FSF SUBSPACE-BASED ALGORITHM FOR JOINT DOA-FOA ESTIMATION

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ABSTRACT

This paper presents a tree-structured subspace-based algorithm for joint estimation of the directions of arrival (DOAs) and frequencies of arrival (FOAs) in wireless communication systems. The proposed approach is a hybrid of one-dimensional (1-D) subspace-based algorithms and spatial/temporal filtering processes, both of which are invoked alternatively to enhance the estimation accuracy. Two temporal and one spatial 1-D subspace-based algorithms are employed alternatively to estimate the FOAs and the DOAs, respectively. Between these subspace-based algorithms, a constrained temporal filtering process and a constrained spatial beamforming process are addressed, which minimize the filtered noise power under a set of linear constraints. These filtering processes aim to effectively partition the incoming rays and to be robust against the propagation errors in the tree-structured estimation scheme so that the overall performance can be enhanced. Furthermore, the estimated FOAs and DOAs are automatically paired without extra computational overhead. Furnished simulations show that the new approach can provide comparable performance with reduced complexity compared with previous works.

1. INTRODUCTION

It is of importance to estimate the parameters embedded in the received signal in wireless communication systems. For example, a precise estimation of the DOA and center of the frequencies of signals can provide exact information of users for further signal processing. As such, joint DOA-FOA estimation has received a flurry of attention recently.

The direct maximum likelihood approach, however, is in general computationally prohibitive. For this, various algorithms for joint DOA-FOA estimation have been addressed. For example, a two-dimensional (2-D) MUltiple SIgnal Classification (MUSIC)-based algorithm was considered in [1], which, however, estimates the parameters via carrying out high dimensional eigendecompositions of the covariance matrices and, in addition, requires a 2-D search on the DOA-FOA plane, thus still calling for enormous computations. To alleviate the computational overhead, several 1-D Estimation of Signal Parameters via Rotational Invariance Technique (ESPRIT)-based algorithms have been proposed. A 1-D ESPRIT-based algorithm was suggested in [2], which, however, needs a special array configuration. Another 1-D ESPRIT-based algorithm was advocated recently in [3]. Despite its high resolution capability, it still calls for higher dimensional singular value decomposition (SVD) apart from an extra pairing procedure to group the separately estimated parameters. On the other hand, [4] makes use of two 1-D ESPRIT algorithms to estimate the parameters separately with a marked subspace technique to overcome the pairing problem, but it can not resolve closely spaced signals.

This paper, we address a low complexity, yet high accuracy tree-structured subspace-based algorithm to jointly estimate the DOAs and FOAs in wireless communication systems. The proposed approach is a hybrid of the 1-D subspace-based algorithms and spatial/temporal filtering processes, both of which are invoked alternatively to enhance the estimation accuracy. More specifically, the proposed approach consists of three 1-D subspace-based algorithms, in which the two temporal and one spatial 1-D subspace-based algorithms are employed alternatively to estimate the FOAs and the DOAs, respectively. Between these subspace-based algorithms, a constrained temporal filtering process and a constrained spatial beamforming process are addressed, which determine the weights based on the linear constrained minimum variance (LCMV) criterion [5]. These filtering processes aim to effectively partition the incoming rays and to be robust against the propagation errors in the tree-structured estimation scheme so that the overall performance can be enhanced. Furthermore, the estimated FOAs and DOAs are automatically paired without extra computational overhead. Furnished simulation results show that the new approach can provide comparable performance but with reduced complexity compared with previous works.

2. SYSTEM MODEL

Consider a uniform linear array (ULA) with P omni-directional antennas. Assume that there are K uncorrelated narrowband sources $s_k(t)$, which are carried by the center frequencies $f_k, k = 1, \cdots, K$, impinging on the ULA. The observed signal at the i^{th} antenna element is given by

$$x_i(t) = \sum_{k=1}^{K} a_i(\theta_k) s_k(t) e^{j2\pi f_k t} + n_i(t),\ i = 1, ..., P \quad (1)$$

where $a_i(\theta_k) = e^{-j2\pi f_k d(i-1) sin\theta_k / c}$ denotes the i^{th} antenna response to the signal from the direction θ_k, c is the wave propagation speed and d is the distance between the adjacent antennas. After sampling the output of each antenna at a rate $f_s = 1/T_s$, and collecting L consecutive temporal samples, we can obtain the data matrix given by

$$\mathbf{X}(t) = \mathbf{A}_s \mathbf{S}(t) \mathbf{A}_t^H + \mathbf{N}(t) \quad (2)$$

where we have used the fact that $s_k(t)'s$ remain roughly the same for all t's due to the narrowband assumption [3], $\mathbf{X}(t) = [\mathbf{x}(t), \mathbf{x}(t-$

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$T_s), ..., \mathbf{x}(t-(L-1)T_s)]$ with $\mathbf{x}(t) = [x_1(t), ..., x_P(t)]^T]$, $\mathbf{A}_s = [\mathbf{a}_s(\mu_1), \mathbf{a}_s(\mu_2), ..., \mathbf{a}_s(\mu_K)]$, in which $\mathbf{a}_s(\mu) = [1, ..., e^{-(P-1)\mu}]$ with $\mu = j2\pi(f sin\theta)d/c$, denotes the $P \times K$ spatial matrix , and $\mathbf{A}_t = [\mathbf{a}_t(\upsilon_1), ..., \mathbf{a}_t(\upsilon_K)]^T$, in which $\mathbf{a}_t(\upsilon) = [1, ..., e^{-(L-1)\upsilon}]^T$ with $\upsilon = j2\pi f/f_s$, denotes the $L \times K$ temporal matrix. The signal matrix is $\mathbf{S}(t) = diag\{\mathbf{s}(t)\}$ with $\mathbf{s}(t) = [s_1(t), ..., s_K(t)]^T$ and the noise matrix is $\mathbf{N}(t) = [\mathbf{n}_s(t), \mathbf{n}_s(t+T_s), ..., \mathbf{n}_s(t+(L-1)T_s)]$, where $\mathbf{n}_s(t) = [n_1(t), ..., n_P(t)]^T$.

3. THE PROPOSED FSF APPROACH

The proposed approach begins with the computation of the *temporal* covariance matrix $\mathbf{R}^t = E[\mathbf{X}(t)^H \mathbf{X}(t)]$, which can be shown as

$$\mathbf{R}^t = \mathbf{A}_t \mathbf{P}_t \mathbf{A}_t^H + \sigma_n^2 \mathbf{I} \tag{3}$$

where $\mathbf{P}_t = E[\mathbf{s}(t)\mathbf{s}(t)^H]$ denotes the signal covariance matrix, and σ_n^2 is the noise power. An eigendecomposition can then be performed on the temporal covariance, and the high-resolution subspace-based algorithm such as the MUSIC [6] or ESPRIT [7] can be applied to estimate the frequencies. It is noteworthy that subspace-based algorithms still can not distinguish very close impinging rays, and here we assume that only q frequencies $(\hat{f}_1, ..., \hat{f}_q)$, referred to as the group frequencies, are estimated, where $q \le K$.

Based on the group frequencies, we can rebuild the temporal steering vectors $\mathbf{a}_t(\hat{\upsilon}_1), ..., \mathbf{a}_t(\hat{\upsilon}_q)$, and then, based on these, determine a set of temporal filters $\mathbf{w}_j^t$'s to minimize the output noise power under a set of the linear constraints. More specifically, the temporal filter for the group corresponding to the $\hat{\upsilon}_j$ is determined by

$$\min_{\mathbf{w}_j^t} \mathbf{w}_j^{tH} \mathbf{R}_n^t \mathbf{w}_j^t \quad \text{under} \quad \mathbf{C}^{tH} \mathbf{w}_j^t = \mathbf{g}_j^t, \;\; j = 1, ..., q \tag{4}$$

where $\mathbf{R}_n^t$ is the temporal noise covariance matrix, $\mathbf{g}_j^t$ is a $q \times 1$ vector with 1 on the j^{th} position and zeros elsewhere, and

$$\mathbf{C}^t = [\mathbf{a}_t(\hat{\upsilon}_1), ..., \mathbf{a}_t(\hat{\upsilon}_q)] \tag{5}$$

The solution of (4) can be readily shown as [5]

$$\mathbf{w}_j^t = (\mathbf{R}_n^t)^{-1} \mathbf{C}^t (\mathbf{C}^{tH} \big(\mathbf{R}_n^t)^{-1} \mathbf{C}^t\big)^{-1} \mathbf{g}_j^t, \;\; j = 1, ..., q \tag{6}$$

Because the noise is white, (6) can be reduced to

$$\mathbf{w}_j^t = \mathbf{C}^t (\mathbf{C}^{tH} \big(\mathbf{C}^t)\big)^{-1} \mathbf{g}_j^t, \;\; j = 1, ..., q \tag{7}$$

Note that there is any interpretation of $\mathbf{w}_j^t$ in (7), which provides an illuminating insight into the temporal filtering process. First, for notational brevity, we move $\mathbf{a}_t(\hat{\upsilon}_j)$ to the first column of $\mathbf{C}^t$ and partition it into two parts as

$$\mathbf{C}^t = [\mathbf{a}_t(\hat{\upsilon}_j), \mathbf{A}_{tj}^{\perp}] \tag{8}$$

where

$$\mathbf{A}_{tj}^{\perp} = [\mathbf{a}_t(\hat{\upsilon}_1), ..., \mathbf{a}_t(\hat{\upsilon}_{j-1}), \mathbf{a}_t(\hat{\upsilon}_{j+1}), ..., \mathbf{a}_t(\hat{\upsilon}_q)] \tag{9}$$

After some manipulations, $\mathbf{w}_j^t$ can be decomposed as

$$\mathbf{w}_j^t = \mathbf{P}_{Ij}^{t\perp} \frac{1}{\lambda_j^t} \mathbf{a}_t(\hat{\upsilon}_j) \tag{10}$$

where

$$\mathbf{P}_{Ij}^{t\perp} = \mathbf{I} - \mathbf{A}_{tj}^{\perp} (\mathbf{A}_{tj}^{\perp H} \mathbf{A}_{tj}^{\perp})^{-1} \mathbf{A}_{tj}^{\perp H} \tag{11}$$

is a projection matrix onto the subspace spanned by $\mathbf{a}_t(\hat{\upsilon}_j)$ and $\lambda_j^t = \mathbf{a}_t(\hat{\upsilon}_j)^H \mathbf{P}_{Ij}^{t\perp} \mathbf{a}_t(\hat{\upsilon}_j)$. Therefore, the data vector after temporal filtering can be expressed as

$$\mathbf{x}_j'(t) \triangleq \mathbf{X}(t) \mathbf{w}_j{}^t \tag{12}$$

$$= \mathbf{X}_j(t) \mathbf{w}_j'{}^t \tag{13}$$

where we have used (10),

$$\mathbf{X}_j(t) = \mathbf{X}(t) \mathbf{P}_{Ij}^{t\perp} \tag{14}$$

and

$$\mathbf{w}_j'{}^t = \frac{1}{\lambda_j^t} \mathbf{a}_t(\hat{\upsilon}_j) \tag{15}$$

As such, the temporal filtering process can be interpreted as the received data is filtered by a projection matrix and then by the temporal steering vector of the desired signal group. Note that $\mathbf{x}_j'(t)$ only contains the signals of a desired group and can be exploited to estimate the DOAs corresponding to this group. The *spatial* covariance matrix of $x_j'(t)$, $\mathbf{R}_j^s = E[\mathbf{x}_j'(t)\mathbf{x}_j'(t)^H]$, can be shown to be

$$\mathbf{R}_j^s = \mathbf{A}_{sj} \mathbf{P}_{sj} \mathbf{A}_{sj}^H + \frac{1}{\lambda_j^t} \sigma_n^2 \mathbf{I}, \quad j = 1, \cdots, r_j \tag{16}$$

where r_j denotes the number of rays corresponding to $\hat{f}_j$, $\mathbf{A}_{sj} = [\mathbf{a}_s(\mu_{j,1}) ..., \mathbf{a}_s(\mu_{j,r_j})]$, and $\mathbf{P}_{sj} = E[\mathbf{s}_j(t)\mathbf{s}_j(t)^H]$ with $\mathbf{s}_j(t) = [s_{j,1}(t), ... s_{j,r_j}(t)]^T$. Based on (16), we can use the 1-D subspace-based algorithm again to estimate μ.

After this, we carry out a spatial beamforming process to partition the signals into groups, yet still possessing the essential temporal information to estimate the frequencies in the succeeding stage. We use $\mathbf{X}_j(t)$ instead of $\mathbf{X}(t)$ to proceed to the next step to make sure that the data only contain the j^{th} group signals. Similar to the temporal filtering process,the spatial filter can be determined by minimizing the output noise power, which is posed as

$$\min_{\mathbf{w}_{j,k}^s} \mathbf{w}_{j,k}^{sH} \mathbf{R}_n^s \mathbf{w}_{j,k}^s \quad \text{under} \quad \mathbf{C}_j^{sH} \mathbf{w}_{j,k}^s = \mathbf{g}_{j,k}^s, \; k = 1, ..., r_j \tag{17}$$

where $\mathbf{R}_n^s$ is the spatial noise covariance matrix, $\mathbf{g}_{j,k}^s$ is an $r_j \times 1$ vector with 1 on the k^{th} position and zeros elsewhere, and

$$\mathbf{C}_j^s = [\mathbf{a}_s(\hat{\mu}_{j,1}), ..., \mathbf{a}_s(\hat{\mu}_{j,r_j})] \tag{18}$$

and the solution of (17) can be readily shown as

$$\mathbf{w}_{j,k}^s = (\mathbf{R}_{nj}^s)^{-1} \mathbf{C}_j^s (\mathbf{C}_j^{sH} \big(\mathbf{R}_{nj}^s)^{-1} \mathbf{C}_j^s\big)^{-1} \mathbf{g}_{j,k}^s \tag{19}$$

It can be readily shown that the noise after temporal filtering is still white, so (19) can also be reduced to

$$\mathbf{w}_{j,k}^s = \mathbf{C}_j^s (\mathbf{C}_j^{sH} \mathbf{C}_j^s)^{-1} \mathbf{g}_{j,k}^s \tag{20}$$

The data, after constrained beamforming, then become

$$\mathbf{x}_{j,k}(t) = (\mathbf{w}_{j,k}^s)^H \mathbf{X}_j(t) \tag{21}$$

which only contains the desired signal we are interested in.

Repeat (3) to obtain the temporal covariance again, but with $\mathbf{X}(t)$ being replaced by $\mathbf{x}_{j,k}(t)$. Based on the new temporal covariance matrix, we use the temporal subspace-based algorithm again to obtain a more precise estimate of the frequency $\hat{f}_{j,k}$. At the same time, we can exploit this frequency and the $\hat{\mu}_{j,k}$ estimated in the previous subspace-based algorithm to estimate the DOA by

$$\hat{\theta}_{j,k} = sin^{-1}(\frac{c \cdot \hat{\mu}_{j,k}}{j2\pi d \hat{f}_{j,k}}) \quad (22)$$

The overall structure of the proposed algorithm is shown in Fig. 1 with each step being described as follows:

Step 1: **Rough FOA Estimation**:

From the received data, estimate the temporal covariance matrix by

$$\hat{\mathbf{R}}^t = \frac{1}{PM} \sum_{m=1}^{M} \mathbf{X}(mT_s)^H \mathbf{X}(mT_s) \quad (23)$$

where M is the total number of samples. We then apply the subspace-based algorithm to $\hat{\mathbf{R}}^t$ to estimate the group frequencies $(\hat{f}_1, ..., \hat{f}_q)$. Exploit (11) to constitute the projection matrix and then use (13) to find $\mathbf{X}_j(mT_s)$.

Step 2: **Temporal Filtering**:

Determine the temporal filters $\mathbf{w'}^t_j, j = 1, ..., q$, by (15) and use (13) to obtain $\mathbf{x'}_j(mT_s)$, which are the partitioned data after the constrained temporal filtering.

Step 3: **Estimation of μ**:

From each $\mathbf{x'}_j(mT_s)$, we can estimate the spatial covariance matrices by

$$\hat{\mathbf{R}}^s_j = \frac{1}{M} \sum_{m=1}^{M} \mathbf{x}'_j(mT_s) \cdot \mathbf{x}'_j(mT_s)^H, \ j = 1, \cdots, r_j \quad (24)$$

We can then apply the subspace based algorithm to each $\hat{\mathbf{R}}^s_j$ to estimate spatial parameter $\{\hat{\mu}_{j,1}, ..., \hat{\mu}_{j,r_j}\}$ in the j^{th} group.

Step 4: **Spatial Beamforming**:

Determine the spatial filters $\mathbf{w}^s_{j,k}, k = 1, ...r_j$, by (20) and use (21) to obtain $\mathbf{x}_{j,k}(mT_s)$ for all j's and k's, which are the data containing only the ray with $\hat{\mu}_{j,k}$.

Step 5: **FOA and DOA Estimation**:

Repeat (3) to obtain the temporal covariance again, but with $\mathbf{X}(mT_s)$ being replaced by $\mathbf{x}_{j,k}(mT_s)$. Based on the new temporal covariance matrix, we use the subspace-based algorithm again to get a more exact frequency $(\hat{f}_{j,k})$. At the same time, we can exploit this estimate and $\hat{\mu}_{j,k}$ to estimate the DOA $\hat{\theta}_{j,k}$ by (22).

4. PERFORMANCE RELATED ISSUES

4.1. Cramer-Rao Lower Bound

In this section, we determine the Cramer-Rao lower bound (CRB) based on the data in (2), which the spatial covariance matrix consists of two parameters, i.e. DOAs and FOAs. The derivations are a modification of those given in [3]. First, note that the log likelihood function of the signal is given by [3]

$$Ł(\mathbf{x}; \gamma) = -\frac{PM}{2} \ln(2\pi\sigma^2) - \frac{1}{2\sigma^2} \sum_{m=1}^{M} [\mathbf{x}(m) - \mathbf{s}(m; \gamma)]^H [\mathbf{x}(m) - \mathbf{s}(m; \gamma)] \quad (25)$$

where $\mathbf{x}(m) = \mathbf{A}_s \mathbf{B} \mathbf{a}_f + \mathbf{n}(m) \stackrel{\triangle}{=} \mathbf{s}(m; \gamma) + \mathbf{n}(m)$, in which $\mathbf{B} = diag\{\beta_i\}_{i=1}^K$ with $\beta_i \in \mathbf{R}^+$ is the amplitude of the i^{th} signal, $\mathbf{a}_f = [e^{-v_1}, ..., e^{-v_K}]^H$, and $\gamma \stackrel{\triangle}{=} [\theta^T, \mathbf{f}^T, \beta^T]^T$ with $\theta = [\theta_1, ..., \theta_K]^T$, $\mathbf{f} = [f_1, ..., f_K]^T$, and $\beta = [\beta_1, ..., \beta_K]^T$. The Fisher information matrix then is given by

$$\mathbf{I}(\gamma) = \frac{1}{\sigma^2} \mathbf{Re}\Big(\sum_{m=1}^{M} \mathbf{D}_m(\gamma)^H \mathbf{D}_m(\gamma) \Big) \quad (26)$$

where $\mathbf{D}_m(\gamma) = [\frac{\partial \mathbf{s}(m;\gamma)}{\partial \theta}, \frac{\partial \mathbf{s}(m;\gamma)}{\partial \mathbf{f}}, \frac{\partial \mathbf{s}(m;\gamma)}{\partial \beta}]$ denotes the gradient of the signal $\mathbf{s}(m; \gamma)$ in which

$$\begin{aligned} \frac{\partial \mathbf{s}(m;\gamma)}{\partial \theta} &= \mathbf{D}_\theta \mathbf{B} \mathbf{F} \\ \frac{\partial \mathbf{s}(m;\gamma)}{\partial \mathbf{f}} &= \mathbf{D}_f \mathbf{B} \mathbf{F} + (-j2\pi m / f_s) \mathbf{A}_s \mathbf{B} \mathbf{F} \\ \frac{\partial \mathbf{s}(m;\gamma)}{\partial \beta} &= \mathbf{A}_s \mathbf{F} \end{aligned} \quad (27)$$

with $\mathbf{D}_\theta = \partial \mathbf{A}_s / \partial \theta$, $\mathbf{D}_f = \partial \mathbf{A}_s / \partial \mathbf{f}$, and $\mathbf{F} = diag\{\mathbf{a}_f\}$. The CRB for θ and $\mathbf{f}$ can then be found from the inverse of the Fisher information as

$$\mathbf{CRB}(\hat{\theta}_i) = [\mathbf{I}^{-1}(\gamma)]_{i,i}, \mathbf{CRB}(\hat{f}_i) = [\mathbf{I}^{-1}(\gamma)]_{i+K,i+K} \quad (28)$$

4.2. Computational Complexity

This subsection determines the arithmetic operations required by the proposed approach using the ESPRIT as the underlying subspace-based algorithm, referred to as FSF-ESPRIT. Note that in general the length of the total samples is greater than the number of the antennas and the size of temporal vector employed, i.e., $M \gg P$, $M \gg L$, the computations required by the FSF-ESPRIT are therefore dictated by (a) estimation of the first and second temporal covariance matrices, which require ML^2P and KML^2 multiplications, respectively; (b) estimation of the spatial covariance matrices, which require qMP^2 multiplications, respectively; (c) temporal and spatial filtering processes, which require $qMLP$ and $KMLP$ multiplications, respectively, and the filtering process by the projection matrices, which require qML^2P multiplications. As a whole, the total number of multiplications required is about $MPL(qL + L + q + K) + qMP^2 + KML^2$. Note that the computational complexity of the proposed approach is in general lower than that of the counterparts such as [3] as the latter stacks the data before the SVD operation and thus involves higher dimensional eigendecomposition.

5. SIMULATIONS AND DISCUSSION

Some simulations are conducted in this section to assess the proposed approach. Assume that there are three users in this system, received by a six-element ULA which spaced a half wavelength apart. The temporal vector length is $L = 6$, and the sampling frequency is $f_s = 500$ MHz. The DOAs of the users are $[-30, -29, 30]^o$ with the center frequencies $[160, 239, 240]$ MHz, respectively.

Three algorithms are carried out for comparison, including the the algorithm in [1], JAFE in [3], and the proposed FSF-ESPRIT. 128 symbols are employed to estimate the temporal and spatial covariances. For each specific SNR, 200 Monte Carlo trials are carried out. For a clear illustration, only the root-mean-square-errors (*RMSEs*) of the DOA and FOA estimates of the second user, which is close to the first user in DOA and the third user in FOA

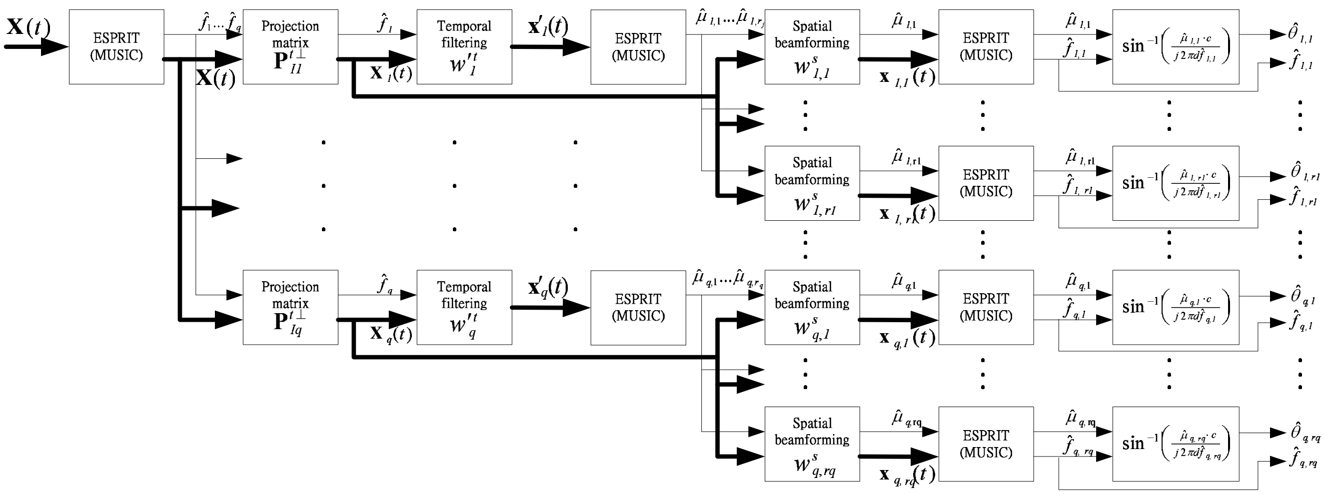

Fig. 1. The structure of the proposed algorithm

are provided, as shown in Figs. 2 and 3.

We can note from Figs. 2 and 3 that the proposed FSF-ESPRIT outperforms [1] in both of the DOA and FOA estimates. The is due to the fact that in [1] the covariance matrix is estimated using only part of the received data. The computational complexity of [1] is also substantially higher than that of the FSF-ESPRIT, as it involves a 2-D search. We can also note that the performance of the proposed algorithm is close to the JAFE. However, in this scenario, apart from the overhead of pairing process, the computational complexity of the FSF-ESPRIT is only about 30 % of that of the JAFE if the R-Bidiagonalization SVD is used.

6. REFERENCES

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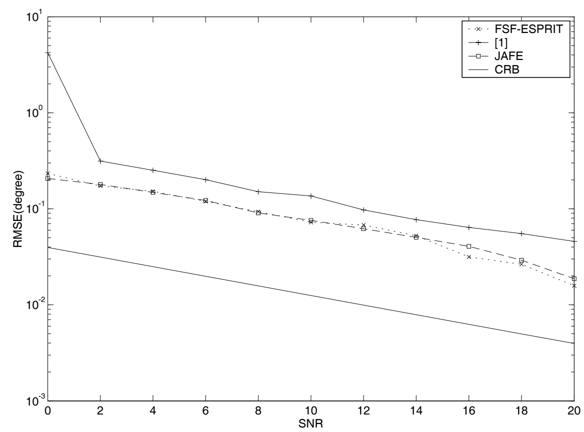

Fig. 2. Comparison of the DOA estimation for various algorithms

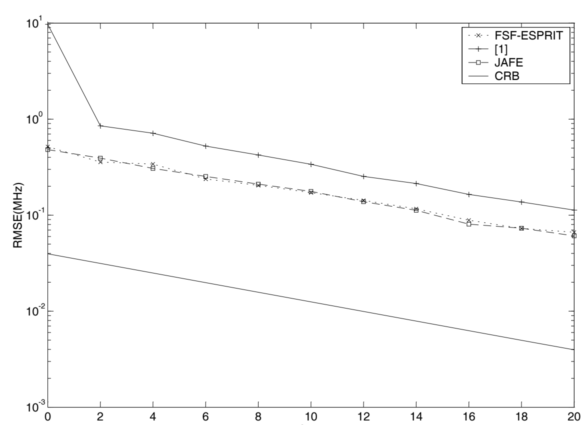

Fig. 3. Comparison of the FOA estimation for various algorithms