DOA ESTIMATION FOR WIDEBAND CYCLOSTATIONARY SIGNALS UNDER MULTIPATH ENVIRONMENT

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ABSTRACT

Many man-made signals encountered in communications, such as BPSK, FSK, AM signals, exhibit cyclostationarity. By exploiting the cyclostationarity, Cyclic MUSIC has been shown to be able to separate the signals with different cycle frequencies, thus, to be able to perform signal selective Direction of Arrival (DOA) estimation. However, both Cyclic MUSIC and the conventional MUSIC suffer performance deterioration when the signals are either wideband or coherent. In this paper we propose a new method that combines the Spatial Smoothing (SS) method with a revised Cyclic MUSIC named Averaged Cyclic MUSIC. Simulation results show the effectiveness of the proposed method in signal selective DOA estimation for wideband signals under a multipath environment.

1. INTRODUCTION

Among subspace based DOA estimation methods, MUSIC [1], [2] is relatively simple and effective, and is thus most studied. To improve the performance of the conventional MUSIC, Cyclic MUSIC [3] is shown to be effective to combat the noise and interference by exploiting cyclostationary property possessed by most man-made signals [4]. However both Cyclic MUSIC and the conventional MUSIC have two shortcomings. One is that they suffer performance deterioration when the signal is wideband. The other is their inability for DOA estimation of coherent signals, i.e., signals resulted from multipath propagation. While some algorithms have been proposed to extend Cyclic MUSIC to wideband signals, such as SC-MUSIC [5] and Wideband Cyclic MUSIC [6], they would fail under a multipath environment when coherent signals exist. Among efforts to DOA estimation for wideband signals under multipath, coherent signal-subspace method [7] uses focusing matrices to "align" steering vectors of different frequency bins. However, this method works only in a small sector of the search space. On the other hand, Hankel Approximation Method (HAM) [9] which incorporates a preprocessing scheme referred to as Spatial Smoothing (SS) [8] is proposed to cope with the problem of coherent cyclostationary signals. This method decorrelates the signals by separating the array of antennas into several overlapping subarrays. However, HAM as proposed in [9] is only for narrowband signals.

In this paper, we propose a DOA estimation method for wideband cyclostationary signals under multipath. First we modify Cyclic MUSIC so that it can perform DOA estimation on wideband cyclostationary signals. Unlike SC-MUSIC and Wideband Cyclic MUSIC, we perform cross cyclic correlation of the array signals. Then in order to formulate in the same way as for narrowband signals, we average the cyclic correlation, thus naming this method as Averaged Cyclic MUSIC. Then SVD can be performed as in the conventional MUSIC algorithm, and finally the DOA of the wideband cyclostationary signal can be found. To cope with the problem of coherent signals, we incorporate the SS scheme into our Averaged Cyclic MUSIC algorithm to decorrelate the coherent wideband cyclostationary signals. For each subarray, the averaged cyclic correlation matrix is calculated and then added up to get a spatially smoothed cyclic correlation matrix. We also note that the spatial smoothed cyclic correlation matrix can be obtained by rearranging the cyclic correlation matrix of the entire array, making the computation more efficient.

2. EXISTING CYCLIC MUSIC ALGORITHM

Consider a Uniform Linear Array (ULA) of size N that receives I_s signals from directions θ_i, $i = 1, \cdots, I_s$. The incident waves are assumed to be plane waves from far field sources. The received narrowband signal can be written as

$$\mathbf{x}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{n}(t) \tag{1}$$

where the vector $\mathbf{s}(t) = [s_1(t), \cdots, s_I(t)]^T$ contains the signals that have cycle frequency α, i.e., the Sources of Interest (SOI) with $I \leq I_s$, the vector $\mathbf{n}(t)$ contains interfering sources and noise, and the matrix $\mathbf{A} = [\mathbf{a}(\theta_1), \cdots, \mathbf{a}(\theta_I)]$ contains the steering vectors $\mathbf{a}(\theta_i)$ defined as $\mathbf{a}(\theta_i) = [1, e^{j2\pi f d \sin\theta_i / c}, \cdots, e^{j2\pi f (N-1) d \sin\theta_i / c}]^T$, for $i = 1, \cdots, I$,

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where d is antenna spacing and c is the speed of propagation.

Instead of calculating the correlation matrix of the received signal in the conventional MUSIC, Cyclic MUSIC calculates the cyclic correlation matrix which is estimated by

$$\mathbf{R}_{\mathbf{x}}^{\alpha}(\tau) = \langle \mathbf{x}(t+\frac{\tau}{2})\mathbf{x}^H(t-\frac{\tau}{2})e^{-j2\pi\alpha t}\rangle \approx \mathbf{A}\mathbf{R}_{\mathbf{s}}^{\alpha}(\tau)\mathbf{A}^H \tag{2}$$

where $\langle\cdot\rangle$ denotes time average and

$$\mathbf{R}_{\mathbf{s}}^{\alpha}(\tau) = \langle \mathbf{s}(t+\frac{\tau}{2})\mathbf{s}^H(t-\frac{\tau}{2})e^{-j2\pi\alpha t}\rangle \tag{3}$$

is the cyclic correlation matrix of the sources. Here $\mathbf{n}(t)$ is neglected as evaluating the cyclic correlation at α retains only those SOI. Now applying SVD to (2), we obtain

$$\mathbf{R}_{\mathbf{x}}^{\alpha}(\tau) = \begin{bmatrix} \mathbf{E}_s & \mathbf{E}_n \end{bmatrix} \begin{bmatrix} \Sigma_s & 0 \\ 0 & \Sigma_n \end{bmatrix} \begin{bmatrix} \mathbf{V}_s & \mathbf{V}_n \end{bmatrix}^H \tag{4}$$

where $\mathbf{E}_s$ spans the signal subspace and $\mathbf{E}_n$ spans the interference and noise subspace which are orthogonal to each other. When the steering vector $\mathbf{a}(\theta)$ is evaluated at the impinging direction θ_i of the SOI, it will be lying in the signal subspace or orthogonal to the noise subspace. Cyclic MUSIC thus finds the maxima of $1/\left\|\mathbf{E}_n^H\mathbf{a}(\theta)\right\|^2$ or $\left\|\mathbf{E}_s^H\mathbf{a}(\theta)\right\|^2$ in terms of θ as the DOA estimate.

This method works well when the impinging signals are narrowband. But if the signals are wideband, the received signal can no longer be written as in equation (1), then the algorithm would fail.

3. AVERAGED CYCLIC MUSIC ALGORITHM

In order to write the cyclic correlation matrix for wideband signals into the form of equation (2), so that we can apply SVD to estimate DOAs, we propose an Averaged Cyclic MUSIC algorithm. We again consider a ULA of size N that receives I signals with the cycle frequency α. We further consider, due to multipath, that there are K_i coherent signals induced from the ith signal. The signal received by the nth antenna will be

$$x_n(t) = \sum_{i=1}^{I}\sum_{k=1}^{K_i}\beta_{ik}s_i(t-\tau_{ik}+(n-1)\Delta_{ik}) \tag{5}$$

where $\beta_{ik}$is the attenuation factor, τ_{ik} is the reference delay, and $\Delta_{ik} = d\sin\theta_{ik}/c$. Note interferences and noise are neglected since most practical interferences and noise can be safely assumed to be cyclically uncorrelated with the SOI.

For simplicity, first let's consider there is only one SOI, i.e. $I = 1$. This corresponds to the case where only one source has the cycle frequency α, which is of interest. Then evaluating the cyclic correlation at this specific cycle frequency retains only one source. We simplify (5) as

$$x_n(t) = \sum_{k=1}^{K_1}\beta_{1k}s_1(t-\tau_{1k}+(n-1)\Delta_{1k}) \tag{6}$$

Then the (p,n)-th element of the cyclic correlation matrix $\mathbf{R}_{\mathbf{x}}^{\alpha}(\tau)$ as defined in equation (2) is

$$\begin{aligned} & r_{x_p x_n}^{\alpha}(\tau) \\ =\ & \langle x_p(t+\tfrac{\tau}{2})x_n^*(t-\tfrac{\tau}{2})e^{-j2\pi\alpha t}\rangle \\ =\ & \langle \sum_{k=1}^{K_1}\beta_{1k}s_1(t+\tfrac{\tau}{2}-\tau_{1k}+(p-1)\Delta_{1k}) \\ & \cdot\sum_{m=1}^{K_1}\beta_{1m}s_1^*(t-\tfrac{\tau}{2}-\tau_{1m}+(n-1)\Delta_{1m})e^{-j2\pi\alpha t}\rangle \\ =\ & \sum_{k=1}^{K_1}\sum_{m=1}^{K_1}\beta_{1k}\beta_{1m}r_{s_1}^{\alpha}(\tau+\tau_{1m}-\tau_{1k}+(p-1)\Delta_{1k} \\ & -(n-1)\Delta_{1m})e^{j\pi\alpha(-\tau_{1m}-\tau_{1k}+(p-1)\Delta_{1k}+(n-1)\Delta_{1m})} \end{aligned} \tag{7}$$

where $r_s^{\alpha}(\tau)$ is defined as

$$r_s^{\alpha}(\tau) = \langle s(t+\tau/2)s^*(t-\tau/2)e^{-j2\pi\alpha t}\rangle \tag{8}$$

and the shift property of cyclic correlation is applied, i.e., if $y(t) = x(t+T)$, then $r_y^{\alpha}(\tau) = r_x^{\alpha}(\tau)e^{j2\pi\alpha T}$. Now average $r_{x_p x_n}^{\alpha}(\tau)$ over τ, we obtain

$$\begin{aligned} \langle r_{x_p x_n}^{\alpha}(\tau)\rangle &= \sum_{k=1}^{K_1}\sum_{m=1}^{K_1}\beta_{1k}\beta_{1m}\langle r_{s_1}^{\alpha}(\tau)\rangle \\ &\quad \cdot e^{-j\pi\alpha(\tau_{1m}+\tau_{1k}-(p-1)\Delta_{1k}-(n-1)\Delta_{1m})} \\ &= \sum_{k=1}^{K_1}\beta_{1k}e^{-j\pi\alpha\tau_{1k}}e^{j\pi\alpha(p-1)\Delta_{1k}}\langle r_{s_1}^{\alpha}(\tau)\rangle \\ &\quad \cdot\sum_{m=1}^{K_1}\beta_{1m}e^{-j\pi\alpha\tau_{1m}}e^{j\pi\alpha(n-1)\Delta_{1m}} \end{aligned} \tag{9}$$

if we define

$$\mathbf{v}_i = \begin{bmatrix} e^{j\pi\alpha(i-1)\Delta_{11}} & \cdots & e^{j\pi\alpha(i-1)\Delta_{1K_1}} \end{bmatrix}^T \tag{10}$$

and

$$\mathbf{b}_1 = \begin{bmatrix} \beta_{11}e^{-j\pi\alpha\tau_{11}} & \cdots & \beta_{1K_1}e^{-j\pi\alpha\tau_{1K_1}} \end{bmatrix}^T \tag{11}$$

equation (9) can be written as

$$\langle r_{x_p x_n}^{\alpha}(\tau)\rangle = \mathbf{v}_p^T\mathbf{b}_1\langle r_{s_1}^{\alpha}(\tau)\rangle\mathbf{b}_1^T\mathbf{v}_n = \mathbf{v}_p^T\mathbf{M}\mathbf{v}_n \tag{12}$$

where

$$\mathbf{M} = \mathbf{b}_1\langle r_{s_1}^{\alpha}(\tau)\rangle\mathbf{b}_1^T \tag{13}$$

(12) is the (p,n)-th element of the N by N matrix $\langle \mathbf{R}_{\mathbf{x}}^{\alpha}(\tau)\rangle$. So putting all elements together, we obtain

$$\begin{aligned}\langle \mathbf{R}_{\mathbf{x}}^{\alpha}(\tau)\rangle &= \left[\begin{array}{ccc}\mathbf{a}(\theta_{11}) & \cdots & \mathbf{a}(\theta_{1K_1})\end{array}\right]\mathbf{M} \\ & \quad \left[\begin{array}{ccc}\mathbf{a}(\theta_{11}) & \cdots & \mathbf{a}(\theta_{1K_1})\end{array}\right]^T \\ &= \mathbf{A}_1\mathbf{M}\mathbf{A}_1^T \end{aligned} \tag{14}$$

where $\mathbf{a}(\theta_{1k}) = \left[\begin{array}{cccc}1 & e^{j\pi\alpha\Delta_{1k}} & \cdots & e^{j\pi\alpha(N-1)\Delta_{1k}}\end{array}\right]^T$, for $k = 1, \cdots, K_1$ are the steering vectors. They form the matrix $\mathbf{A}_1$. We notice that the averaged cyclic correlation matrix, $\langle \mathbf{R}_{\mathbf{x}}^{\alpha}(\tau)\rangle$, has a form like (2). Then it is possible to use SVD to estimate the DOAs as long as $\mathbf{M}$ is full rank. But for the above case, it is obvious that $\mathbf{M}$ has only rank 1, so $\langle \mathbf{R}_{\mathbf{x}}^{\alpha}(\tau)\rangle$ has rank 1 too. Therefore although the Averaged Cyclic MUSIC can perform wideband signal selection, the multipaths cannot be separated by SVD. This problem is solved in the next section.

4. SPATIAL SMOOTHING

To decorrelate the signals under a multipath environment, we divide the array into L overlapping subarrays, such that the lth to $(M+l-1)$th antennas form the lth subarray. There are M antennas in each subarray and the number of total antennas is $N = M + L - 1$. We denote the signal received in the lth subarray as

$$\mathbf{x}^{(l)}(t) = \left[\begin{array}{ccc}x_l(t) & \cdots & x_{l+M-1}(t)\end{array}\right]^T \tag{15}$$

and the averaged cyclic correlation matrix for this subarray as $\langle \mathbf{R}_{\mathbf{x}}^{(l)\alpha}(\tau)\rangle$. Similar to (14),

$$\langle \mathbf{R}_{\mathbf{x}}^{(l)\alpha}(\tau)\rangle = \mathbf{A}_1\mathbf{M}^{(l)}\mathbf{A}_1^T \tag{16}$$

where $\mathbf{M}^{(l)}$ is similar to (13)

$$\mathbf{M}^{(l)} = \mathbf{b}_1^{(l)}\langle r_{s_1}^{\alpha}(\tau)\rangle \mathbf{b}_1^{(l)T} \tag{17}$$

and $\mathbf{b}_1^{(l)}$ is defined as

$$\mathbf{b}_1^{(l)} = \left[\begin{array}{c}\beta_{11}e^{-j\pi\alpha\tau_{11}}e^{j\pi\alpha(l-1)\Delta_{11}} \\ \vdots \\ \beta_{1K_1}e^{-j\pi\alpha\tau_{1K_1}}e^{j\pi\alpha(l-1)\Delta_{1K_1}}\end{array}\right] \tag{18}$$

The extra exponential factors are due to the fact that the lth subarray starts from the lth sensor in the entire array and the vector $\mathbf{b}_1$ is actually $\mathbf{b}_1^{(l)}$ when $l = 1$. Another difference is here the columns of $\mathbf{A}_1$, i.e. the steering vectors have M instead of N elements as the number of antennas of each subarray is M.

Although $\mathbf{M}^{(l)}$ is still rank one, by adding all L such cyclic correlation matrices from all L subarrays, we obtain

$$\langle \mathbf{R}_{\mathbf{x}}^{\alpha}(\tau)\rangle_{sub} = \sum_{l=1}^{L}\langle \mathbf{R}_{\mathbf{x}}^{(l)\alpha}(\tau)\rangle = \mathbf{A}_1\sum_{l=1}^{L}\mathbf{M}^{(l)}\mathbf{A}_1^T \tag{19}$$

substitute (17) into (19), we obtain

$$\begin{aligned}\langle \mathbf{R}_{\mathbf{x}}^{\alpha}(\tau)\rangle_{sub} &= \mathbf{A}_1\sum_{l=1}^{L}\mathbf{b}_1^{(l)}\langle r_{s_1}^{\alpha}(\tau)\rangle\mathbf{b}_1^{(l)T}\mathbf{A}_1^T \\ &= \mathbf{A}_1\mathbf{B}_1\mathbf{I}_{s_1}\mathbf{B}_1^T\mathbf{A}_1^T\end{aligned} \tag{20}$$

where $\mathbf{B}_1 = \left[\begin{array}{ccc}\mathbf{b}_1^{(1)} & \cdots & \mathbf{b}_1^{(L)}\end{array}\right]$ and $\mathbf{I}_{s_1} = \langle r_{s_1}^{\alpha}(\tau)\rangle\mathbf{I}_L$. $\mathbf{I}_L$ is the identity matrix of size L.

We can see $\mathbf{B}_1\mathbf{I}_{s_1}\mathbf{B}_1^T$ could be of full rank if L is greater than or equal to K_1. Consequently, the rank of $\langle \mathbf{R}_{\mathbf{x}}^{\alpha}(\tau)\rangle_{sub}$ could be K_1 if $min\{L, M\} \geq K_1$. Thus when the number of subarrays and the number of elements in each subarray is greater than or equal to K_1, all paths could be detected by evaluating the steering vectors and the smoothed averaged cyclic correlation matrix at $\alpha/2$.

This algorithm could be extended to several signals with the same cycle frequency. If we define

$$\mathbf{A} = \left[\begin{array}{ccc}\mathbf{A}_1 & \cdots & \mathbf{A}_I\end{array}\right] \tag{21}$$

$$\mathbf{B} = \left[\begin{array}{cccc}\mathbf{B}_1 & \mathbf{0}_{K_1L} & & \mathbf{0}_{K_1L} \\ \mathbf{0}_{K_2L} & \mathbf{B}_2 & \ddots & \mathbf{0}_{K_2L} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0}_{K_IL} & \mathbf{0}_{K_IL} & & \mathbf{B}_I\end{array}\right] \tag{22}$$

and

$$\mathbf{I}_S = \left[\begin{array}{cccc}\mathbf{I}_{S_1} & \mathbf{0}_{LL} & & \mathbf{0}_{LL} \\ \mathbf{0}_{LL} & \mathbf{I}_{S_2} & \ddots & \mathbf{0}_{LL} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0}_{LL} & \mathbf{0}_{LL} & & \mathbf{I}_{S_I}\end{array}\right] \tag{23}$$

where $\mathbf{A}_i = \left[\begin{array}{ccc}\mathbf{a}(\theta_{i1}) & \cdots & \mathbf{a}(\theta_{iK_i})\end{array}\right]$, $\mathbf{I}_{s_i} = \langle r_{s_i}^{\alpha}(\tau)\rangle\mathbf{I}_L$ and $\mathbf{B}_i = \left[\begin{array}{ccc}\mathbf{b}_i^{(1)} & \cdots & \mathbf{b}_i^{(L)}\end{array}\right]$, for $i = 1, \cdots, I$. Then (20) could be extend to

$$\langle \mathbf{R}_{\mathbf{x}}^{\alpha}(\tau)\rangle_{sub} = \mathbf{A}\mathbf{B}\mathbf{I}_s\mathbf{B}^T\mathbf{A}^T \tag{24}$$

which is of full rank as long as $min\{L, M\} \geq max\{K_i\}$. So theoretically all the steering vectors from I signals and their multipaths can be detected.

We also note that the averaged cyclic correlation matrix for the lth subarray, $\langle \mathbf{R}_{\mathbf{x}}^{(l)\alpha}(\tau)\rangle$, is actually the lth submatrix along the main diagonal of the averaged cyclic correlation matrix for the entire array. So the algorithm could be summarized as

1. Compute $\mathbf{R}_{\mathbf{x}}^{\alpha}(\tau)$ for the entire array.
2. Average $\mathbf{R}_{\mathbf{x}}^{\alpha}(\tau)$ over τ to obtain $\langle \mathbf{R}_{\mathbf{x}}^{\alpha}(\tau)\rangle$.
3. Take the lth submatrix of size M by M along the main diagonal of $\langle \mathbf{R}_{\mathbf{x}}^{\alpha}(\tau)\rangle$ and add them up to obtain $\langle \mathbf{R}_{\mathbf{x}}^{\alpha}(\tau)\rangle_{sub}$.
4. Apply SVD to $\langle \mathbf{R}_{\mathbf{x}}^{\alpha}(\tau)\rangle_{sub}$and find the DOA as in the conventional MUSIC.

5. SIMULATION RESULTS

5.1. Two Signal Case

The following simulation is tested on two BPSK signals with raised cosine pulse shape. The baud rate is 5 MHz, center frequency is 10 MHz, and the Signal to Noise Ratio (SNR) is 10 dB, for both signals. We consider a simulated system with a sampling rate of 80 MHz. 15 antennas were assumed to receive one signal from a DOA of 30° and its multipath from a DOA of 60° and the other signal from a DOA of -60° and its multipath DOA of -20°. 3200 snapshots are used in the computations. Fig.1 is obtained from 20 runs. It is seen that all wideband source DOAs and their multipath DOAs are correctly detected.

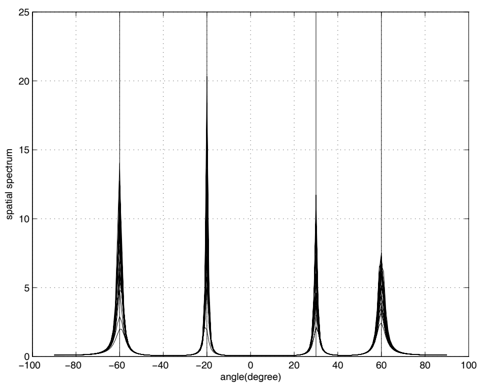

Fig.1. One direct path from -60° and multipath from -20°, another direct path from 30° and multipath from 60°, are all detected

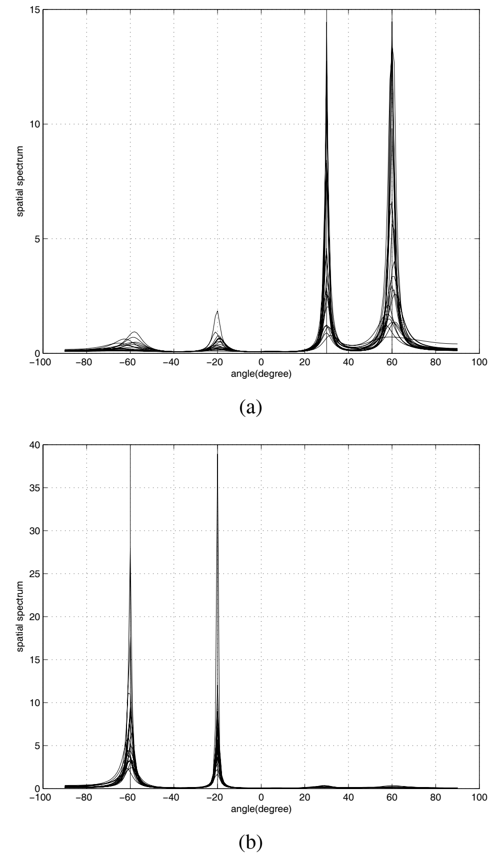

Fig.2. (a) One BPSK signal from the direct path 30° and its multipath from 60° is detected, (b) The other BPSK signal from the direct path -60° and its multipath from -20° is detected

5.2. Signal Selectivity

This simulation is done to test the signal selectivity of the algorithm. Signals used here are the same as in section 5.1 except the baud rate of one signal is changed to 2 MHz. As cycle frequency of the two signals are different, if we choose either signal as SOI, the other signal will be suppressed, then only two paths associated with the SOI will be detected. The results are shown in Fig.2.

6. REFERENCES

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