RANGE AND BEARING ESTIMATION OF WIDEBAND SOURCES USING AN ORTHOGONAL BEAMSPACE PROCESSING STRUCTURE

Darren B. Ward
Dept. of Electrical and Electronic Engineering
Imperial College London
London SW7 2BT, United Kingdom

Thushara D. Abhayapala
Dept. of Telecommunications Engineering, RSISE
The Australian National University
Canberra ACT 0200, Australia

ABSTRACT

We propose a new method for range and bearing estimation of wideband sources based on a novel beamspace structure. The proposed structure consists of an orthogonal set of beamspace processors that have a frequency-invariant beampattern property, and can be steered easily to any nearfield range while maintaining the frequency-invariance of the spatial response. These properties mean that the proposed structure can perform coherent signal subspace processing for nearfield wideband sources, making it ideal for wideband range and bearing estimation. Performance of the estimator is demonstrated through Monte Carlo simulations.

1. INTRODUCTION

Passive range and bearing estimation has received considerable attention over the past two decades (see [1] for a review). While there has been a plethora of range/bearing estimators reported, having them operate well for wideband and nearfield sources remains a challenging problem. In this paper we use a general beamspace processing structure (based on a modal beampattern decomposition) to develop a wideband range/bearing estimator. The proposed beamspace structure has the following key properties: (i) a set of fixed frequency-independent beamformers transforms array data from element space to a reduced-dimension beamspace; and (ii) a single parameter is used to steer these beamformers to any desired radial distance (outside the array aperture).

A common approach to wideband direction-of-arrival (DOA) estimation is the coherent signal subspace method [2]. The idea is to use a set of focusing matrices that transform the narrowband data in each frequency bin to a single reference frequency bin, where a focused covariance matrix is formed and used with a conventional narrowband DOA estimator (e.g., MUSIC [3]).

The advantage of beamspace processing (as distinct from conventional elementspace processing), is that the dimensionality of the data can be greatly reduced, leading to computational savings. As shown in [4, 5], beamspace processors that exhibit a frequency-invariant beampattern property perform analogously to focusing matrices in the coherent signal subspace method. The beamspace structure in this paper has this frequency-invariance property, meaning that it can be used straightforwardly for coherent wideband DOA estimation. Moreover, because of the explicit radial-focusing ability of our proposed structure, it has the added advantage that the frequency-invariance property can be maintained at any radial distance. The beamspace structure in this paper can therefore be used for wideband DOA *and* range estimation. An elementspace processing structure for nearfield coherent focusing was recently presented in [6, 7]. To the best of our knowledge, no previously proposed *beamspace* processor can achieve wideband focusing in nearfield as well as farfield.

2. PROBLEM FORMULATION

Consider a signal environment in which D wideband source signals impinge on a linear array of Q sensors located at points $\ell_1, \ldots, \ell_Q$ along a straight line. Assuming that D is known, the problem is to determine the location (bearing and range) of each wideband source.

At each sensor, time-domain data is collected over M samples and a windowed M-point DFT used to form a frequency-domain snapshot. The signal received in the mth frequency bin at the qth sensor ($q = 1, \ldots, Q$) is

$$x_q(f_m) = \sum_{d=1}^{D} v_q(r_d, \theta_d, f_m)\, s_d(f_m) + n_q(f_m), \qquad (1)$$

where $s_d(f_m)$ is the dth source signal located at (r_d, θ_d), $n_q(f_m)$ is additive white noise, and

$$v_q(r, \theta, f_m) = r e^{ik_m r} \frac{e^{-ik_m \sqrt{\ell_q^2 + r^2 - 2r\ell_q \cos\theta}}}{\sqrt{\ell_q^2 + r^2 - 2r\ell_q \cos\theta}} \qquad (2)$$

is the array manifold with $k_m = 2\pi f_m / c$ the wavenumber, c is the speed of wave propagation, and f_m is the frequency. For a farfield source, the array manifold becomes

$$v_q(\theta, f_m) \triangleq \lim_{r \to \infty} v_q(r, \theta, f_m) = e^{ik_m \ell_q \cos\theta}. \qquad (3)$$

In beamspace processing, P beamformers ($D \leq P < Q$) are created from these sensor signals, and the pth beamformer output is

$$y_p(f_m) = \mathbf{w}_{p,m}^H \mathbf{x}_m, \quad p = 0, \ldots, P-1, \tag{4}$$

where

$$\mathbf{w}_{p,m} = [w_{p,1}(f_m), \ldots, w_{p,Q}(f_m)]^H$$

is the array weight vector for the pth beamformer (with $w_{p,q}(f_m) \in \mathbb{C}$ the qth weight of the pth beamformer), and

$$\mathbf{x}_m = [x_1(f_m), \ldots, x_Q(f_m)]^T$$

is the array data vector. We use $\cdot^T$ to denote transpose, and $\cdot^H$ to denote Hermitian (complex conjugate) transpose. Stack the outputs of the P beamformers to form the $P \times 1$ beamspace vector

$$\mathbf{y}_m = \mathbf{W}_m^H \mathbf{x}_m, \tag{5}$$

where

$$\mathbf{W}_m = [\mathbf{w}_{0,m} \cdots \mathbf{w}_{P-1,m}] \tag{6}$$

is the $Q \times P$ weight matrix. Thus $\mathbf{W}_m$ transforms the Q-dimensional elementspace data vector $\mathbf{x}_m$ to the P-dimensional beamspace data vector $\mathbf{y}_m$.

Beamspace processors often are designed such that the set of beamformers (4) covers a chosen spatial region within which the desired sources are assumed to lie [8]. In this paper, however, the beamspace processing is based on a particular set of orthogonal beamformers whose design is motivated by the modal representation of beampatterns. By adopting this approach, we demonstrate that the resulting beamspace processors have the unique property that they perform wideband coherent focusing at *any* radial distance outside the array aperture.

3. MODAL REPRESENTATION OF BEAMPATTERNS

The spatial response of the pth beamformer to a point source at polar coordinate (r, θ) is

$$b_{p,m}(r,\theta) = \mathbf{w}_{p,m}^H \mathbf{v}_m(r,\theta), \tag{7}$$

where $\mathbf{v}_m(r,\theta) = [v_1(r,\theta,f_m), \ldots, v_Q(r,\theta,f_m)]^T$ is the array manifold vector. We champion the use of the specific set of beamformers where at a radial distance r, the pth beamformer has a spatial response:

$$b_{p,m}(r,\theta) = Y_p(\theta), \quad m \in \mathcal{M}, \tag{8}$$

where $\mathcal{M}$ is the set of indices for the frequency bins that fall within the frequency band of interest $[f_L, f_U]$,

$$Y_p(\theta) \triangleq \sqrt{\frac{2p+1}{2}}\, L_p(\cos\theta), \quad p = 0, \ldots, P-1, \tag{9}$$

are a set of modal beampatterns, and $L_p(\cos\theta)$ are the Legendre functions. Because of the orthogonality property of Legendre functions, these beampatterns form an orthonormal basis set on $[0, \pi]$.

This particular set of beampatterns arises from the modal representation of beampatterns [9], which can be expressed as follows. Consider the spatial response of an arbitrary beamformer created from a linear array:

$$b(r,\theta) \triangleq \sum_{q=1}^{Q} w_q v_q(r,\theta). \tag{10}$$

For a radial distance r that is outside the aperture of the array (i.e., $r > \max|\ell_q|$), the Jacobi-Anger expansion [10] can be used to show that any arbitrary beampattern that can be written in the form (10), can be represented using the modal expansion:

$$b(r,\theta) = \sum_{n=0}^{\infty} \alpha_n g_n(r) Y_n(\theta), \tag{11}$$

where

$$\alpha_n = i^n \sqrt{4n+2} \sum_{q=1}^{Q} w_q j_n(k\ell_q), \tag{12}$$

are a set of modal weighting terms,

$$g_n(r) \triangleq \frac{kr}{i^{n+1}}\, e^{ikr} h_n^{(2)}(kr), \tag{13}$$

and $Y_n(\theta)$ is given by (9). In this expansion, n is a non-negative integer to index modes, $h_n^{(2)}(\cdot)$ are the spherical Hankel functions of the second kind, and $j_n(\cdot)$ are the spherical Bessel functions.

The key to the use of this expansion is that, for a given wavenumber, each mode of the beampattern has two separable terms: the first term $g_n(r)$ is a *radial steering* term that is independent of angle, and the second term $Y_n(\theta)$ is a valid beampattern that is *independent of frequency*. This unique separability property provides us with the capability to construct frequency-independent beamformers that can readily be steered to any radial distance. Hence, this beamspace structure extends the wideband beamspace coherent signal subspace approach in [4, 5] to nearfield distances, thereby providing a very general means for wideband nearfield range and bearing estimation.

A further useful property of this modal expansion arises by noting that (11) can be viewed as a standard Fourier transform type expansion of the orthonormal basis set $Y_n(\theta), n = 0, \ldots, \infty$. Thus, for any beampattern that can be represented by (11), the best approximation using a finite number of terms, N, is given by truncating the expansion to use the terms $n = 0, \ldots, N-1$. Hence, using the set of beampatterns $Y_p(\theta), p = 0, \ldots, P-1$, to form a beamspace processor is a well-posed approach.

Assume that one has designed a set of farfield beamformers that produce the required beampatterns in the farfield (this can be achieved through any one of a multitude of fixed farfield design techniques). Let $\mathbf{W}_m^{\infty}$ denote the set of array weights that produce the farfield responses $b_{p,m}(\infty, \theta) = Y_p(\theta)$. It can be shown [11] that the set of array weights that produce the nearfield responses $b_{p,m}(r, \theta) = Y_p(\theta)$ are given by

$$\mathbf{W}_m(r) = \mathbf{W}_m^{\infty}\, \mathbf{G}_m^H(r), \tag{14}$$

where $\mathbf{G}_m(r) = \text{diag}[1/g_0(r), \ldots, 1/g_{P-1}(r)]$, with $g_p(r)$ given by (13). The fact that $\mathbf{G}_m(r)$ is a diagonal matrix demonstrates a key property of our beamspace structure: once the farfield weight matrix $\mathbf{W}_m^{\infty}$ has been designed, the beamspace processor can be steered to any arbitrary radial distance simply by multiplying the farfield beamformer outputs by a scalar.

4. WIDEBAND RANGE AND BEARING ESTIMATION

We now describe how the proposed structure may be used for wideband range/bearing estimation using the well-known MUSIC algorithm [3].

From (1), the $Q \times Q$ elementspace spatial spectrum matrix is

$$\mathbf{S}_m^{\text{ES}} = \mathbf{V}_m(\Theta)\mathbf{S}_m\mathbf{V}_m^H(\Theta) + \sigma^2\mathbf{I}_Q, \tag{15}$$

where $\mathbf{S}_m$ is the $D \times D$ source covariance matrix, σ^2 is the variance of the additive noise, $\Theta = [(r_1, \theta_1) \ldots (r_D, \theta_D)]$ is the set of D source ranges and bearings, and the columns of $\mathbf{V}_m(\Theta)$ are the array manifold vectors at the respective source locations. After transforming the received data to beamspace through (5) and (14), the $P \times P$ beamspace covariance matrix becomes:

$$\begin{aligned}\mathbf{S}_m^{\text{BS}} &= \mathbf{W}_m^H(r)\mathbf{S}_m^{\text{ES}}\mathbf{W}_m(r)\\ &= \mathbf{W}_m^H(r)\mathbf{V}_m(\Theta)\mathbf{S}_m\mathbf{V}_m^H(\Theta)\mathbf{W}_m(r)\\ &\quad +\sigma^2\mathbf{S}_m^{\text{NOISE}}(r),\end{aligned} \tag{16}$$

where $\mathbf{S}_m^{\text{NOISE}}(r) = \mathbf{W}_m^H(r)\mathbf{W}_m(r)$ is the beamspace noise covariance matrix.

Because the beamspace processing is designed to maintain a frequency-independent spatial response within the design band, at any radial distance r outside the array aperture we have the following nearfield focusing property:

$$\mathbf{W}_m^H(r)\mathbf{v}_m(r, \theta) = \mathbf{W}_0^H(r)\mathbf{v}_0(r, \theta), \quad m \in \mathcal{M}, \forall\theta, \tag{17}$$

where $\mathbf{v}_0(r, \theta)$ is the array manifold vector evaluated at the mid-band frequency $f_0 = (f_U + f_L)/2$. Thus, one can combine the radially-focused data across the frequency band of interest, without destroying the spatial information contained in the data covariance matrix.

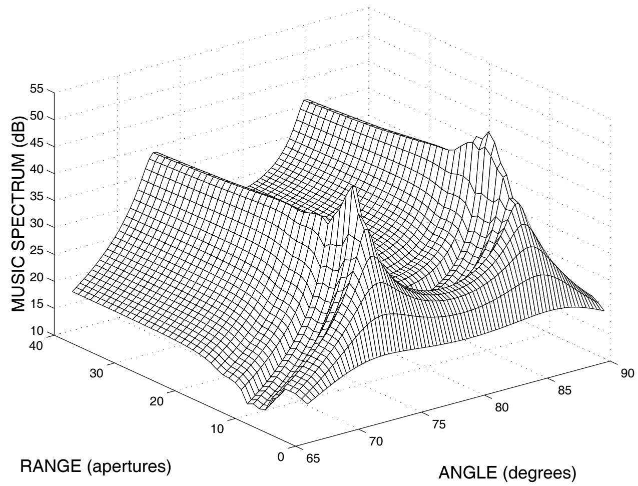

Fig. 1. Asymptotic wideband beamspace MUSIC spectrum obtained for two nearfield sources.

The coherent wideband MUSIC spatial spectrum is formed as

$$\Phi_{\text{COH}}(r, \theta) = \frac{\tilde{\mathbf{v}}_0^H(r, \theta)\tilde{\mathbf{v}}_0(r, \theta)}{\tilde{\mathbf{v}}_0^H(r, \theta)\mathbf{E}(r)\mathbf{E}^H(r)\tilde{\mathbf{v}}_0(r, \theta)}, \tag{18}$$

where $\tilde{\mathbf{v}}_0(r, \theta) = \mathbf{W}_0^H(r)\mathbf{v}_0(r, \theta)$ is the focussed beamspace manifold vector evaluated at the mid-band frequency, and $\mathbf{E}(r)$ is the matrix of eigenvectors corresponding to the $(P - D)$ smallest eigenvalues of the generalized eigendecomposition of $[\bar{\mathbf{S}}^{\text{BS}}(r), \bar{\mathbf{S}}^{\text{NOISE}}(r)]$. The wideband beamspace data and noise covariance matrices are, respectively,

$$\bar{\mathbf{S}}^{\text{BS}}(r) = \sum_{m \in \mathcal{M}} \mathbf{S}_m^{\text{BS}}(r), \tag{19}$$

$$\bar{\mathbf{S}}^{\text{NOISE}}(r) = \sum_{m \in \mathcal{M}} \mathbf{S}_m^{\text{NOISE}}(r). \tag{20}$$

As with the elementspace estimator described in [6], the iso-contours of the wideband MUSIC spatial spectrum (18) are aligned with the (r, θ) axes. This property is demonstrated in Fig. 1, which shows the asymptotic MUSIC spectrum for two wideband sources with SNR of 10 dB, located at bearings of $85°$ and $73°$ and normalized ranges of 10 and 7.5 times the array aperture (this is the same source scenario as in [6]). The source signals covered the normalized frequency range from $f_L = 0.2$ to $f_U = 0.4$, with a normalized sampling frequency of 1. The array consisted of 20 elements with a uniform spacing corresponding to a half-wavelength at a normalized frequency of 0.5, and used a beamspace processor with $P = 8$ beamformers designed according to Section 3.[1]

[1]The farfield weight matrix $\mathbf{W}_m^{\infty}$ was designed using a least squares farfield beampattern design method.

Not only does the spatial spectrum exhibit unique peaks at the correct ranges and bearings, it also has "ridges" that remain at the correct bearings for increasing range. As described in [6], this structure in the wideband MUSIC spectrum means that the 2D search for the source locations can be performed by two one-dimensional searches: first by searching for bearing with a farfield beamformer (i.e., with $\mathbf{G}_m(\infty) = \mathbf{I}_P$ in (14)); then by searching over r using these bearings.

5. SIMULATIONS

To demonstrate the performance of the proposed range/bearing estimator, we created synthetic element data for a wideband source covering the normalized frequency band of $f_L = 0.2$ to $f_U = 0.4$ (with a normalized sampling frequency of 1), impinging on a uniform linear array of $Q = 20$ sensors with an inter-sensor separation corresponding to half a wavelength at a normalized frequency of 0.5. Beamspace processing was performed using a set of $P = 8$ modal beamformers as described in Section 3. There was a single source (with a flat frequency spectrum) present at $65°$ at a radial distance of 2 aperture lengths (where the aperture length is defined as $\max \ell_q - \min \ell_q$). The frequency domain data was obtained with a DFT of length 256, resulting in 51 frequency bins within the design band. Random realizations of the elementspace data were created, and an estimate of $\mathbf{S}_m^{\text{ES}}$ was obtained from 50 snapshots. Performance of the estimator was evaluated using 100 Monte Carlo simulations. This was repeated for the source at a radial distance of 5 apertures, keeping all other parameters the same. Results of the range estimation are shown in Fig. 2, which shows the root mean-square-error (normalized with respect to the aperture size) versus the SNR.[2]

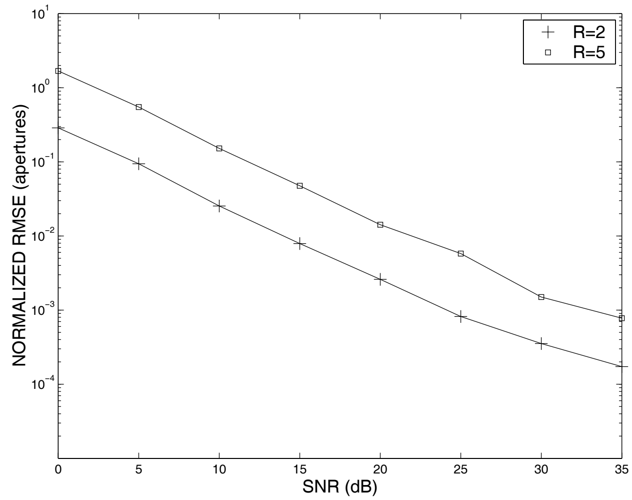

Fig. 2. Performance of the proposed beamspace processor in estimating range as a function of SNR.

6. CONCLUSIONS

We have presented a new beamspace processing structure having the unique property that it can maintain a frequency-invariant spatial response while being steered to any radial distance. Using this property, we showed how the beamspace structure can be used for wideband range/bearing estimation. Results from Monte Carlo simulations indicate that the proposed technique can accurately locate wideband nearfield sources.

7. REFERENCES

[1] H. Krim and M. Viberg, "Two decades of array signal processing research: The parametric approach," *IEEE Signal Processing Mag.*, vol. 13, no. 4, pp. 67–94, July 1996.

[2] H. Wang and M. Kaveh, "Coherent signal-subspace processing for the detection and estimation of angles of arrival of multiple wide-band sources," *IEEE Trans. Acoust., Speech, Signal Processing*, vol. ASSP-33, no. 4, pp. 823–831, Aug. 1985.

[3] R.O. Schmidt, "Multiple emitter location and signal parameter estimation," *IEEE Trans. Antennas Propagat.*, vol. AP-34, no. 3, pp. 276–280, Mar. 1986.

[4] T.S. Lee, "Efficient wideband source localization using beamforming invariance technique," *IEEE Trans. Signal Processing*, vol. 42, no. 6, pp. 1376–1387, June 1994.

[5] D.B. Ward, Z. Ding, and R.A. Kennedy, "Broadband DOA estimation using frequency invariant beamforming," *IEEE Trans. Signal Processing*, vol. 46, no. 5, pp. 1463–1469, May 1998.

[6] R. Jeffers, K.L. Bell, and H.L. Van Trees, "Broadband passive range estimation using MUSIC," in *Proc. IEEE Int. Conf. Acoust., Speech, Signal Processing (ICASSP-2002)*, May 2002, vol. III, pp. 2921–2924.

[7] R. Jeffers and K.L. Bell, "Broadband signal subspace processing for range estimation," in *Proc. IEEE Sensor Array and Multichannel Workshop*, Aug. 2002, pp. 495–498.

[8] X.L. Xu and K.M. Buckley, "Reduced-dimension beamspace broadband source localization: Preprocessor design and evaluation," in *Proc. Fourth Annual ASSP Workshop on Spectrum Estimation and Modeling*, Aug. 1988, pp. 22–27.

[9] T.D. Abhayapala, R.A. Kennedy, and R.C. Williamson, "Nearfield broadband array design using a radially invariant modal expansion," *J. Acoust. Soc. Am.*, vol. 107, no. 1, pp. 392–403, Jan. 2000.

[10] D. Colton and R. Kress, *Inverse Acoustic and Electromagnetic Scattering Theory*, Springer, New York, second edition, 1998.

[11] D.B. Ward and T.D. Abhayapala, "Wideband beamspace processing using orthogonal modal beamformers," *IEEE Trans. Signal Processing*, 2003, (submitted).

[2] We only present the range estimation results, since bearing results have previously been analyzed for frequency-invariant farfield beamspace processors in [4, 5].