{9}$$

where R_i is the radius of the ith ring, $v_j = 2\pi j/N_i$ is the jth element's azimuth angle, $k = 2\pi/\lambda$ and λ is the wavelength of the signal. The delay-and-sum beamforming can reduce certain amount of noise and interference. Strong interference will be removed more effectively by the adaptive weight vector $\mathbf{w}$.

Note that the linear constraints specified in (2) is applied on $\mathbf{v}$. To derive the linear constraints applied to $\mathbf{w}$ for the design of partially adaptive array, we first partition $\mathbf{C}$ as:

$$\mathbf{C} = [\mathbf{C}_1^H, \mathbf{C}_2^H, \ldots, \mathbf{C}_M^H]^H \tag{10}$$

where $\mathbf{C}_i$ is a $N_i \times Q$ matrix, with N_i being the number of elements in the ith ring.

Putting (10) into (2) and replacing $\mathbf{v}$ by (8) yields:

$$\overline{\mathbf{C}}^H \mathbf{w} = \mathbf{f} \tag{11}$$

where $\overline{\mathbf{C}} = [\mathbf{C}_1^H \mathbf{h}_1, \mathbf{C}_2^H \mathbf{h}_2, \ldots, \mathbf{C}_M^H \mathbf{h}_M]^H$.

The proposed partial adaptive array in the form of GSC structure is shown in Fig.4. In the diagram, $\mathbf{y}(l)$ is an $M \times 1$ vector that contains the outputs from each ring at time instant l. $\mathbf{w}_q$ in the upper branch specifies the quiescent pattern of the partial adaptive array and it is:

$$\mathbf{w}_q = \overline{\mathbf{C}}(\overline{\mathbf{C}}^H \overline{\mathbf{C}})^{-1} \mathbf{f}. \tag{12}$$

In [4], we proposed a method to compute $\mathbf{w}$ so that a desired deterministic beampattern can be achieved for a given concentric ring array. As an alternative to (12), $\mathbf{w}_q$ can be computed using the deterministic design method to improve the quiescent beampattern in the upper branch. Fig. 3 shows a quiescent beampattern obtained using this method, together with a quiescent beampattern obtained through (12) when $\mathbf{C}$ contains only the unit gain constraint on the desired signal's look direction. The former has an overall lower sidelobe level of $-30dB$. Quiescent pattern design is useful for applications in rapidly changing signal environment. However, doing so requires an additional linear constraint to be added into $\mathbf{C}$ and the degrees of freedom of $\mathbf{w}$ is reduced by 1 [5]. Hence the proposed quiescent beampattern gives better results for larger M.

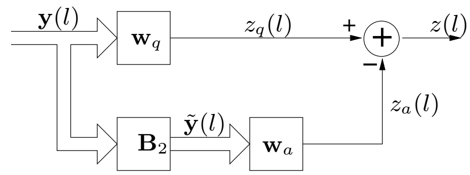

Fig. 4. Partial adaptive GSC structure of a concentric ring array.

In the lower branch of the diagram, $\mathbf{B}_2$ is a $M \times (M - Q)$ blocking matrix satisfying $\mathbf{B}_2^H \overline{\mathbf{C}} = \mathbf{0}$. $\mathbf{w}_a$ is the adaptive weight vector of size $(M - Q) \times 1$. Hence the partial adaptive array has $M - Q$ degrees of freedom and it is capable of cancelling up to $M - Q$ additional interferences besides those priorly specified in the linear constraint matrix $\mathbf{C}$.

There are many adaptive algorithms available to obtain $\mathbf{w}_a$. A popular one is the LMS [6] algorithm that minimizes the instantaneous output $z^*(l)z(l)$ through iterating:

$$z_a(l) = \mathbf{w}_a^H(l-1)\tilde{\mathbf{y}}(l) \tag{13}$$

$$\mathbf{w}_a(l) = \mathbf{w}_a(l-1) + \mu z^*(l)\tilde{\mathbf{y}}(l) \tag{14}$$

where μ is the step size. Larger value of μ results in faster convergence rate but also raises the fluctuation noise level in steady-state.

For some broad-band beamforming applications, the incoming signal's energy may concentrate in certain frequency range. In such cases, the spatial correlation matrices of the signal components in those frequency ranges have relatively large eigenvalue spreads, resulting in a very slow overall convergence rate. The convergence rate in LMS can be increased by enlarging the step size μ but the steady state performance will suffer.

Due to the small number of adaptive weights in the proposed partial adaptive array, more advanced adaptive algorithm such as RLS [6] algorithm can be used. RLS algorithm recursively computes the inverse of the correlation matrix, which yields faster convergence rate when compared to the LMS algorithm. The RLS algorithm applied to the proposed partial adaptive array is summarized as follow:

$$\mathbf{G}(l) = \frac{\mathbf{P}(l-1)\tilde{\mathbf{y}}(l)}{\gamma + \tilde{\mathbf{y}}^H(l)\mathbf{P}(l-1)\tilde{\mathbf{y}}(l)} \tag{15}$$

$$z(l) = z_q(l) - \mathbf{w}_a^H(l-1)\tilde{\mathbf{y}}(l) \tag{16}$$

$$\mathbf{P}(l) = \frac{1}{\gamma}[\mathbf{P}(l-1) - \mathbf{G}(l)\tilde{\mathbf{y}}^H(l)\mathbf{P}(l-1)] \tag{17}$$

$$\mathbf{w}_a(l) = \mathbf{w}_a(l-1) + \mathbf{G}(l)z^*(l) \tag{18}$$

where γ is a constant, and its typical value is between 0.98 and 1.

Comparing the fully adaptive array in Fig. 2 with the proposed partial adaptive array, the number of adaptive coefficients is reduced from $K - Q$ to $M - Q$. The number of complex multiplications required to update the weight vector for each of them is listed in Table 1 for comparison. Typically, $K \gg M$. Hence the computation is reduced by a factor of 3 in LMS and $3K$ in RLS as can be seen from the table.

	Fully adaptive	Partial adaptive
LMS	$O(2(K-Q)+K)$	$O(2(M-Q)+K)$
RLS	$O(3(K-Q)^2+K)$	$O(3(M-Q)^2+K)$

Table 1. Number of required complex multiplications.

4. EXPERIMENTAL RESULTS

To demonstrate the performance of the proposed partial adaptive array, we present a design example for the reception of speech signal in the frequency range of $100Hz - 4kHz$.

The received array signal is simulated by a computer, which contains a desired speech signal coming from ($\theta_0 = 45°, \phi_0 = 60°$). The interference is a speech signal coming from ($\theta_1 = 90°, \phi_1 = 70°$). The background noise is omni-directional. The signal to interference ratio is $-15dB$ and the signal to background noise ratio is $0dB$. To demonstrate the adaptation process, interference and background noise appear starting at about $t = 0.4s$.

The concentric ring array is composed of 9 rings grouped into 2 sub-arrays centered around $2kHz$ and $1kHz$ respectively. The radii of the 9 rings normalized by the first radius are: $[1, 2, 3, 4, 5, 6, 8, 10, 12]$. The largest ring's radius is $0.475m$. The first sub-array is composed of the $1 - 6th$ rings. Through the compound ring design [4], half of the rings in the first sub-array are reused in the second sub-array, thus the second sub-array is composed of the the $2, 4, 6, 7, 8, 9th$ rings. The number of array elements on each ring are: $[7, 14, 19, 23, 27, 32, 23, 27, 32]$. The total number of array elements are 204, of which 122 elements are in the first sub-array and 151 are in the second sub-array.

Speech signal is rich in low frequency content, thus spatial correlation matrices in low frequency components have large eigenvalue spreads. When implementing the adaptive algorithm, weights for frequencies lower than $1500Hz$ are updated using RLS algorithm to speed up convergence rate, other weights are updated using LMS algorithm for its simplicity. For comparison, we also implemented a fully adaptive array using LMS algorithm. Both fully and partial adaptive array use only one linear constraint, which is the unit gain constraint on the desired signal's DOA. Using Table 1, the number of complex multiplications in the partial adaptive array using the mix of LMS and RLS is found to be about 44% of that in the fully adaptive array using LMS.

The processing results are presented in Fig. 5. It can be seen that the outputs of both arrays are degraded heavily when interference appears at $0.4s$ but are improving afterwards as adaptation begins to cancel out the interference. The error signals in (e) and (f) show that due to the relatively slower convergence rate of the fully adaptive array, it is unable to remove interference as completely as in the case of partial adaptive array. However, the fully adaptive array demonstrate better performance for ambient noise cancellation towards the end. This is because the fully adaptive array eventually yields globally optimized weight vector to reduce the background noise when the interference disappears.

5. SUMMARY

In this paper, we proposed a partially adaptive concentric ring array for the beamforming of a broad-band signal. The partial adaptive array is derived by partitioning the array weight vector into two components: weights within each ring and weights among different rings. Only the weights among different rings are adapted to cancel out interference. Comparing to a fully adaptive array with the same configuration, the partial adaptive design has the advantage of having faster convergence rate and being less computationally expensive. The partial adaptive array is implemented in a GSC structure. Among many typical adaptive algorithms that can be used, we applied the LMS and the more advanced RLS for implementation. RLS gives faster convergence rate with only moderate increase in computation since the number of adaptive coefficients in the proposed beamformer is small. The proposed partial adaptive array is suitable for signal environment with rapidly changing characteristics because of its fast convergence rate. Experimental results demonstrate the superiority of the proposed technique. We plan to compare our method with beamspace partially adaptive array.

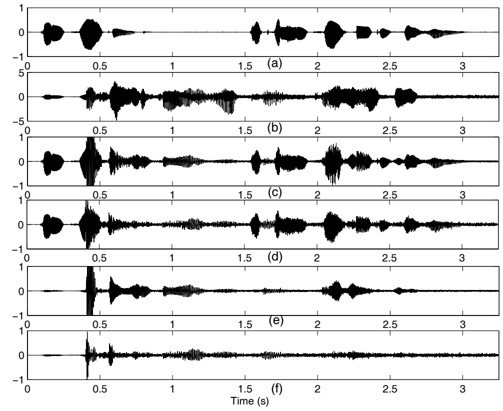

Fig. 5. Processing results: (a) Desired signal $s(l)$; (b) Received noisy signal in one channel; (c) Fully adaptive array output $z_1(l)$; (d) Partial adaptive array output $z_2(l)$; (e) Error signal: $e_1(l) = z_1(l) - s(l)$; (f) Error signal: $e_2(l) = z_2(l) - s(l)$.

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