# A ROBUST ADAPTIVE BLIND MULTICHANNEL IDENTIFICATION ALGORITHM FOR ACOUSTIC APPLICATIONS

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## ABSTRACT

In this paper, we propose a robust adaptive blind multichannel identification algorithm in frequency-domain. It utilizes the fast Fourier transform (FFT) to reduce the computational complexity when the channel impulse response (IR) is long. Moreover, the Newton-LMS algorithm is obtained in frequency domain with small computational load to improve the convergence speed. The advantage of the proposed method is its robustness to input noise, especially when the channel IR is long, e.g., the room acoustic IR with a length up to hundreds or thousands taps. The conventional methods cannot obtain estimate with acceptable accuracy and low computational load. The situation becomes worse when the input signal-to-noise ratio (SNR) is low. The simulation results show that the proposed method is suitable to estimate long multichannel IRs in practical environments.

## 1. INTRODUCTION

System identification is a fundamental technique in building a mathematical model of a dynamic system. It has many applications in the area of digital signal processing, digital communication, etc. If the system input/output data are available, the conventional system identification methods [1] can be used to find the system model. However, in some applications ,the input data are unobservable or expensive to acquire, and only output data are available. Typical cases include acoustic dereverberation, wireless communication and time delay estimation. Conventional system identification method cannot be applied. The solution is inevitably blind multichannel identification (BCI) method.

At the first sight, BCI is impossible since the input and the channel are both unknown. However, the pioneer work done by Sato [2] indicated the possibility of BCI under some assumptions. Since then, many BCI algorithms have been proposed in literature ([3,4] and the references therein). These algorithms can generally be classified into two classes: one is based on second-order statistics (SOS) and the other is based on higher-order statistics (HOS). HOS based methods are not so practical as SOS based methods because of their slow convergence speed and non-convex optimization cost function. As pointed out in [5], the BCI can be simply solved by SOS. SOS based method has potentially fast convergence. Therefore, the focus of BCI shifted to SOS methods.

Unfortunately, most of the SOS based methods are difficult to implement in adaptive mode [6]. They need eigenvalue decomposition (EVD) or singular value decomposition (SVD) of the covariance or data matrix, which is generally intensive in computaMeng Hwa Er

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tion, especially when the IR length is large. Moreover, some SOS based algorithms have the assumption that the input signal should be white. This is not the case for many applications, such as acoustic devereberation.

Considering the source signal characteristics and long room IR in acoustic applications, we find that the normalized multi-channel frequency domain LMS (NMCFLMS) algorithm [7] is possible to adaptively estimate the room acoustic IR of large length. However, the NMCFLMS algorithm has a drawback that it requires high input SNR, since the estimated instantaneous gradient vector of NMCFLMS is very sensitive to the input noise. If a robust gradient vector can be found, the resulting adaptive method will be insensitive to the input noise. In this paper, we propose a new cost function for derivation of a robust adaptive method, called normalized blind frequency-domain least mean square (NBFLMS) method. The computer simulation shows that the proposed method can produce acceptable estimate of multichannel IR even in SNR as low as 0dB. Therefore, the proposed algorithm is practical for applications.

This paper is organized as follows. In Section 2, the system model and notation used in this paper are discussed. In Section 3, the proposed method is derived in detail. Simulation results are shown in Section 4 to illustrate the performance of the proposed method. Brief conclusion is given in Section 5.

#### 2. SYSTEM MODEL

Notations used in this paper are defined before we formulate the problem and develop the algorithm.  $E\{\cdot\}$ ,  $(\cdot)^*$ ,  $(\cdot)^T$ ,  $(\cdot)^H$ ,  $\star$  and  $||\cdot||$  stand for mathematical expectation, complex conjugate, vector/matrix transpose, vector/matrix Hermitian transpose, linear convolution, and Euclidean norm, respectively. The identity matrix is **I**.

There are M sensors used. Each sensor picks up the target signal as well as the environment noise. The target signal s(k) propagates through the *i*th channel with IR  $h_{i,k}$ ,  $i = 1, 2, \dots, M$ , and is corrupted by an additive environment noise  $n_i(k)$ . The received signal  $x_i(k)$  of *i*th channel is expressed as

$$x_i(k) = h_{i,k} \star s(k) + n_i(k), \quad i = 1, 2, \cdots, M.$$
 (1)

In most applications, the IR  $h_{i,k}$  can be approximated as FIR filter with length L and coefficient vector

$$\mathbf{h}_{i} = [h_{i,0} \ h_{i,1} \ \cdots \ h_{i,L-1}]^{T}.$$
 (2)

Here, we assume that all the IRs are fixed or changing very slowly although the time variation can be tracked with the proposed method. The estimate of IR coefficient vector  $\hat{\mathbf{h}}_i$  of *i*th channel at *m*th iteration is expressed as

$$\hat{\mathbf{h}}_{i}(m) = [\hat{h}_{i,0}(m) \ \hat{h}_{i,1}(m) \ \cdots \ \hat{h}_{i,L-1}(m)]^{T}.$$
 (3)

#### 3. PROPOSED BLIND MULTICHANNEL IDENTIFICATION METHOD

Before the detail derivation of the method, some matrices are first defined and discussed. Due to space limitation, the proofs are omitted.

**Definition 1** The discrete Fourier transform (DFT) matrix  $F_L$  is a L by L matrix, whose (p,q)th element  $(F_L)_{p,q} = e^{-j2\pi pq/L}, p, q = 0, 1, \cdots, L-1$ .

**Lemma 1** The inverse matrix of  $F_L$  is  $F_L^{-1} = \frac{1}{L}F_L^H$ .

**Definition 2** The windowing matrices are defined as

$$\begin{split} \mathbf{W}_{L\times N'}^{10} &= [\mathbf{I}_{L\times L} \ \mathbf{0}_{L\times (N-1)}], \quad \mathbf{W}_{N'\times L}^{10} &= (\mathbf{W}_{L\times N'}^{10})^T, \\ \mathbf{W}_{N\times N'}^{01} &= [\mathbf{0}_{N\times (L-1)} \ \mathbf{I}_{N\times N}], \quad \mathbf{W}_{N'\times N}^{01} &= (\mathbf{W}_{N\times N'}^{01})^T, \\ \mathbf{W}_{N'\times N'}^{01} &= \begin{bmatrix} \mathbf{0}_{(L-1)\times (L-1)} & \mathbf{0}_{(L-1)\times N} \\ \mathbf{0}_{N\times (L-1)} & \mathbf{I}_{N\times N} \end{bmatrix}, \\ \mathbf{W}_{N'\times N'}^{10} &= \begin{bmatrix} \mathbf{I}_{L\times L} & \mathbf{0}_{L\times (N-1)} \\ \mathbf{0}_{(N-1)\times L} & \mathbf{0}_{(N-1)\times (N-1)} \end{bmatrix}, \end{split}$$

where N' = N + L - 1.

Definition 3 The transformed windowing matrices are defined as

$$\mathcal{W}_{L\times N'}^{10} = F_L \mathbf{W}_{L\times N'}^{10} F_{N'}^{-1}, \quad \mathcal{W}_{N\times N'}^{01} = F_N \mathbf{W}_{N\times N'}^{01} F_{N'}^{-1}, \mathcal{W}_{N'\times L}^{10} = F_{N'} \mathbf{W}_{N'\times L}^{10} F_L^{-1}, \quad \mathcal{W}_{N'\times N}^{01} = F_{N'} \mathbf{W}_{N'\times N}^{01} F_N^{-1}, \mathcal{W}_{N'\times N'}^{10} = F_{N'} \mathbf{W}_{N'\times N'}^{10} F_{N'}^{-1}, \quad \mathcal{W}_{N'\times N'}^{01} = F_{N'} \mathbf{W}_{N'\times N'}^{01} F_{N'}^{-1}.$$

**Lemma 2** The transformed windowing matrices have the following properties

$$\boldsymbol{\mathcal{W}}_{L\times N'}^{10} = \frac{N'}{L} (\boldsymbol{\mathcal{W}}_{N'\times L}^{10})^{H}, \quad \boldsymbol{\mathcal{W}}_{N'\times N}^{01} = \frac{N}{N'} (\boldsymbol{\mathcal{W}}_{N\times N'}^{01})^{H}$$

The derivation of NBFLMS is based on cross relation (CR) criteria [8,9] in frequency domain using overlap-save method [10]. The linear filtering of a signal by a filter can be obtained in block mode using circular convolution. Refer to [7] for details. Here we define the posteriori filtered signal block  $\mathbf{y}_{i,j}(n,m)$  of length N, which is produced by filtering *n*th signal block of *i*th channel by IR estimate at *m*th iteration *j*th channel. It is expressed in matrix notation as

$$\mathbf{y}_{ij}(n,m) = \mathbf{W}_{N \times N'}^{01} \breve{\mathbf{X}}_i(n) \tilde{\mathbf{h}}_j(m)$$
(4)

where  $n, m(n \le m)$  stand for data block index, the matrix  $\mathbf{\tilde{X}}_i(m)$  is a circulant matrix with its first column  $\mathbf{\tilde{x}}_i(n)$ , and

$$\tilde{\mathbf{x}}_i(n) = [x_i(nN - L + 1) \cdots x_i(nN + N - 1)]^T,$$
  

$$\mathbf{y}_{ij}(n,m) = [y_{ij}(mN) \cdots y_{ij}(mN + N - 1)]^T,$$
  

$$\tilde{\mathbf{h}}_j(m) = \mathbf{W}_{N' \times L}^{10} \hat{\mathbf{h}}_j(m).$$
(5)

The signal block of the posteriori error in frequency domain based on CR criteria between *i*th and *j*th channel is determined as

$$\boldsymbol{e}_{ij}(n,m) = \mathbf{F}_{N} \left( \mathbf{y}_{ij}(n,m) - \mathbf{y}_{ji}(n,m) \right)$$
$$= \boldsymbol{\mathcal{W}}_{N \times N'}^{01} [\boldsymbol{\mathcal{D}}_{x_{i}}(n) \boldsymbol{\mathcal{W}}_{N' \times L}^{10} \hat{\boldsymbol{h}}_{j}(m) \qquad (6)$$
$$- \boldsymbol{\mathcal{D}}_{x_{j}}(n) \boldsymbol{\mathcal{W}}_{N' \times L}^{10} \hat{\boldsymbol{h}}_{i}(m)]$$

where

$$\mathcal{D}_{x_i}(n) = \mathbf{F}_{N'} \breve{\mathbf{X}}_i(n) \mathbf{F}_{N'}^{-1}, \ \hat{\mathbf{h}}_i(m) = \mathbf{F}_L \hat{\mathbf{h}}_i(m)$$
(7)

**Lemma 3** The matrix  $\mathcal{D}_{x_i}(m)$  in (7) is a diagonal matrix whose diagonal elements are given by the DFT of the first column of  $\check{\mathbf{X}}_i(n)$ .

With the derived error signal in (6), the squared error  $\epsilon(n,m)$  is defined as

$$\epsilon(n,m) = \sum_{i=1}^{M-1} \sum_{j=i+1}^{M} \boldsymbol{e}_{ij}^{H}(n,m) \boldsymbol{e}_{ij}(n,m)$$
(8)

Define the cost function for derivation of NBFLMS as

$$J(m) = E\{\sum_{n=m-K}^{m} \beta^{m-n} \epsilon(n,m)\}$$
(9)

where  $0 < \beta \le 1$  is the forgetting factor. When K = 0, the cost function is similar to the one used in [7], which minimizes the least mean square of the instantaneous error. In the case of K > 0, the cost function in (9) aims to minimize the least mean square of not only the instantaneous error, but also the weighted posteriori errors. The gradient vector estimated by cost function in [7] is very sensitive to the input noise, causing NMCFLMS method requiring high input SNR to estimate multichannel IRs with long taps. The proposed method is less sensitive to input noise, which will be illustrated by the simulation result.

The LMS algorithm is constructed as

$$\hat{\boldsymbol{h}}_k(m) = \hat{\boldsymbol{h}}_k(m-1) - \mu \bigtriangledown J_k(m) \tag{10}$$

where  $\mu$  is the stepsize.  $\bigtriangledown J_k(m)$  is the estimated gradient vector. To achieve fast convergence, the Newton method [11] is used

$$\hat{h}_{k}(m) = \hat{h}_{k}(m-1) - \rho E\{\nabla^{2} J_{k}(m)\}^{-1} \nabla J_{k}(m) \quad (11)$$

where  $\nabla^2 J_k(m)$  is the Hessian matrix and  $\rho$  is the stpesize.

In the derivation of LMS type algorithm, the gradient vector  $\nabla J_k(m)$  and the Hessian matrix  $\nabla^2 J_k(m)$  should be estimated first. They are given in Lemma 4.

**Lemma 4** The gradient vector  $\nabla J_k(m)$  and Hessian matrix  $\nabla^2 J_k(m)$ of J(m) to  $\hat{h}_k(m)$  are given by

$$\nabla J_k(m) = \frac{\partial J(m)}{\partial \hat{h}_k^*(m)} = \sum_{n=m-K}^m \beta^{m-n}$$

$$\sum_{i=1}^M (\mathcal{W}_{N\times N'}^{01} \mathcal{D}_{x_i}(n) \mathcal{W}_{N'\times L}^{10})^H e_{ik}(n,m)$$

$$\nabla^2 J_k(m) = \frac{\partial^2 J(m)}{\partial \hat{h}_k^*(m) \partial \hat{h}_k^T(m)} = \sum_{n=m-K}^m \beta^{m-n}$$

$$\sum_{i=1, i \neq k}^M \mathcal{W}_{L\times N'}^{10} \mathcal{D}_{x_i}^H(n) \mathcal{W}_{N'\times N'}^{01} \mathcal{D}_{x_i}(n) \mathcal{W}_{N'\times L}^{10} )$$

It is obvious that the calculation of  $\nabla J_k(m)$  and  $\nabla^2 J_k(m)$  is computational intensive, especially when K is large. In this paper, we use approximation of  $\nabla J_k(m)$  and  $\nabla^2 J_k(m)$  in algorithm derivation.

#### Lemma 5

$$\mathcal{W}_{N' \times N'}^{01} \approx \frac{N}{N'} \mathbf{I}_{2L \times 2L}, \text{ when } N' \text{ is large.}$$
  
 $\mathcal{W}_{N' \times N'}^{10} \approx \frac{L}{N'} \mathbf{I}_{2L \times 2L}, \text{ when } N' \text{ is large.}$ 

**Lemma 6** When N' is large,  $\nabla J_k(m)$  and  $\nabla^2 J_k(m)$  can be approximated as

$$\nabla J_{k}(m) \approx \frac{L}{N'} \boldsymbol{\mathcal{W}}_{L \times N'}^{10} \sum_{i=1}^{M} \left[ \boldsymbol{\mathcal{R}}_{x_{i}x_{i}}(m) \boldsymbol{\mathcal{W}}_{N' \times L}^{10} \hat{\boldsymbol{h}}_{k}(m) - \boldsymbol{\mathcal{R}}_{x_{i}x_{k}}(m) \boldsymbol{\mathcal{W}}_{N' \times L}^{10} \hat{\boldsymbol{h}}_{i}(m) \right]$$
$$\nabla^{2} J_{k}(m) \approx \boldsymbol{\mathcal{W}}_{L \times N'}^{10} \left[ \sum_{i=1, i \neq k}^{M} \boldsymbol{\mathcal{R}}_{x_{i}x_{i}}(m) \right] \boldsymbol{\mathcal{W}}_{N' \times L}^{10}$$

where

$$\mathcal{R}_{x_i x_i}(m) = \sum_{n=m-K}^{m} \beta^{m-n} \mathcal{D}_{x_i}^{H}(n) \mathcal{D}_{x_i}(n)$$
$$\mathcal{R}_{x_i x_k}(m) = \sum_{n=m-K}^{m} \beta^{m-n} \mathcal{D}_{x_i}^{H}(n) \mathcal{D}_{x_k}(n)$$

**Lemma 7** When N' and K is large,  $\mathcal{R}_{x_ix_i}(m)$  and  $\mathcal{R}_{x_ix_k}(m)$  can be recursively estimated by

$$\mathcal{R}_{x_i x_i}(m) \approx \beta \mathcal{R}_{x_i x_i}(m-1) + \mathcal{D}_{x_i}^H(m) \mathcal{D}_{x_i}(m)$$
$$\mathcal{R}_{x_i x_k}(m) \approx \beta \mathcal{R}_{x_i x_k}(m-1) + \mathcal{D}_{x_i}^H(m) \mathcal{D}_{x_k}(m)$$

Lemma 8

$$E\{\nabla^2 J(m)\} = \boldsymbol{\mathcal{W}}_{L \times N'}^{10} \sum_{i=1, i \neq k}^M \boldsymbol{\mathcal{R}}_{x_i x_i} \boldsymbol{\mathcal{W}}_{N' \times D}^{10}$$

where  $\mathcal{R}_{x_i x_i} = E\{\mathcal{R}_{x_i x_i}(m)\}$  is estimated as

$$\begin{aligned} \mathcal{R}_{x_i x_i}(0) &= \mathcal{R}_{x_i x_i}(0) \\ \hat{\mathcal{R}}_{x_i x_i}(m) &= \lambda \hat{\mathcal{R}}_{x_i x_i}(m-1) + \mathcal{R}_{x_i x_i}(m) \end{aligned}$$

where  $\lambda$  (0 <  $\lambda$  < 1) is the exponential forgeting factor.

The inverse of Hessian matrix is simplified as

**Lemma 9** Let  $\mathcal{P} = \sum_{i=1, i \neq k}^{M} \mathcal{R}_{x_i x_i}$ , we have

$$\boldsymbol{\mathcal{W}}_{N'\times L}^{10} E\{\nabla^2 J(m)\}^{-1} \boldsymbol{\mathcal{W}}_{L\times N'}^{10} = \frac{N'}{L} \boldsymbol{\mathcal{W}}_{N'\times N'}^{10} \boldsymbol{\mathcal{P}}^{-1} \quad (12)$$

With the estimated  $\hat{\mathcal{R}}_{x_i x_i}(m)$ , we have

$$\boldsymbol{\mathcal{W}}_{N'\times L}^{10} E\{\nabla^2 J(m)\}^{-1} \boldsymbol{\mathcal{W}}_{L\times N'}^{10} = \frac{N'}{L} \boldsymbol{\mathcal{W}}_{N'\times N'}^{10} \boldsymbol{\mathcal{P}}^{-1}(m)$$

where

$$\mathcal{P}(m) = \sum_{i=1, i \neq k}^{M} \hat{\mathcal{R}}_{x_i x_i}(m)$$

Finally, we obtain the NBFLMS algorithm in the following Theorem 1.

Theorem 1 The constrained NBFLMS algorithm is

$$\bar{\boldsymbol{h}}_{k}(m) = \tilde{\boldsymbol{h}}_{k}(m-1) - \rho \boldsymbol{\mathcal{W}}_{N' \times N'}^{10} \boldsymbol{\mathcal{P}}^{-1}(m)$$
$$\sum_{i=1}^{M} \left[ \boldsymbol{\mathcal{R}}_{x_{i}x_{i}}(m) \tilde{\boldsymbol{h}}_{k}(m) - \boldsymbol{\mathcal{R}}_{x_{i}x_{k}}(m) \tilde{\boldsymbol{h}}_{i}(m) \right]$$
$$k = 1, 2, \cdots, M$$

The unconstrained NBFLMS algorithm is

$$\bar{\boldsymbol{h}}_{k}(m) = \bar{\boldsymbol{h}}_{k}(m-1) - \rho \boldsymbol{\mathcal{P}}^{-1}(m)$$

$$\sum_{i=1}^{M} \left[ \boldsymbol{\mathcal{R}}_{x_{i}x_{i}}(m)\tilde{\boldsymbol{h}}_{k}(m) - \boldsymbol{\mathcal{R}}_{x_{i}x_{k}}(m)\tilde{\boldsymbol{h}}_{i}(m) \right]$$

$$k = 1, 2, \cdots, M,$$

where  $\tilde{\mathbf{h}}_k(m) = \mathbf{F}_{N'} \tilde{\mathbf{h}}_k(m)$ . To avoid the trivial solution, the updated filter coefficient vectors are normalized to vector with unit norm.

$$\tilde{\boldsymbol{h}}_k(m) = \frac{\boldsymbol{h}_k(m)}{||\boldsymbol{h}(m)||}, \quad \boldsymbol{h}(m) = [\bar{\boldsymbol{h}}_1(m) \cdots \bar{\boldsymbol{h}}_M(m)]^T$$

The computational load of NBFLMS in Theorem 1 is low since the matrices  $\mathcal{R}_{x_i x_k}(m)$  and  $\mathcal{P}(m)$  are both diagonal.

#### 4. NUMERICAL STUDY

In this section, we asses the performance of the proposed method. There are five microphones used in simulation. The acoustic enclosure is a small office room with dimension  $(x \times y \times z) = (2.8m \times 3.2m \times 2.2m)$ , wall reflection coefficients 0.8 and floor celling reflection coefficients 0.4. The position of each microphone is given in Table I. A source signal is placed in the poistion (1.0m, 1.5m, 1.4m). The IR relating speech source and each microphone is calculated using image method [12] with sampling rate 8kHz. A white background noise is used. The length of IR is set as L = 256, so that most of the reverberation is taken into account.

In Fig. 1, one of the estimated channel IR and its associated frequency response are compared with its true IR and frequency response. It is obvious that the estimated channel response is very close to the true one.

The normalized root mean square projection misalignment (NRM-SPM) in decibel is also used as a performance measure of estimation accuracy versus different input SNR. The NRMSPM is defined as

$$NRMSPM = 20 \log_{10} \left[ \frac{1}{||\mathbf{h}||} \sqrt{\frac{1}{N} \sum_{i=1}^{N} ||\xi^{(i)}||^2} \right]$$
(13)  
$$\xi = \mathbf{h} - \frac{\mathbf{h}^T \hat{\mathbf{h}}}{\hat{\mathbf{h}}^T \hat{\mathbf{h}}} \hat{\mathbf{h}}$$

	1	2	3	4	5
Х	0.80	0.80	1.00	1.20	1.20
у	2.0	2.0	2.0	2.0	2.0
z	1.6	1.2	1.6	1.6	1.2

Table 1. Position (x,y,z) of five microphones (in meter)



**Fig. 1**. The comparison of the estimated IRs and the real IRs (5dB input SNR)

where *N* is the number of Monte Carlo runs,  $(\cdot)^{(i)}$  denotes a value obtained for the *i*th run.  $\xi$  is the projection misalignment vector [13]. The NRMSPM of NRFLMS algorithm after 200 Monte Carlo runs is shown in Fig. 2. The NBFLMS algorithm can obtain acceptable IR estimate at low input SNR. For NMCFLMS, the estimate with reasonable accuracy is achieved with SNR up to 50dB [7]. It is also shown in Fig. 2 that NMCFLMS has large NRMSPM. Therefore, the NMCFLMS cannot be applied directly in practical applications with low input SNR, while the proposed NBFLMS can.

#### 5. CONCLUSION

A robust adaptive blind multichannel identification algorithm in frequency domain is proposed. It uses a new cost function in algorithm derivation. With some approximation, no extra computational load is introduced. The proposed method has the advantage of robustness to input noise, especially when the IR length is large. The simulation results show the effectiveness of the proposed method.



**Fig. 2.** Comparison of NRMSPM between NBFLMS and NM-CFLMS at 200 Monte Carlo runs versus different input SNR

#### 6. REFERENCES

- L. Ljung, System Identification. Englewood Cliffs, NJ: Prentice-Hall, Inc., 1987.
- [2] Y. Sato, "A method of self-recovering equalization for multilevel amplitude-modulation," *IEEE Trans. Commun.*, vol. 23, no. 6, pp. 679–682, June 1975.
- [3] Z. Ding and Y. Li, *Blind Equalization and Identification*. New York: Marcel Dekker, Inc., 2001.
- [4] G. B. Giannakis, Y. Hua, P. Stoica, and L. Tong, Eds., Signal Processing Advances in Wireless and Mobile Communication: Trends in Channel Estimation and Equalization. Upper Saddle River, NJ: Prentice Hall PTR, 2001, vol. I.
- [5] L. Tong, G. Xu, and T. Kailath, "A new approach to blind identification and equalization of multipath channels," in *Proc. 25th Asilomar Conf. Signals, Syst. Comput.*, vol. 2, 1991, pp. 856–860.
- [6] L. Tong and S. Perreau, "Multichannel blind identification: From subspace to maximum likelihood mehtods," *Proc. IEEE*, vol. 86, no. 10, pp. 1951–1968, Oct. 1998.
- [7] Y. Huang and J. Benesty, "A class of frequency-domain adaptive approaches to blind multichannel identification," *IEEE Trans. Signal Processing*, vol. 51, pp. 11–24, Jan. 2003.
- [8] H. Liu, G. Xu, and L. Tong, "A deterministic approach to blind equalization," in *IEEE Conference Record of The Twenty-Seventh Asilo*mar Conference on Signals, Systems and Computers, vol. 1, Nov. 1993, pp. 751–755.
- [9] G. Xu, H. Liu, L. Tong, and T. Kailath, "A least-square approach to blind channel identification," *IEEE Trans. Signal Processing*, vol. 43, pp. 2982–2993, Dec. 1995.
- [10] A. V. Oppenheim and R. W. Schafer, *Discrete-Time Signal Process*ing. Englewood Cliffs, N.J.: Prentice-Hall, Inc., 1989.
- [11] T. K. Moon and W. C. Stirling, Mathematical Methods and Algorithms. Upper Saddle River, NJ: Prentice-Hall, 1999.
- [12] J. B. Allen and D. A. Berkley, "Image method for efficiently simulating small room acoustics," *J. Acoust. Soc. Am.*, vol. 65, no. 4, pp. 943–950, April 1979.
- [13] D. R. Morgan, J. Benesty, and M. M. Sondhi, "On the evaluation of estimated impulse responses," *IEEE Singal Processing Lett.*, vol. 5, pp. 174–176, July 1998.