ARRAY SIGNAL PROCESSING USING GARCH NOISE MODELING

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ABSTRACT

In this paper we propose a new method for modeling practical non-Gaussian and non-stationary noise in array signal processing. GARCH (Generalized Autoregressive Conditional Heteroscedasticity) models are introduced as the feasible model for the heavy tailed probability density functions (PDFs) and time varying variances of stochastic processes. We use GARCH noise model in the maximum likelihood approach for the estimation of Direction-Of-Arrivals (DOAs). Our analysis exploits time varying variance and spatially non-uniform noise in sensor array signal processing. We show through simulations that this GARCH modeling is suitable for high-resolution source separation and noise suppression in a non-Gaussian environment.

1. INTRODUCTION

Array signal processing problems such as estimation of the parameters of superimposed complex signals corrupted by noise, Time Difference Of Arrival(TDOA) and DOAs at sensors of arrays can all be reduced to general case of estimation of the unknown parameters. Due to their excellent asymptotic and threshold properties, Maximum Likelihood (ML) methods play an important role in array signal processing [5, 6]. In these methods, the key assumption is the noise model; i.e., additive noise covariance, that is used in estimation of unknown parameters. In the natural environment, the measurements of ambient noise shows that we have non-Gaussian and non-stationary process[4]. Non-Gaussian impulsive noise has attracted considerable attention from researchers in various fields of applied signal processing due to their close fit to a variety of underlying physical processes. Such is the case in the communications channels for additive ambient noise and noises being generated from different natural and man made sources and their effects when propagating to the receiver. All the above mentioned factors give a stochastic *volatility* nature to the received signal. Hence, we accept a model in which some kind of changing variance (*volatility*) is included. Thus, a proper model presentation which could best and simply describe the different features of the actual ambient noise affecting the desired signal is an essential part of an array sensor processing. Besides, in many actual applications, such as those receivers having non-ideal hardware, involving sparse sensors with prevailing external noise or non-ideal hardware receivers, see [3] and the references therein, the assumed noise model may be simplified by different sensor noise variances. In the last decade, after the seminal works by Engle [1] and Bullerslev [2] there has been a growing interest in time series modeling of changing variance or volatility. These models have found a great application in financial time series analysis. Generalized Autoregressive Conditional Heteroscedasticity (GARCH) [2] is a time series modeling technique that uses past variances and the past variance forecasts to forecast future variances. A series is defined to be heteroscedastic if its variance changes over time. GARCH models account for two main characteristics, excess kurtosis; i.e., heavy tailed probability distribution, and the volatility clustering; i.e., large changes tend to follow large changes and small changes tend to follow small ones, compatible to a large extent to the ambient noises in a natural environment. We now suggest a more realistic dynamic model for the ambient noise largely in parts due to the facts that 1) the commonly used model for environmental additive noise exhibits heavier tail than the standard normal distribution [4], and 2) the volatility clustering suggests a time series model in which successive disturbances are uncorrelated but dependent, that is, a more logical modeling for the dynamic of the ambient noise. Hence, the GARCH model is a good candidate for the additive noise model in the DOA problem for array signal processing, therefore, in this paper, we propose to assume a GARCH noise model. This paper is organized as follows: Section 2 contains the mathematical formulation of the problem, the data model and in section 3 the GARCH modeling for array signal processing. Section 4 considers maximum likelihood estimation in array signal processing for estimation of DOAs of impinging sources. A new method of direction finding from ML in conjunction with GARCH noise modeling is given in section 5, and section 6 is devoted to the simulation results of applying the GARCH noise model to GARCH and underwater acoustic ambient noises. Conclusions are provided in section 7.

2. PROBLEM FORMULATION

We assume an array of L sensors receive D ($D<L$) narrow-band source signals with unknown DOAs $\{\psi_1, ..., \psi_D\}$. The kth snapshot vector of sensor array outputs can be modelled as

$$\mathbf{x}(k) = \mathbf{V}(\boldsymbol{\psi})\mathbf{f}(k) + \mathbf{n}(k), \quad k = 1, ..., K, \tag{1}$$

where,

$$\mathbf{V}(\boldsymbol{\psi}) \triangleq [\mathbf{v}(\psi_1), ..., \mathbf{v}(\psi_D)], \tag{2}$$

is the $L{\times}D$ steering matrix composed of the signal direction vectors $\mathbf{v}(\psi_i)$, $i = 1, ..., D$, $\boldsymbol{\psi}{\triangleq}\{\psi_1, ..., \psi_D\}^T$ is the $D{\times}1$ vector of the unknown signal DOA, $\mathbf{f}(k)$ is the $D{\times}1$ vector of the source waveforms, $\mathbf{n}(k)$ is the $L{\times}1$ vector of sensor noise, K is the number of snapshots, and $(\cdot)^T$ stands for the transpose operation. In a more compact notation, (1) can be rewritten as

$$\mathbf{X} = \mathbf{V}(\boldsymbol{\psi})\mathbf{F} + \mathbf{N}, \tag{3}$$

where,

$\mathbf{X}{\triangleq} [\mathbf{x}(1), ..., \mathbf{x}(K)]$, $L{\times}K$ array data matrix;
$\mathbf{F}{\triangleq} [\mathbf{f}(1), ..., \mathbf{f}(K)]$, $D{\times}K$ source waveform matrix;
$\mathbf{N}{\triangleq} [\mathbf{n}(1), ..., \mathbf{n}(K)]$, $L{\times}K$ sensor noise matrix;

In general, the sensor noise is assumed to be a zero-mean spatially and temporally white Gaussian process with the unknown diagonal covariance matrix

$$\mathbf{Q} \triangleq E\{\mathbf{n}(k)\mathbf{n}^H(k)\} = diag\{\sigma_1^2, \sigma_2^2, ..., \sigma_L^2\}. \tag{4}$$

In what follows, the signal waveforms will be assumed to be deterministic unknown processes. The signal snapshots are assumed to satisfy the following model:

$$\mathbf{x}(k) \sim \mathcal{CN}(\mathbf{V}\mathbf{f}(k), \mathbf{Q}) \tag{5}$$

And,

$$\mathbf{R} \triangleq E\{\mathbf{x}(k)\mathbf{x}^H(k)\} = \mathbf{V}\mathbf{P}\mathbf{V}^H + \mathbf{Q} \tag{6}$$

is the array covariance matrix, $\mathbf{P} \triangleq E\{\mathbf{f}(k)\mathbf{f}^H(k)\}$ is the source waveform covariance matrix, $\mathcal{CN}$ stands for the complex Gaussian distribution, and $(\cdot)^H$ for the Hermitian transpose operation.

3. GARCH MODEL

GARCH stands for Generalized Autoregressive Conditional Heteroscedasticity, generally speaking, in Heteroscedasticity we consider time series with time varying variance; i.e., volatility, conditional implies a dependence on the observation of the immediate past, and autoregressive describes a feedback mechanism that incorporates past observations into the present. GARCH then is a mechanism that includes past variance in the explanation of the future variance. However in [1, 2, 8, 9], it is shown that a time-varied $\sigma^2(k)$ over time is more useful than a constant for modeling non-Gaussian and non-stationary phenomena such as economic series. GARCH models account for heavy tailed PDF as excess kurtosis and volatility clustering a type of heteroscedasticity. Now, in order to formulate the time-series volatility and the dependence on the past observations we let $\epsilon(k)$ denote a real-valued discrete-time stochastic process, the GARCH (p, q) process is then given by [2],

$$\epsilon(k) = \eta(k)\sigma(k), \qquad \eta(k) \sim \mathcal{N}(0, 1), \tag{7}$$

$$\sigma^2(k) = \alpha_0^2 + \sum_{i=1}^{q} \alpha_i^2 \epsilon^2(k-i) + \sum_{i=1}^{p} \beta_i^2 \sigma^2(k-i), \tag{8}$$

where $\eta(k)$ is a sequence of independent and identically distributed random variables with zero mean and variance of one, and $\mathcal{N}$ denotes the standard normal probability density function. In practice, $\eta(k)$ is often assumed to follow the standard normal or a student-t distribution.

4. MAXIMUM LIKELIHOOD ESTIMATION

Under the above assumption the joint probability density function of the observed snapshots from the array given the signal and noise parameters is expressed as

$$\mathbf{f}_{\mathbf{X}|\boldsymbol{\theta}}(X) = \prod_{k=1}^{K} \frac{1}{\det[\pi \boldsymbol{Q}(\boldsymbol{\theta})]} \exp\{-[\mathbf{x}(k) - \mathbf{V}(\psi)\mathbf{f}(k)]^H \mathbf{Q}^{-1}(\boldsymbol{\theta})[\mathbf{x}(k) - \mathbf{V}(\psi)\mathbf{f}(k)]\} \tag{9}$$

where

$$\boldsymbol{\theta} = [\mathbf{f}^T(1), ..., \mathbf{f}^T(K), \boldsymbol{\psi}^T, \boldsymbol{\sigma}^{2^T}]^T \tag{10}$$

is the vector of unknown signal and noise parameters, and $\boldsymbol{\sigma}^{\mathbf{2}} = [\sigma_1^2, \sigma_2^2, ..., \sigma_L^2]^T$. Hence, the Log-Likelihood function for Deterministic Maximum Likelihood (DML) [7] is expressed as

$$L(\boldsymbol{\theta}) = -\sum_{k=1}^{K} \ln(\det[\pi \mathbf{Q}(\boldsymbol{\theta})]) + \sum_{k=1}^{K} \{[\mathbf{x}(k) - \mathbf{V}(\psi)\mathbf{f}(k)]^H \mathbf{Q}^{-1}(\boldsymbol{\theta})[\mathbf{x}(k) - \mathbf{V}(\psi)\mathbf{f}(k)]\}. \tag{11}$$

In the following section we exploit GARCH model noise where (11) and (8) are jointly used to estimate the parameters of the signal and noise.

5. PROPOSED METHOD

Signal processing concerns itself primarily with the treatment of signal in noise which for various reasons, is most often assumed to be Gaussian. However, measurement of background or ambient noise in natural environments shows that the noise can sometimes be significantly non-Gaussian [4]. Therefore signal processing algorithms that are optimized for Gaussian noise may degrade significantly in a non-Gaussian and non-stationary environment. On the other hand in array signal processing algorithms such as DOA estimation the noise models significantly effect performance. Under the above assumption we use the GARCH model for noise in DOA estimation and source localization. We employ the DML approach, equations (1-6), we use the vector representation of the GARCH (p, q) model for the noise which can be written as

$$\sigma_\ell^2(k) = \alpha_{\ell,0}^2 + \sum_{i=1}^{p} \alpha_{\ell,i}^2 n_\ell^2(k-i) + \sum_{j=1}^{q} \beta_{\ell,j}^2 \sigma_\ell^2(k-j), \quad (12)$$

where $\ell = 1, 2, .., L$, $k = 1, 2, ..., K$, index ℓ denotes sensors index and k stands for snapshot index. Consequently, we note that in this model, noise is not uniform across L sensors which is a realistic modeling resting on the assumption of non-uniformity [3], and non-stationarity ; i.e., time-varying variance. By using equation (12) in (11) it can be shown that the following holds for log-likelihood :

$$L_p(\boldsymbol{\theta}) = -\sum_{k=1}^{K}\sum_{\ell=1}^{L} \ln(\sigma_\ell^2(k)) + \sum_{k=1}^{K}\{[\mathbf{x}(k) - \mathbf{V}(\psi)\mathbf{f}(k)]^H \mathbf{Q}^{-1}(\boldsymbol{\theta})[\mathbf{x}(k) - \mathbf{V}(\psi)\mathbf{f}(k)]\} \quad (13)$$

where

$$\boldsymbol{\theta} = \{\mathbf{f}^T(1), ..., \mathbf{f}^T(K), \boldsymbol{\psi}^T, \alpha_{\ell,o}^2, \alpha_{\ell,i}^2, \beta_{\ell,j}^2\}, \quad (14)$$

$\ell = 1, 2, .., L$, $i = 1, 2, ..., p$, $j = 1, 2, ..., q$, and $L_p(\cdot)$ stands for the proposed Log-likelihood function to be maximized over the vector of unknown parameters $\boldsymbol{\theta}$ through ML approach by

$$\hat{\boldsymbol{\theta}} = arg \max_{\boldsymbol{\theta}} \{L_p(\boldsymbol{\theta})\}. \quad (15)$$

The following section shows the simulation for the proposed method using (12,13,15).

6. SIMULATION AND RESULTS

In our simulation, we assume a uniform linear array with omnidirectional sensors and half-wavelength inter-element spacing. We conducted two experiments to show the performance of our proposed method with different values of SNR. In experiment one, we generate the GARCH(1,1) noise model but in the former we use underwater ambient noise[7, 10]. In all our experimental results, conventional Bartlet and MUSIC DOA estimation approaches [7] have been compared with the proposed method. First we assume two equally powered sources with DOA $= [-5°, 10°]$ relative to broadside. Figure 1, (a) and (b) show root-mean-square-errors (RMSEs) versus SNR for experiment 1 and 2. In the following we assume two equally powered sources with DOA $= [5°, 10°]$ at SNR=0 dB and simulate the above mentioned experiments. Normalized spectra versus DOA can be seen in Figure 2, (a) , (b) for the two experiments. We see the proposed method has resolved the targets better than two other methods, and the RMSEs of the proposed method are less than the others.

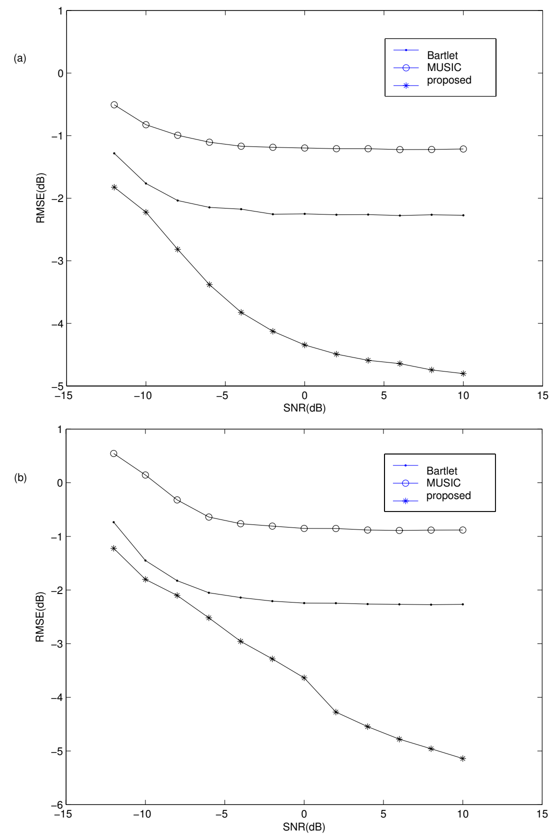

Figure 1: RMSEs (dB) vs. SNR(dB) for Bartlet, MUSIC, and proposed method with two targets $[-5°, 10°]$ in (a) GARCH(1,1) , (b) underwater ambient noise

7. CONCLUSIONS

In this paper we propose a new method for noise modeling in array signal processing . We utilized GARCH noise modeling in the ML approach to estimate DOAs of sources. This model accounts for heavy tails PDFs with excess kur-

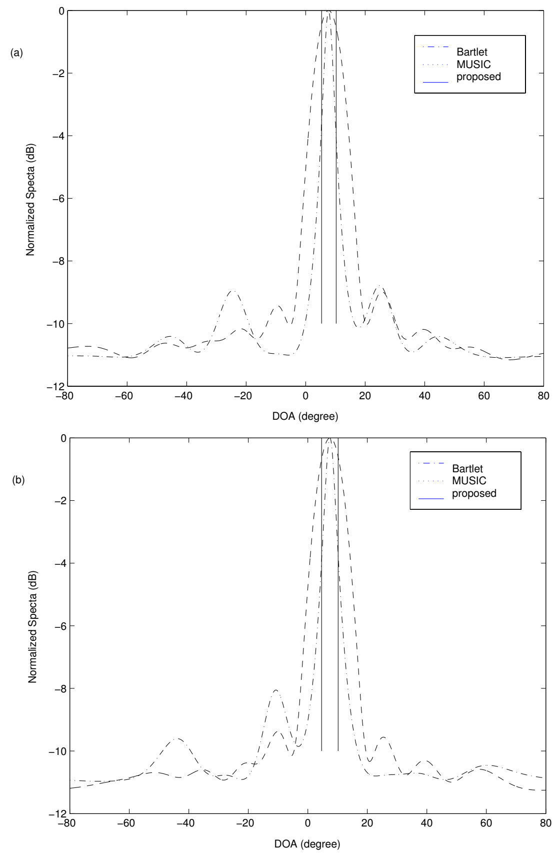

Figure 2: Normalized Spectra(dB) vs. DOA(degree) for Bartlet, MUSIC, Proposed methods, two targets $[5^\circ, 10^\circ]$ in (a) GARCH(1,1) , (b) underwater ambient noise

tosis and volatility clustering a type of heteroscedasticity . We simulated this approach for two different experiments (GARCH and underwater ambient noise) to show the performance of our proposed method. The results of these simulations verify that the proposed method is suitable for high-resolution source separation and noise suppression in a the realistic non-Gaussian environment.

8. REFERENCES

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