DIRECT POSITION DETERMINATION OF MULTIPLE RADIO SIGNALS

Alon Amar, and *Anthony J. Weiss*

Dept. Electrical Engineering, Tel Aviv University, Tel Aviv, Israel

ABSTRACT

The most common methods for localization of radio signal emitters are based on measuring a specified parameter such as signal Angle-of-Arrival (AOA) or Time-of-Arrival (TOA). The measured parameters are then used to estimate the transmitter location. Since the measurements are done at each base station independently, without using the constraint that all AOA/TOA estimates must correspond to the same transmitter they are sub-optimal. Moreover, if the number of array elements at each base station is *M* the number of co-channel simultaneous transmitters that can be localized is *M-1*. We propose a technique that uses exactly the same data as the common AOA/TOA methods but the position determination is direct. The proposed method can handle *LM-1* co-channel simultaneous signals, using *L* base stations. Although there are many stray parameters only two-dimensional search is required for a planar geometry and a three-dimensional search for the general case. The technique exploits the principles of the MUSIC algorithm, and provides a natural solution to the measurements-sources association problem that is encountered in AOA/TOA based systems.

1. INTRODUCTION

The problem of emitter location attracts much interest in the Signal Processing, Vehicular Technology and Underwater Acoustics literature. Defense oriented location systems have been reported since world war I. Civilian systems are now in use for the localization of cellular phone caller, spectrum monitoring and law enforcement. Perhaps the first paper on the mathematics of emitter location, using Angle-of-Arrival (AOA), is due to Stansfield [1]. Many other publications followed including a fine review paper by Torrieri [2]. The papers by Krim and Viberg [3] and Wax [4] are comprehensive review papers on antenna array processing for location by AOA. Recently, Van-Trees [5] published a book that is fully devoted to Array Processing. Positioning by Time-of-Arrival (TOA) and its derivatives (DTOA, EOTD) is used extensively in cellular phone localization, radar systems [6], and underwater acoustics [7]. In underwater acoustics Matched-Field Processing (MFP) is viewed as a promising procedure for source localization [8]. MFP can be interpreted as the Maximum *a Posteriori* (MAP) estimate of location given the observed signal at the output of an array of sensors [8, 9]. Other interpretation of MFP is the well-known beamforming extended to non-planar wave fields and unknown environmental parameters. The majority of the literature on MFP focuses on single source localization.

In this correspondence we discuss a method that solves the localization problem using the data collected at all sensors at all base stations *together*, in contradiction to the traditional AOA/TOA approach that is composed of two separate steps: 1) AOA/TOA independent estimates and 2) triangulation based on the results of the first step. It is rather obvious that measuring AOA/TOA at each base station separately and independently is suboptimal since this approach ignores the constraint that all measurements must correspond to the same source. Moreover, the base stations are geographically separated and therefore often not all base stations observe the same transmitter. Thus the system must somehow ensure that all AOA/TOA measurements used to locate a specific source correspond to the same source. In the case of co-channel simultaneous sources, the localization system confronts the association problem of deciding which of the multiple AOA/TOA estimates reported by the base stations corresponds to which source.

The Direct Position Determination (DPD) method that we propose takes advantage of the rather simple propagation assumptions that are usually used for Radio Frequency (RF) signals. We assume line of sight propagation with unknown complex attenuation at each base station. We also assume that all base stations are time synchronized to the level provided by GPS (approximately 50 nanoseconds.) The assumptions that we make are realistic and have been verified with real data. The proposed method uses both the array response at each station and the time of arrival at each station. We derive the Maximum Likelihood Estimate (MLE) of the sources position [11]. However, the cost function associated with MLE requires a prohibitive multidimensional search in the multiple source case. Thus, we resort to the ideas of R.O. Schmidt [10] also known as the MUSIC algorithm. We show that for a planar geometry of sources and base stations a two-dimensional search is sufficient to localize all sources. For a general geometry only a three dimensional search is needed. A side benefit of the DPD is its ability to determine the positions of more sources than the number of sensors at each base station. The DPD technique requires the transmission of the received signals (possibly sampled) to a central processing location. However, AOA and TOA require only the transmission of the measured parameters to the central processing location. This is the cost of employing DPD. The paper focuses on the multiple signal case.

2. PROBLEM FORMULATION AND ALGORITHM

Consider a transmitter and L base stations intercepting the transmitted signal. Each base station is equipped with an antenna array consisting of M elements. Denote the transmitter position by the vector of coordinates, $\mathbf{p}$. The signal observed by the l-th base station array is given by

$$\mathbf{r}_l(t) = b_l\mathbf{a}_l(\mathbf{p})s(t-\tau_l(\mathbf{p})-t_0)+\mathbf{n}_l(t); \tag{1}$$

where $\mathbf{r}_l(t)$ is a time-dependent $M\times 1$ vector, b_l is an *unknown* complex scalar representing the channel effect (attenuation), $\mathbf{a}_l(\mathbf{p})$ is the l-th array response to a signal transmitted from position $\mathbf{p}$, and $s(t-\tau_l(\mathbf{p})-t_0)$ is the signal waveform, transmitted at time t_0 and delayed by $\tau_l(\mathbf{p})$. The vector $\mathbf{n}_l(t)$ represents noise and interference, including multipath, observed by the array. The observed signal can be partitioned into K sections and each section can be Fourier transformed. The result of this process is given by the following equation

$$\mathbf{r}_l(\omega,k) = b_l\mathbf{a}_l(\mathbf{p})s(\omega,k)e^{-j\omega[\tau_l(\mathbf{p})+t_k]}+\mathbf{n}_l(\omega,k) \tag{2}$$

where $\mathbf{r}_l(\omega,k)$ is Fourier transform of the k-th section of the observed signal, $s(\omega,k)$ is the Fourier transform of the k-th section of the signal, t_k represent the transmit time of the k-th section and finally, $\mathbf{n}_l(\omega,k)$ represent the Fourier transform of the k-th section of the noise waveform.

For easy exhibition we define the following vectors and scalars,

$$\begin{aligned}\bar{s}(\omega,k) &\triangleq s(\omega,k)e^{-j\omega t_k}\\ \bar{\mathbf{a}}_l(\mathbf{p},b_l) &\triangleq b_l\mathbf{a}_l(\mathbf{p})e^{-j\omega\tau_l(\mathbf{p})}\end{aligned} \tag{3}$$

which leads to the following representation of equation (2),

$$\mathbf{r}_l(\omega,k) = \bar{\mathbf{a}}_l(\mathbf{p},b_l)\bar{s}(\omega,k)+\mathbf{n}_l(\omega,k) \tag{4}$$

We observe that all information about the transmitter position is embedded in the vector $\bar{\mathbf{a}}_l(\mathbf{p},b_l)$. If the number of emitters is $q>1$ equation (4) becomes,

$$\begin{aligned}\mathbf{r}_l(\omega,k) &= \mathbf{A}_l\bar{\mathbf{s}}(\omega,k)+\mathbf{n}_l(\omega,k)\\ \mathbf{A}_l &\triangleq \left[\bar{\mathbf{a}}_l(\mathbf{p}_1,b_{l1}),\cdots\bar{\mathbf{a}}_l(\mathbf{p}_q,b_{lq})\right]\\ \bar{\mathbf{s}}(\omega,k) &\triangleq \left[\bar{s}_1(\omega,k),\cdots\bar{s}_q(\omega,k)\right]^T\end{aligned} \tag{5}$$

Since the vector $\bar{\mathbf{s}}(\omega,k)$ is the same at all base stations we can concatenate the observed vectors at all base stations and form the following equation that encompasses all the data and all the information of the location system at hand,

$$\begin{aligned}\mathbf{r}(\omega,k) &= \mathbf{A}\bar{\mathbf{s}}(\omega,k)+\mathbf{n}(\omega,k)\\ \mathbf{r}(\omega,k) &\triangleq \left[\mathbf{r}_1^T(\omega,k),\cdots\mathbf{r}_L^T(\omega,k)\right]^T\\ \mathbf{n}(\omega,k) &\triangleq \left[\mathbf{n}_1^T(\omega,k),\cdots\mathbf{n}_L^T(\omega,k)\right]^T\\ \mathbf{A} &\triangleq \left[\mathbf{A}_1^T,\cdots\mathbf{A}_L^T\right]^T\end{aligned} \tag{6}$$

It is straightforward to write the probability density function, under appropriate assumptions, of the observations presented in (6) as a function of the unknown parameters. The unknown parameters include the complex attenuation factor of each signal at each base station, the signal waveforms, and the location of each transmitter. The Maximum Likelihood Estimator will therefore require a prohibitive multidimensional search.

It is now clear that one can follow the steps leading to the MUSIC algorithm. First note that

$$\begin{aligned}\mathbf{R}_r(\omega) &\triangleq \mathrm{E}\left\{\mathbf{r}(\omega,k)\mathbf{r}^H(\omega,k)\right\} = \mathbf{A}\mathbf{R}_s(\omega)\mathbf{A}^H+\sigma^2\mathbf{I}\\ \mathbf{R}_s(\omega) &\triangleq \mathrm{E}\left\{\bar{\mathbf{s}}(\omega,k)\bar{\mathbf{s}}^H(\omega,k)\right\}\\ \mathrm{E}&\left\{\mathbf{n}(\omega,k)\mathbf{n}^H(\omega,k)\right\} = \sigma^2\mathbf{I}\end{aligned} \tag{7}$$

where we assumed that the noise is temporally and spatially white and uncorrelated between sensors and frequencies and uncorrelated with the signals. Thus, the column vectors of $\mathbf{A}$ are orthogonal to the noise subspace of $\mathbf{R}_r(\omega)$ and contained in the signal subspace. Following the MUSIC algorithm we propose the following cost function,

$$\begin{aligned}Q(\mathbf{p},\mathbf{b}) &= \sum_{\omega}\bar{\mathbf{a}}^H(\mathbf{p},\mathbf{b})\mathbf{U}_s(\omega)\mathbf{U}_s^H(\omega)\bar{\mathbf{a}}(\mathbf{p},\mathbf{b})\\ \bar{\mathbf{a}}(\mathbf{p},\mathbf{b}) &\triangleq \left[\bar{\mathbf{a}}_1^T(\mathbf{p},b_1),\cdots\bar{\mathbf{a}}_L^T(\mathbf{p},b_L)\right]^T\\ \mathbf{b} &\triangleq \left[b_1,\cdots b_L\right]^T\end{aligned} \tag{8}$$

where $\mathbf{U}_s$ is a $ML\times q$ matrix consisting of the eigenvectors of $\mathbf{R}_r$ corresponding to the q largest eigenvalues. Recall that the vectors $\bar{\mathbf{a}}(\mathbf{p},\mathbf{b})$ contain the L unknown complex attenuation coefficients b_l in addition to the unknown location $\mathbf{p}$. The minimum points of

$Q(\mathbf{p}, \mathbf{b})$ depend on all unknowns and therefore require a L+2 dimensional search. In order to reduce this search we propose to represent $\bar{\mathbf{a}}(\mathbf{p}, \mathbf{b})$ as follows,

$$\begin{aligned}
&\bar{\mathbf{a}}(\mathbf{p}) = \mathbf{\Lambda H b} \\
&\mathbf{\Lambda} \triangleq \text{diag}\left\{\mathbf{a}_1^T(\mathbf{p})e^{-j\omega\tau_1(\mathbf{p})}, \cdots \mathbf{a}_L^T(\mathbf{p})e^{-j\omega\tau_L(\mathbf{p})}\right\} \quad (9) \\
&\mathbf{H} \triangleq \mathbf{I}_L \otimes \mathbf{J}_M
\end{aligned}$$

where $\mathbf{\Lambda}$ is a diagonal matrix whose elements are the elements of the response vectors of the array in all base stations, $\mathbf{I}_L$ stands for the $L \times L$ identity matrix, $\mathbf{J}_M$ stands for a $M \times 1$ column vector of ones, and finally $\otimes$ stands for the Kronecker product. Substituting equation (9) in (8) we get,

$$Q(\mathbf{p}) = \mathbf{b}^H \mathbf{H}^H \left[\sum_{\omega} \mathbf{\Lambda}^H \mathbf{U}_s(\omega) \mathbf{U}_s^H(\omega) \mathbf{\Lambda} \right] \mathbf{H b} \quad (10)$$

Without loss of generality, we assume that the norm of $\mathbf{b}$ is one. Hence for any assumed position $\mathbf{p}$ the maximum of $Q(\mathbf{p})$ corresponds to the maximal eigenvalue of the matrix $\mathbf{D}$ defined by,

$$\mathbf{D} \triangleq \mathbf{H}^H \left[\sum_{\omega} \mathbf{\Lambda}^H \mathbf{U}_s(\omega) \mathbf{U}_s^H(\omega) \mathbf{\Lambda} \right] \mathbf{H} \quad (11)$$

Therefore, equation (10) reduces to

$$Q(\mathbf{p}) = \lambda_{\max}(\mathbf{D}) \quad (12)$$

where the right side of (12) denotes the largest eigenvalue of $\mathbf{D}$, and the matrix $\mathbf{D}$ is a function of the data, the array response at each base station, the location of the base stations and the unknown emitter location $\mathbf{p}$. It is clear that the maximization of (12) requires only a two-dimensional search for planar geometries or 3-D search in the general case. It is interesting to note that the dimensions of the matrix $\mathbf{D}$ are $L \times L$ which are usually rather small.

3. NUMERICAL RESULTS

In order to examine the performance of the advocated method and compare it with the traditional approach we performed extensive Monte-Carlo simulations. Some examples are shown here. Consider 4 base-stations placed at the corners of a 4 Km $\times$ 4 Km square. Each base-station is equipped with a uniform linear array of only 3 antenna elements. The two transmitters are located at coordinates (0, 1.5) and (0, -1.5) Km. Each location determination is based on 30 snapshots of 4 Fourier coefficients. The SNR is varied between –10 dB and +10 dB at 2.5 dB steps. At each SNR value we performed 50 experiments in order to obtain the statistical properties of

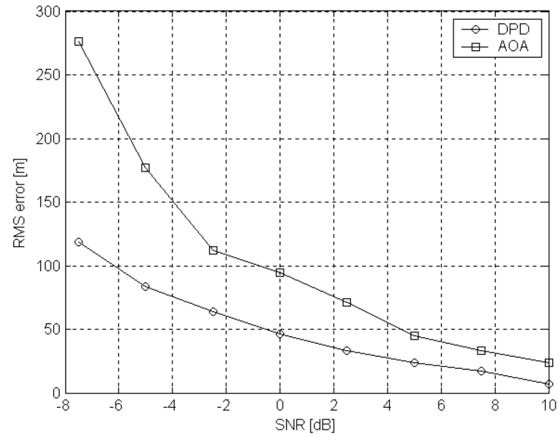

Figure 1: RMS miss distance for DPD and traditional AOA.

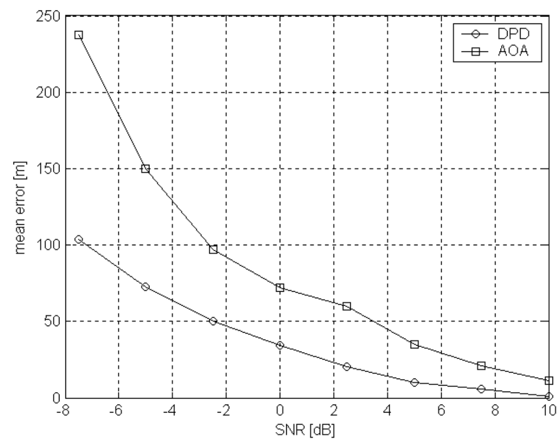

Figure 2: Mean miss distance for DPD and traditional AOA.

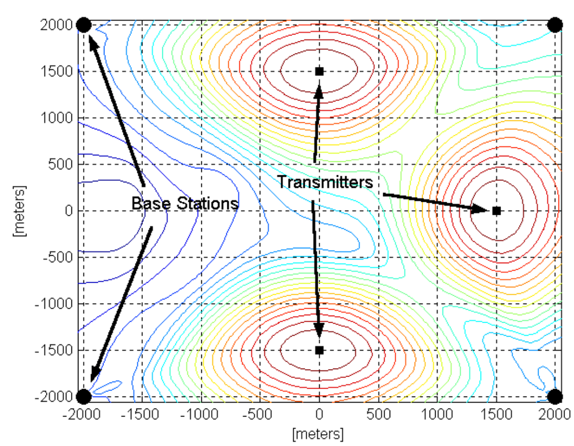

Figure 3: DPD cost function, 3 transmitters, 4 base stations, each equipped with a linear antenna array of 3 elements.

the performance. The path-loss attenuation magnitude is selected at random using normal distribution (mean=1, std=0.1) and the attenuation phase is uniformly distributed in $[-\pi, \pi]$. We applied 2 different techniques in order to locate the transmitter:

1. Angle of Arrival estimation using MUSIC at each base station independently.
2. Direct Position Determination (DPD) according to equation (12)

The performance evaluation is based on the statistics of the miss distance i.e., the distance between the true emitter position and the estimated emitter position.
We used 2 different criteria:

1. Root Mean Square (RMS) of miss distance (Figure 1)
2. Mean of miss distance (Figure 2)

The plots indicate that DPD is superior to the traditional approach of independent estimates at each base station. The advantage of DPD is at low SNR. At high SNR both methods give excellent results.

Obviously, the traditional approach cannot localize 3 co-channel simultaneously emitters with only 3 antenna elements at each base station. However, the DPD can handle this situation. Figure 3 shows the DPD cost function for 3 separate emitters.

4. CONCLUSIONS

We have proposed a direct position determination technique for localizing multiple narrowband radio frequency sources. The technique can locate more sources than the traditional AOA approach. Moreover, DPD provides better accuracy than traditional AOA and it does not encounter the association problem of independent AOA measurements at each base station. Surprisingly, the DPD does not impose higher computation load than traditional methods since the traditional approach includes not only AOA/TOA estimation but also a rather heavy triangulation algorithm (fix algorithm) that must reject wild measurements and must find the right association of different AOA measurements.

The proposed technique uses the MUSIC approach in order to reduce the complexity of the algorithm. The advantages of DPD do not come without a price. While in traditional methods only AOA/TOA estimates must be transferred to a central processing location for triangulation, the DPD requires raw signal data to be transferred to a common processor.

Finally, the extension of the DPD approach to known signal waveforms and multipath scenarios will be reported in the near future.

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