A NOVEL JOINT ESTIMATOR OF MULTIPLE UNDERWATER SOURCES WITH MULTIPLE PARAMETERS

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ABSTRACT

In this paper, a new generalized eigenstructure-based 3-dimensional joint estimator (GETJE) for direction, frequency, and time-delay of multiple sources is presented. For the 2-dimensional joint parameter estimation, a ESPRIT-based algorithm has been proposed in [3]. Now we extend it to 3-dimensional. Using a single echo wave, the bearings, frequencies and time-delays of multiple reflectors are jointly estimated. We construct 2 sub-arrays like ESPRIT, and then conduct generalized eigen-decomposition for their auto-correlation matrix and cross-correlation matrix. Bearing parameters can be estimated from eigenvalues, while frequency parameters can also be obtained from corresponding eigenvectors. After modifying the time-delay vectors of the envelop of emitting signal according to the estimated frequencies, the time-delay parameters can be obtained simultaneously. Computer simulations show that the GETJE can carry out the 3-dimensional joint parameter estimation without additional pairing in the case of lower SNR. Preferable results are also obtained in water tank experiments, which indicate that the GETJE is robust for sensor array errors and has the potential applications.

1. INTRODUCTION AND MODELING

The applications of multiple targets tracking in sonar and radar require the information about bearing, ranging, and speed of multiple reflectors. This requirement not only involves high-resolution estimation of these 3 parameters, but also includes pairing of multi-dimensional parameters, namely determining which parameters of bearing, time-delay and frequency to correspond to the same reflector. To accomplish this multi-reflector multi-dimensional joint parameter estimation, traditional methods reduce the multiple dimensions of joint estimation into one dimension using the separability of multiple reflectors. For example, if frequencies of multiple reflectors show comparatively big differences, their echo waves will be separated by filters. Only 2-dimensional joint estimation of bearing and time-delay is needed to process the output of each filter. In the many cases of underwater environment, the duration of emitted signal is comparatively long (difficult to separate in time domain), the aperture of sensor array is limited (difficult to separate in spatial domain), and difference between speeds of reflectors is small(difficult separation in frequency domain). In addition, the underwater data rate is very low, so the joint estimation results within a single echo wave are always expected. All these reasons make the multi-reflector joint estimation more difficult.

In this paper, we present new generalized eigenstructure-based 3-dimensional joint estimator (GETJE) for direction, frequency, and time-delay of multiple sources. It solves the problems mentioned above preferably. In the case of that multi-reflector waveforms can not be separated in either time domain, frequency domain, or spatial domain, this novel method can estimate the parameters of bearing, frequency and time-delay within a single echo wave, while no traditional pairing is needed. It shows a new way to the underwater multi-reflector localization and joint parameter estimation.

A uniform linear array with M sensors equal-spaced in d is supposed to receive echoes. $s(t)$ is the envelope of emitted signal, λ is the length of waveform. If there are p reflectors, the received complex envelope can be written as

$$\tilde{\mathbf{x}}(t)=\sum_{i=1}^{p}s(t-\tau_i)b_i\exp[j(2\pi f_{di}t+\varphi_i)]\mathbf{a}^T(\theta_i)+\mathbf{n}(t) \quad (1)$$

which can also be expressed in matrix form as

$$\tilde{\mathbf{X}}(\mathbf{t})=\mathbf{S}'(\mathbf{t})\mathbf{\Phi}\mathbf{A}^T(\theta)+\mathbf{N}(\mathbf{t}) \quad (2)$$

where $\mathbf{A}(\theta)=[\mathbf{a}(\theta_1),\mathbf{a}(\theta_2),\cdots,\mathbf{a}(\theta_p)]$ is a M×P Vandermonde matrix called direction matrix, each column of which $\mathbf{a}(\theta_i)$ is called direction vector of the sensor array. For a uniform linear array, $\mathbf{a}(\theta_i)=\{1,\exp(-j\omega_i),\cdots,\exp[-j(M-1)\omega_i]\}^T$, in which $\omega_i=2\pi\frac{d\sin(\theta_i)}{\lambda}$, $i=1,2,\cdots,p$ denotes the spatial frequency of each reflector.

$\mathbf{\Phi}=diag[b_1\exp(j\varphi_1),b_2\exp(j\varphi_2),\cdots,b_p\exp(j\varphi_p)]$, where b_i,φ_i are the amplitude and original phase of each reflector respectively.

$\mathbf{S}'(\mathbf{t})=[\mathbf{s}(\mathbf{t}-\tau_1)\exp(j2\pi f_{d1}\mathbf{t}),\mathbf{s}(\mathbf{t}-\tau_2)\exp(j2\pi f_{d2}\mathbf{t}),\cdots,\mathbf{s}(\mathbf{t}-\tau_p)\exp(j2\pi f_{dp}\mathbf{t})]$ is the matrix of the normalized time-delay and frequency of envelop, i-th column of which $\mathbf{s}(\mathbf{t}-\tau_i)\exp(j2\pi f_{di}\mathbf{t})$ is the vector of time-delay and frequency for the i-th reflector. $\mathbf{N}$(t) is the noise matrix.

ESPRIT requires two identical sub-arrays. In the case of uniform linear array, the sub-arrays are usually constructed as follows: Taking 1~(M-1)th sensors as one sub-array, the data matrix of the sub-array can be written as

$$X_1(t)=\tilde{S}(t)A_-^T(\theta) \quad (3)$$

where $A_-(\theta)$ is a sub-matrix of $A(\theta)$ in Equation (2) by removing the last row of $A(\theta)$. In the same way, taking 2~Mth sensors as another sub-array, the data matrix of this sub-array can be written in

$$X_2(t)=\tilde{S}(t)\Phi A_-^T(\theta) \quad (4)$$

where $\Phi=diag[\exp(j\omega_1),\exp(j\omega_2),\cdots,\exp(j\omega_p)]$ is a diagonal matrix containing the DOA information of reflectors.

2. THE PRINCIPLE OF GETJE METHOD

In 1986, R. Roy proposed ESPRIT method [1], which gives high-resolution DOA estimation via the generalized eigenvalues of auto-correlation and cross-correlation matrix of 2 sub-arrays. It was found that the generalized eigenvectors in ESPRIT algorithm are not utilized sufficiently. So some researchers extended ESPRIT to 2-D joint estimation successfully [2-4]. In this paper, we extend it to 3-D joint parameter estimation named generalized eigenstructure-based 3-D joint estimator (GETJE).

The procedures of GETJE are given as follows:

(a) Construct the auto-correlation matrix and cross-correlation matrix of above 2 sub-arrays respectively:

$$\mathbf{Y}_0 = \mathbf{X}_1(t)\mathbf{X}_1^H(\mathbf{t}) = \tilde{\mathbf{S}}(t)\mathbf{A}_-^T(\theta)\mathbf{A}_-^*(\theta)\tilde{\mathbf{S}}^H(\mathbf{t}) \tag{5}$$

$$\mathbf{Y}_1 = \mathbf{X}_1(t)\mathbf{X}_2^H(\mathbf{t}) = \tilde{\mathbf{S}}(\mathbf{t})\mathbf{A}_-^T(\theta)\mathbf{A}_-^*(\theta)\mathbf{\Phi}^H\tilde{\mathbf{S}}^H(\mathbf{t}) \tag{6}$$

Ignoring the noise, $\mathbf{Y}_0$, $\mathbf{Y}_1$ are non full-rank matrix, the ranks of which are both M. But they also satisfy the equation of traditional ESPRIT:

$$\mathbf{Y}_0\tilde{\mathbf{S}}(\mathbf{t})\mathbf{\Phi}^H = \mathbf{Y}_1\tilde{\mathbf{S}}(\mathbf{t}) \tag{7}$$

So columns of $\tilde{\mathbf{S}}(\mathbf{t})$ are the generalized eigenvectors of $\mathbf{Y}_0$ and $\mathbf{Y}_1$, which contain the time-delay and frequency information of multiple reflectors. And the diagonal components of $\mathbf{\Phi}$ correspond to generalized eigenvalues, which contain bearing information of multiple reflectors.

(b) Conduct an eigen-decomposition of $\mathbf{Y}_0$:

$$\mathbf{Y}_0 = \mathbf{V}\mathbf{E}\mathbf{V}^H \tag{8}$$

where each column of $\mathbf{V}$ is an eigenvector. $\mathbf{E} = diag(\lambda_1, \lambda_2, \cdots, \lambda_N)$, $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_N$ correspond to eigenvalues in descent order.

(c) Determine the number of reflectors using eigenvalues.

There are several order-determining methods using eigenvalues of correlation matrix, such as AIC, MDL[5], EGM[6], etc.. We use Akaike Information Criteria (AIC) here. The AIC is given as

$$AIC(p) = -\log[LR(p)] + p(2M - p) \tag{9}$$

where $LR(p) = \left\{ \dfrac{\sqrt[M-p]{\prod_{i=p+1}^{M} \lambda_i}}{\dfrac{1}{M-p}\sum_{i=p+1}^{M} \lambda_i} \right\}^{(M-p)N}$. Choose the p as the number of reflectors when AIC(p) reaches its minimum.

(d) Construct a projection matrix of signal subspace

The projection matrix of signal subspace is constructed by using principal eigenvectors in Equation (8) as

$$\mathbf{Y}^{\#} = \sum_{i=1}^{p} \lambda_i^{-1}\mathbf{v}_i\mathbf{v}_i^H \tag{10}$$

(e) Conduct eigen-decomposition of the matrix $Y_1Y^{\#}$.

$Y_1Y^{\#}$ is the projection of Y_1 on the signal subspace of Y_0. It can be decomposed as

$$\mathbf{Y}_1\mathbf{Y}^{\#} = \mathbf{U}\mathbf{E}_1\mathbf{U}^H \tag{11}$$

where each column of $\mathbf{U}$ (denote as $\mathbf{u}_i$) is the generalized eigenvector in Equation (7). The non-zero elements γ_i in $E_1 = diag(\gamma_1, \gamma_2, \cdots, \gamma_N)$ are corresponding to the generalized eigenvalues in Equation (7).

(f) Estimate the bearings of reflectors.

The bearings of reflectors are given by the phases of γ_i as

$$2\pi\frac{d}{\lambda}\sin(\theta_i) = \angle\gamma_i, \qquad i = 1, 2, \cdots, p \tag{12}$$

(g) Estimate the frequencies of reflectors.

Calculate the power spectrum of eigenvector $\mathbf{u}_i$ corresponding to the non-zero eigenvalue:

$$P_i(f) = \left|\mathrm{FT}(\mathbf{u}_i)\right|^2 = \left|\sum_{n=0}^{N-1} u_i(n)\exp(-j2\pi\frac{f}{f_s}n)\right|^2, \quad \mathrm{i} = 1, 2, \cdots, \mathrm{p} \tag{13}$$

The f corresponding to the maximum of $P_i(f)$ is the frequency estimation $\hat{f}_i$ of i-th reflector.

(h) Estimate the time-delays of reflectors.

Compensate the frequency shift of eigenvector $\mathbf{u}_i$, using the $\hat{f}_i$ estimated in last step and calculate the cross-correlation between the compensated eigenvector and emitted signal:

$$r_i(\tau) = \frac{1}{N}\sum_{n=0}^{N-1} s(n+\tau)u_i(n)\exp(-j2\pi\frac{\hat{f}_i}{f_s}n) \tag{14}$$

Take the τ corresponding to the maximum of $r_i(\tau)$ as the time-delay estimation $\hat{\tau}_i$.

In this algorithm, since the eigenvalues and eigenvectors are paired, the parameter estimations of bearing, frequency and time-delay are paired automatically. The additional pairing algorithm is not needed.

3. SIMULATIONS

Consider a sine wave modulated by a ladder-shape as the emitting signal with the duration of 75ms. A 16-sensor uniform linear array with a half-wavelength spaced is employed to receive the echoes. There are 3 reflectors with equal power. Their time-delay and direction parameters are shown in Table 1. The time-delay intervals between neighbor reflectors are chosen as half of the time resolution of ambiguity function, while their direction intervals are approximately 2/3 of beam-width. These three reflectors can be resolved in neither time domain nor spatial domain by classical methods.

Table 1 True parameters of 3 reflectors

	Time-delay (second)	Bearing (^{o})	Frequency shift (Hz)
Reflector 1	0.902	-3.67	92.4
Reflector 2	0.933	0.00	138.5
Reflector 3	0.968	3.42	115.4

The sampling duration used in GETJE is chosen to cover the echoes of these 3 reflectors, within which 100 snapshots are used. Then a 100×16 data matrix is formed. The SNR in sensor output is 15dB.

The bearing estimates of traditional beamforming method is shown in Fig.1(a), and the time-delay estimates of traditional cross-correlation method is shown in Fig.1(b). Obviously, both of them can not resolve these three reflectors.

We apply GETJE method to the same data. The data matrix is divided into two sub-matrices. Their auto-correlation and cross-correlation matrices are calculated. After employing eigen-decomposition on the auto-correlation matrix, the number of sources is determined to be 3 by AIC.

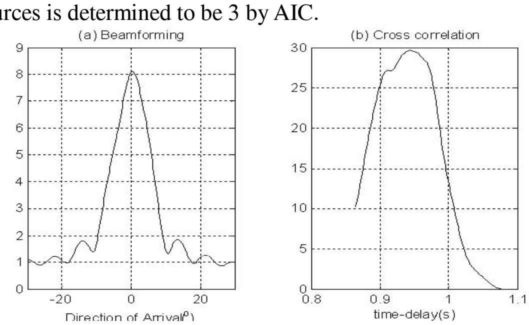

Figure 1 Traditional bearing and time-delay estimation

The generalized eigenvalues and eigenvectors can be calculated according to above procedures of GETJE. The bearing estimates are given by Equation (15) as -3.95°, 0.08°, 3.46°. The spectrums of 3 non-zero eigen-vectors are shown in Fig.2. The maximum peak of each spectral curve indicates the frequency estimate of a reflector. The cross-correlation curves given by Equation (17) are shown in Fig.3. Each curve shows a peak clearly, which indicates the time-delay estimate of a reflector. The values of all these 3-D parameters of 3 reflectors are given in Table 2. Comparing the estimated parameters in Table 2 with the true parameters in Table 1, it is clear that the GETJE can give accurate estimates with correct pairing.

Table 2 Estimated parameters of 3 reflectors

	Time-delay (second)	Bearing (°)	Frequency shift (Hz)
Reflector 1	0.901	-3.95	92.9
Reflector 2	0.934	0.08	139.8
Reflector 3	0.968	3.46	115.4

The statistical performance of proposed method is evaluated in various SNRs. Table 3 shows the probabilities of correct reflector number determining, resolution, and correct pairing. All statistical results are conducted using 100 trials. According to Table 3, when SNR>=5dB, the probability of correct number determining is close to 100%, and the probability of bearing resolution is greater than 87%. When SNR>=10dB, the probabilities of frequency resolution and time-delay resolution are both greater than 83%, the correct pairing probabilities are also greater than 83%. The root mean square errors (RMS) of GETJE in various SNRs are shown in Fig.4, which indicates that GETJE method can satisfy the demand of underwater localization and joint parameter estimation as long as SNR>=7.5dB.

Table 3 Probability of correct reflector number determining, resolution, and pairing

SNR(dB)	15	10	7.5	5	2.5	0
Prob. of correct order-determining (%)	99	96	100	97	47	10
Prob. of bearing resol.(%)	99	96	100	87	27	4
Prob. of frequency resol.(%)	98	83	63	58	29	1
Prob. of correct pairing between bearing and freq. (%)	98	83	62	39	10	1
Prob. of time-delay resol. (%)	98	83	63	53	16	5
Prob. of correct pairing between bearing and time-delay (%)	98	83	62	39	10	1

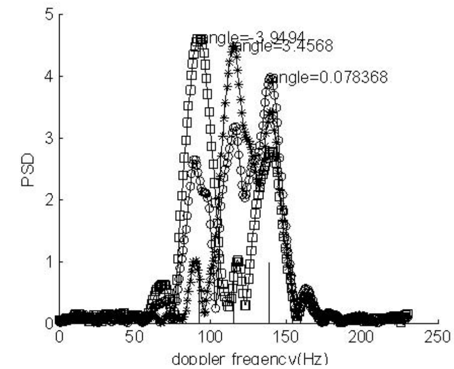

Figure 2 Frequency estimation

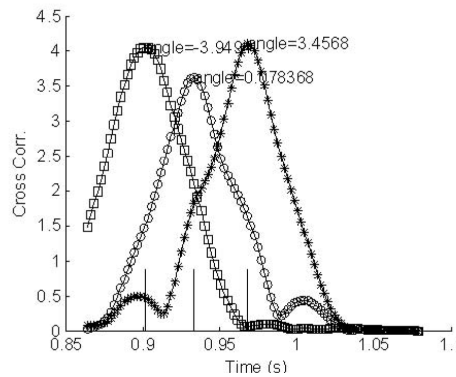

Figure 3 Time-delay estimation

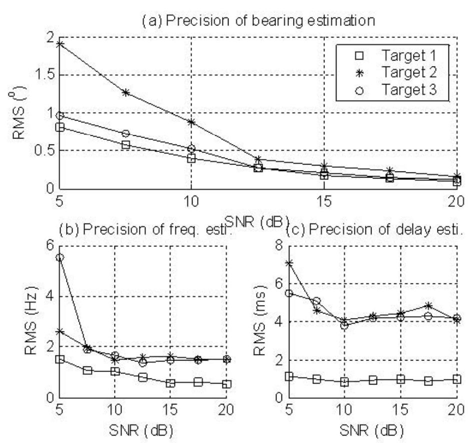

Figure 4 RMS of parameter estimation

4. VALIDATION IN WATER TANK EXPERIMENTS

To evaluate the efficiency of GETJE in the case of a real array, we have carried out a series of joint estimation experiments in water tank. The experiment system is composed shown in Fig.5. The sensor array is a 14-element uniform linear array. The duration of emitting signal is 20ms, repeat period of which is 120ms.

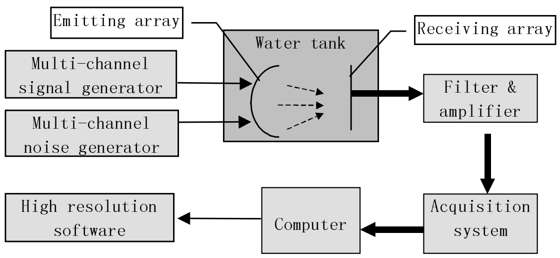

Figure 5 The structure of multi-reflector multi-dimensional joint estimation experiment system

Joint estimation experiments of 2 reflectors are carried out firstly. The bearing interval between 2 reflectors is specified as 3/4 of beam-width. 4 experiments are completed with different time-delay interval and the results are shown in Table 4. From this table, 3-D parameters in all experiments can be resolved preferably. The biases of bearing estimates are smaller than 0.5^o, while the biases of time-delay estimates are smaller than 0.3ms. Even in the case of time-delay interval equal to 1/12 of emitting pulse duration, GETJE can still resolve time-delay parameters perfectly to show a high-resolution time-delay performance.

Joint estimation experiments of 3 reflectors are carried out secondly. The bearing parameters are specified with intervals close to 3/4 and 1/2 of beam-width. 2 experiments are completed with different time-delay interval and the results are shown in Table 5. In first experiment, relative time-delay intervals are 1/4 and 1/8 of pulse duration respectively. 3 reflectors can be well resolved with accurate frequency and time-delay estimates, while bearing estimation error is appreciably great as 3^o. In second experiment, relative time-delay intervals are 1/4 and 1/3 of pulse duration respectively. That is the time-delay interval between nearest reflectors is equal to 1/12 of pulse duration. 3 reflectors can still be resolved.

5. CONCLUSION

A new generalized eigenstructure-based 3-dimension joint estimator (GETJE) for direction, frequency, and time-delay of multiple sources is proposed. A special correlation matrix is constructed which is different from traditional ESPRIT method. Via generalized eigen-decomposition, the bearing parameters are estimated using generalized eigenvalues, and the frequency and time-delay parameters are estimated using generalized eigenvectors. The parameters can be paired automatically by the relationship between eigenvalue and eigenvector. Computer simulations show that GETJE can obtain preferably high probability of correct resolution and correct pairing when SNR>=7.5dB. In this case, the RMS of bearing, frequency, and time-delay estimation are smaller than 1.2°, 2Hz, 6ms respectively. The proposed method is validated by water tank experiments. In the experiments, GETJE succeeds in resolving 3 reflectors with bearing interval equal to half beam width, time-delay interval equal to 1/12 of pulse duration. The experiment results show GETJE possesses attractive prospect in applications of underwater localization and multi-dimensional joint parameter estimation.

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Table 4 Joint estimation experiment results of 2 reflectors

Time-delay (pulse dura.)	Actual value			Estimated value		
	bearing(^{o})	Freq.(KHz)	Time-delay(ms) *	bearing(^{o})	Freq.(KHz)	Time-delay(ms)
1/3	-3.6 2.8	29.9 30.0	0.0, 6.7	-3.1, 3.3	29.9, 30.0	0.0, 7.0
1/4			0.0, 5.0	-3.1, 3.3	29.9, 30.0	0.0, 5.0
1/8			0.0, 2.5	-3.2, 3.3	29.9, 30.0	0.0, 2.5
1/12			1.7, 0.0	-3.2, 3.3	29.9, 30.0	2.0, 0.0

*Notes: Values listed here is the relative time-delay between 2 reflectors.

Table 5 Joint estimation experiment results of 3 reflectors

Time-delay (pulse dura.)	Actual value			Estimated value		
	bearing(^{o})	Freq.(KHz)	Time-delay(ms)	bearing(^{o})	Freq.(KHz)	Time-delay(ms)
0	2.0 -3.6 -7.6	29.9 30.0 30.1	0.0	5.0	29.9	0.0
1/4			5.0	-1.8	30.0	5.5
1/8			2.5	-8.6	30.1	3.0
0			0.0	4.8	29.9	0.0
1/4			5.0	-1.6	30.0	3.0
1/3			6.7	-8.5	30.1	4.5