DIFFERENTIAL SPACE-TIME CODING BASED ON GENERALIZED MULTI-CHANNEL AMPLITUDE AND PHASE MODULATION

Hongbin Li

Stevens Institute of Technology
Department of Electrical and Computer Engineering
Hoboken, New Jersey 07030, USA
E-mail: hli@stevens-tech.edu

ABSTRACT

We present a new differential space-time coding scheme based on generalized multi-channel amplitude and phase modulation. Each code matrix employed by our scheme consists of an amplitude and a phase component, and can be thought of as a space-time multi-channel generalization of the scalar amplitude and phase shift keying (APSK) constellation. The amplitude component takes a scalar coefficient that controls the total transmission power, while the phase component is a unitary matrix formed from PSK symbols. Both the amplitude and phase components are differentially encoded and admit efficient differential decoding. We show that the maximum likelihood (ML) decoding of the amplitude coefficient and phase matrix is decoupled. Moreover, the phase matrix, when constructed from orthogonal designs, is amenable to decoupled differential decoding of the phase entries, which further simplifies the decoding complexity significantly. Simulation results show that the proposed amplitude-phase differential space-time modulation scheme achieves a performance very close to its phase-only counterpart, while providing higher spectral efficiency offered by amplitude modulation.

1. INTRODUCTION

Utilizing multi-antenna transmission, space-time coding can offer both diversity and coding gain to the receiver. While coherent detection of space-time codes requires multi-channel estimation, a challenging task especially in fading environments, differential space-time modulation/coding circumvents this difficulty. A number of differential space-time modulation schemes have been proposed for both flat-fading [1, 2, 3, 4] and frequency-selective fading [5, 6] channels. All of the above schemes utilize unitary code matrices formed by phase-shift-keying (PSK) entries. These unitary code matrices can be thought of as multi-channel extensions of the scalar PSK constellation. Therefore, we may call these schemes as generalized phase modulation based differential space-time techniques.

This work was supported in part by the Army Research Office under Contract DAAD19-03-1-0184, and by the New Jersey Commission on Science and Technology.

It is known that PSK becomes energy inefficient when transmission rate is high. This has motivated the use of multi-level constellations, such as amplitude and phase shift keying (APSK), for differential transmission in single-antenna systems (see [7] and references therein). Differential space-time modulation using multi-level constellations has been recently examined in several studies. Specifically, Tao and Cheng [8] proposed a scheme that forms space-time code matrices with multi-level entries from orthogonal designs. Since the code matrix carries non-uniform energy (Frobenius norm), their decoding technique requires an estimate of the energy of the previous code matrix to decode the current one. As a result, error propagation may occur. Another method introduced by Xia [9] utilizes APSK constellation for systems equipped with two transmit antennas. It draws two APSK symbols at a time that are used to form an Alamouti code matrix [10]. The code matrix has constant energy due to a design constraint that one of the symbol pair is always picked from the inner ring and the other from the outer ring of the APSK constellation (see Figure 1 for an example of 16-APSK). The code matrix is differentially encoded, similarly to the differential Alamouti scheme [5]. In addition, a one-bit amplitude coefficient, which is differentially encoded by differential ASK, is used to control the overall energy transmitted from the two transmit antennas. Both the Alamouti code matrix and the amplitude coefficient can be differentially decoded, thus without incurring error propagation.

In this paper, we introduce a new differential space-time coding scheme based on generalized multi-channel amplitude and phase modulation. The proposed scheme utilizes code matrices having an amplitude and a phase component. These code matrices can be thought of as multi-channel generalizations of the APSK constellation. The amplitude component is a scalar that controls the total transmission power, while the phase component is a unitary matrix formed from PSK symbols. Both the amplitude and phase components are differentially encoded and allow efficient differential decoding. Unlike the method of [9] which works for

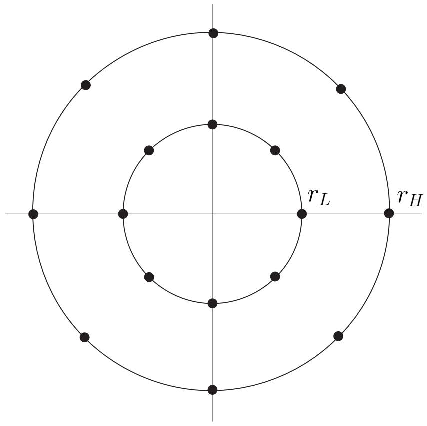

Fig. 1. 16-APSK constellation.

systems with two transmit antennas, the proposed scheme can accommodate systems with an arbitrary number of antennas. The maximum likelihood (ML) decoding of the amplitude coefficient and phase matrix is shown to be decoupled; furthermore, the phase code matrix, if constructed by orthogonal designs, offers decoupled differential decoding of the phase entries, thus further reducing the decoding complexity. The proposed scheme yields full spatial diversity. Simulation results show that the proposed amplitude-phase differential space-time modulation scheme achieves a performance very close to its phase-only counterpart, while providing higher spectral efficiency offered by amplitude modulation.

Notation: Vectors (matrices) are denoted by boldface lower (upper) case letters; all vectors are column vectors; superscripts $(\cdot)^*$, $(\cdot)^T$, $(\cdot)^H$ denote the complex conjugate, transpose, and conjugate transpose, respectively; $\mathbf{I}_N$ denotes the $N \times N$ identity matrix; $\|\cdot\|$ denotes the Frobenius norm; and $\Re(\cdot)$ takes the real part of the argument.

2. PROPOSED SCHEME

2.1. Scalar APSK Constellation

We briefly review the notation associated with the scalar APSK constellation, which forms the basis of the proposed amplitude-phase differential space-time coding scheme. Consider an $2M$-APSK constellation that consists of a combination of an independent M-PSK: $\exp\{j2\pi m/M\}, m = 0, 1, \ldots, M-1$, and a binary ASK (2-ASK): r_L and r_H with $(r_L^2 + r_H^2)/2 = 1$ [7]. Let

$$\gamma \triangleq r_H / r_L. \tag{1}$$

Then, it is ready to show that $r_L = \sqrt{2/(\gamma^2+1)}$. Figure 1 depicts an example of the 16-APSK constellation. Note that each M-APSK symbol carries $\log_2 M + 1$ bits of information, with 1 bit carried by the 2-ASK while $\log_2(M)$ bits by the M-PSK.

2.2. Differential Encoding: Generalized Multi-Channel Amplitude-Phase Modulation

Consider a space-time modulation system utilizing n_T transmit antennas and n_R receive antennas. For simplicity of presentation, we consider in the following $n_T = 2$. The extension to $n_T > 2$ can be made in a manner similar to that in [11]. At time $2nT$, where T denotes the symbol duration and n the code matrix index, the space-time encoder takes a total of $2\log_2 M + 1$ bits of information and map them to a 2×2 unitary matrix $\mathbf{C}_n$, formed from a pair of M-PSK symbols, and a one-bit coefficient $\alpha_n \in \{1, \gamma, 1/\gamma\}$, where γ is defined in (1). The composite space-time code matrix $\alpha_n \mathbf{C}_n$ can be thought of as a multi-channel extension of the scaler APSK constellation, with α_n being denoted as the *amplitude coefficient* and $\mathbf{C}_n$ the *phase matrix*. The space-time code matrix $\alpha_n \mathbf{C}_n$ is then differentially modulated (to be specified) and transmitted over a period of $2T$ seconds, yielding a spectral efficiency of $(\log_2 M + 0.5)$ bits/sec/Hz.

The unitary phase matrix $\mathbf{C}_n$ can be formed in various ways. For efficient decoding, we consider the one based on orthogonal designs [12], which reduces to the Alamouti scheme for $n_T = 2$ [10]. In particular, we map the first $2\log_2 M$ bits of information to two M-PSK symbols $c_{n,1}$ and $c_{n,2}$. The phase code matrix is formed as follows:

$$\mathbf{C}_n = \frac{1}{\sqrt{2}} \begin{bmatrix} c_{n,1} & -c_{n,2}^* \\ c_{n,2} & c_{n,1}^* \end{bmatrix}, \tag{2}$$

where the scaling factor of $1/\sqrt{2}$ is to ensure that $\mathbf{C}_n$ is unitary and we assume that $c_{n,1}$ and $c_{n,2}$ are drawn from a unit-energy M-PSK constellation. The last information bit is mapped to the amplitude coefficient α_n as described below.

- *Initialization:*
  For $n = 0$, let

$$\mathbf{D}_0 = \sqrt{2}\mathbf{I}_2, \tag{3}$$

$$\beta_0 = r_L. \tag{4}$$

  The first transmitted space-time code matrix is

$$\mathbf{S}_0 = \beta_0 \mathbf{D}_0. \tag{5}$$

- *Differential Encoding:*
  For $n > 0$, the encoder takes a total of $2\log_2 M + 1$ bits of information, in which the first $2\log_2 M$ bits are mapped to $\mathbf{C}_n$, formed as in (2), and the last bit, denoted by b_n, is mapped to the amplitude coefficient α_n. The amplitude coefficient is assigned and differentially encoded according to the following rule:

$$\alpha_n = \begin{cases} 1, & \text{if } b_n = 0, \\ \gamma, & \text{if } b_n = 1 \text{ and } \beta_{n-1} = r_L, \\ 1/\gamma, & \text{if } b_n = 1 \text{ and } \beta_{n-1} = r_H, \end{cases} \tag{6}$$

$$\beta_n = \beta_{n-1}\alpha_n. \tag{7}$$

The phase matrix is differentially encoded as follows:

$$\mathbf{D}_n = \mathbf{D}_{n-1}\mathbf{C}_n. \tag{8}$$

The transmitted space-time code matrix $\mathbf{S}_n$ is given by

$$\mathbf{S}_n = \beta_n \mathbf{D}_n. \tag{9}$$

2.3. Differential Decoding

We assume frequency non-selective fading channels. The received signal at the n_R receive antennas are given by

$$\mathbf{Y}_n = \sqrt{\rho_R}\mathbf{H}\mathbf{S}_n + \mathbf{W}_n, \tag{10}$$

where $\mathbf{Y}_n \in \mathbb{C}^{n_R\times 2}$ denotes the received data matrix, $\rho_R \triangleq \rho/2$ and ρ denotes the SNR per receive antenna, $\mathbf{H} \in \mathbb{C}^{n_R\times 2}$ denotes the channel matrix, and $\mathbf{W}_n \in \mathbb{C}^{n_R\times 2}$ denotes the channel noise with independent and identically distributed (i.i.d.) complex Gaussian entries of zero mean and unit variance (i.e., $\mathcal{CN}(0,1)$-distributed). Substituting (9) into (10) leads to

$$\mathbf{Y}_n = \alpha_n \mathbf{Y}_{n-1}\mathbf{C}_n + \mathbf{V}_n, \tag{11}$$

where $\mathbf{V}_n \triangleq \mathbf{W}_n - \alpha_n \mathbf{W}_{n-1}\mathbf{C}_n$. Since $\mathbf{C}_n$ is unitary, it is ready to show that $\mathbf{V}_n$ consists of i.i.d. complex Gaussian entries with zero mean and variance $1+\alpha_n^2$.

Consider ML detection. The likelihood function of $\mathbf{Y}_n$, conditioned on α_n, $\mathbf{C}_n$ and $\mathbf{Y}_{n-1}$, is given by

$$\begin{aligned} p(\mathbf{Y}_n|\mathbf{Y}_{n-1},\alpha_n,\mathbf{C}_n) &= \frac{1}{\pi^{2n_R}(1+\alpha_n^2)^{2n_R}} \\ &\times \exp\left\{-\frac{1}{1+\alpha_n^2}\left\|\mathbf{Y}_n - \alpha_n\mathbf{Y}_{n-1}\mathbf{C}_n\right\|^2\right\} \end{aligned} \tag{12}$$

Maximizing the likelihood function is equivalent to minimizing

$$f(\alpha_n, \mathbf{C}_n) \triangleq 2n_R \log(1+\alpha_n^2) + \frac{\|\mathbf{Y}_n - \alpha_n \mathbf{Y}_{n-1}\mathbf{C}_n\|^2}{1+\alpha_n^2}. \tag{13}$$

After some manipulations, we notice that

$$f(\alpha_n, \mathbf{C}_n) = f_1(\alpha_n) - f_2(\alpha_n) f_3(\mathbf{C}_n), \tag{14}$$

where

$$f_1(\alpha_n) \triangleq 2n_R \log(1+\alpha_n^2) + \frac{\|\mathbf{Y}_n\|^2 + \alpha_n^2\|\mathbf{Y}_{n-1}\|^2}{1+\alpha_n^2}, \tag{15}$$

$$f_2(\alpha_n) \triangleq \frac{2\alpha_n}{1+\alpha_n^2}, \tag{16}$$

$$f_3(\mathbf{C}_n) \triangleq \Re\big\{ \mathrm{tr}(\mathbf{Y}_n^H \mathbf{Y}_{n-1}\mathbf{C}_n)\big\}. \tag{17}$$

Equation (14) indicates that the decoding of α_n and $\mathbf{C}_n$ is decoupled. In particular, we can first decode $\mathbf{C}_n$ by maximizing $f_3(\mathbf{C}_n)$, and then substitute the maximizing $\mathbf{C}_n$ back into (14) to decode α_n.

Therefore, we first decode the phase matrix by maximizing $f_3(\mathbf{C}_n)$ over all possible phase matrices. Since $\mathbf{C}_n$ is obtained by orthogonal designs, the decoding process can be further simplified. Specifically, it is easy to see that maximizing $f_3(\mathbf{C}_n)$ is equivalent to minimizing

$$f_3'(\mathbf{C}_n) = \|\mathbf{Y}_n - \mathbf{Y}_{n-1}\mathbf{C}_n\|^2. \tag{18}$$

Let $\mathbf{y}_{n,1}$ and $\mathbf{y}_{n,2}$ be the first and second column of $\mathbf{Y}_n$, and $\mathbf{y}_{n-1,1}$ and $\mathbf{y}_{n-1,2}$ are similarly defined for $\mathbf{Y}_{n-1}$. We can write f_3' as follows (see (2))

$$\begin{aligned} & f_3'(c_{n,1}, c_{n,2}) \\ =& \|\mathbf{y}_{n,1} - c_{n,1}\mathbf{y}_{n-1,1} - c_{n,2}\mathbf{y}_{n-1,2}\|^2 \\ &+ \|\mathbf{y}_{n,2} + c_{n,2}^*\mathbf{y}_{n-1,1} - c_{n,1}^*\mathbf{y}_{n-1,2}\|^2 \\ =& \|\mathbf{y}_{n,1} - c_{n,1}\mathbf{y}_{n-1,1} - c_{n,2}\mathbf{y}_{n-1,2}\|^2 \\ &+ \|\mathbf{y}_{n,2}^* - c_{n,1}\mathbf{y}_{n-1,2}^* + c_{n,2}\mathbf{y}_{n-1,1}^*\|^2 \\ =& \left\|\tilde{\mathbf{y}}_n - \tilde{\mathbf{Y}}_{n-1}\mathbf{c}_n\right\|^2, \end{aligned} \tag{19}$$

where in the second equality, we took the conjugation of the second term, which does not affect the norm, and in the third, we used the following definitions

$$\tilde{\mathbf{y}}_n \triangleq \begin{bmatrix}\mathbf{y}_{n,1}^T, & \mathbf{y}_{n,2}^H\end{bmatrix}^T, \tag{20}$$

$$\mathbf{c}_n \triangleq \begin{bmatrix}c_{n,1}, & c_{n,2}\end{bmatrix}^T, \tag{21}$$

$$\tilde{\mathbf{Y}}_{n-1} \triangleq \begin{bmatrix}\mathbf{y}_{n-1,1} & \mathbf{y}_{n-1,2} \\ \mathbf{y}_{n-1,2}^* & -\mathbf{y}_{n-1,1}^*\end{bmatrix}. \tag{22}$$

It is ready to verify that $\tilde{\mathbf{Y}}_{n-1}$ has orthogonal columns with $\tilde{\mathbf{Y}}_{n-1}^H\tilde{\mathbf{Y}}_{n-1} = (\|\mathbf{y}_{n-1,1}\|^2 + \|\mathbf{y}_{n-1,2}\|^2)\mathbf{I}_2$. Therefore, the the phase angles of $\mathbf{c}_n$, i.e., $\boldsymbol{\theta}_n \triangleq \arg(\mathbf{c}_n)$, can be estimated by computing $\arg\left(\tilde{\mathbf{Y}}_{n-1}^H\tilde{\mathbf{y}}_n\right)$ followed by rounding to the nearest multiple of $2\pi/M$. Clearly, the decoding of $c_{n,1}$ and $c_{n,2}$ is decoupled.

Once we have the decoded symbols $\hat{c}_{n,1}$ and $\hat{c}_{n,2}$, we use them to form $\hat{\mathbf{C}}_n$, substitute it back to (14) and decode α_n as follows:

$$\hat{\alpha}_n = \arg\min_{\alpha_n\in\{1,\gamma,1/\gamma\}} f(\alpha_n, \hat{\mathbf{C}}_n). \tag{23}$$

3. NUMERICAL RESULTS

We consider a system equipped with $n_T = 2$ transmit antennas and $n_R = 1$ receive antenna. The underlying channel is flat Rayleigh fading, i.e., the channel coefficients in $\mathbf{H}$ are generated as i.i.d. complex Gaussian variables with zero mean and unit variance, varying independently from trial to trial. We consider two differential space-time coding (DSTC) schemes, namely the proposed one based on generalized multi-channel amplitude-phase modulation, referred to as *DSTC/Amplitude-Phase*, and the one based on only

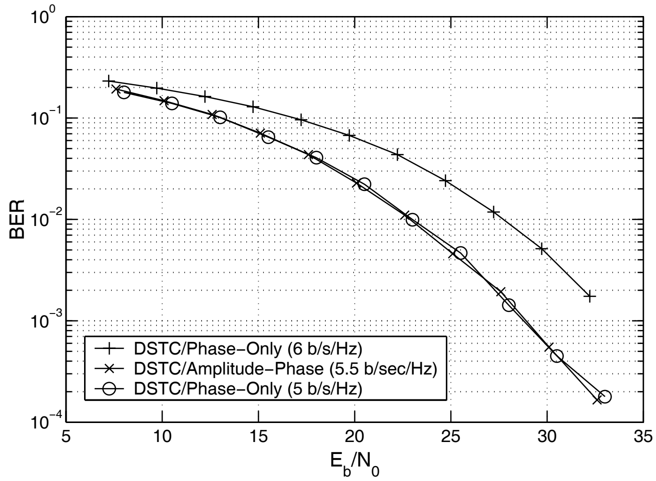

Fig. 2. Bit error rate (BER) as a function of E_b/N_0 for a phase-only and the proposed amplitude-phase differential space-time modulation scheme in Rayleigh fading channels when $n_T = 2$ and $n_R = 1$.

phase modulation [1], referred to as *DSTC/Phase-Only*. Due to the additional amplitude bit used in our scheme, we cannot match the data rate for both schemes exactly. Instead, we compare the two schemes for the nearest possible data rates. The performance measure is the bit error rate (BER) as a function of E_b/N_0, where E_b denotes the total energy per bit used in the transmission. Figures depicts the BER of the DSTC/Phase-Only scheme built from 64PSK and 32PSK constellations with the associated data rate of 6 bits/sec/Hz and 5 bits/sec/Hz, respectively. Also shown there is the BER of the proposed DSTC/Amplitude-Phase scheme built from 2ASK and 32PSK with a rate of 5.5 bits/sec/Hz. The 2ASK uses $\gamma = 1.3$, which was found to provide good performance for our scheme. It is seen that the proposed scheme achieves almost identical BER to the lower-rate DSTC/Phase-only with rate 5 bits/sec/Hz, and significantly outperforms the higher-rate DSTC/Phase-only with rate 6 bits/sec/Hz. Also noted is that all schemes yield full spatial diversity.

4. CONCLUSIONS

We have presented a differential space-time coding scheme based on generalized multi-channel amplitude and phase modulation. We have shown that the proposed scheme admits decoupled decoding of the amplitude coefficient and phase matrix, as well as decoupled decoding of the phase entries of the phase matrix, given that the latter is formed by orthogonal designs. The proposed amplitude-phase differential space-time coding scheme achieves a performance very close to its counterpart based only on phase modulation, while offering higher spectral efficiency provided by amplitude modulation. Although we only discussed the case with $n_T = 2$ transmit antennas. Extension of the proposed scheme to arbitrary n_T can be made by following a procedure in in [11].

5. REFERENCES

[1] V. Tarokh and H. Jafarkhani, "A differential detection scheme for transmit diversity," *IEEE Journal on Selected Areas in Communications*, vol. 18, no. 7, pp. 1169–1174, July 2000.

[2] B. M. Hochwald and W. Sweldens, "Differential unitary space-time modulation," *IEEE Transactions on Communications*, vol. 48, no. 12, pp. 2041–2052, December 2000.

[3] B. L. Hughes, "Differential space-time modulation," *IEEE Transactions on Information Theory*, vol. 46, no. 7, pp. 2567–2578, November 2000.

[4] G. Ganesan and P. Stoica, "Differential detection based on space-time block codes," *Wireless Personal Communications*, vol. 21, pp. 163–180, 2002.

[5] S. N. Diggavi, N. Al-Dhahir, A. Stamoulis, and A. R. Calderbank, "Differential space-time coding for frequency-selective channels," *IEEE Communications Letters*, vol. 6, no. 6, pp. 253–255, June 2002.

[6] H. Li, "Differential space-time-frequency modulation over frequency-selective fading channels," *IEEE Communications Letters*, vol. 7, no. 8, pp. 349–351, August 2003.

[7] F. Adachi and M. Sawahashi, "Decision feedback differential detection of differentially encoded 16APSK signals," *IEEE Transactions on Communications*, vol. 44, no. 4, pp. 416–418, April 1996.

[8] M. Tao and R. S. Cheng, "Differential space-time block codes," in *Proceedings of the IEEE GLOBECOM*, San Antonio, TX, November 2001, pp. 1098–1102.

[9] X. G. Xia, "differentially en/decoded orthogonal space-time block codes with APSK signals," *IEEE Communications Letters*, vol. 6, no. 4, pp. 150–152, April 2002.

[10] S. M. Alamouti, "A simple transmit diversity techniques for wireless communications," *IEEE Journal on Selected Areas in Communications*, vol. 16, no. 8, pp. 1451–1458, October 1998.

[11] H. Li, "Differential space-time modulation with full spatio-spectral diversity and arbitrary number of transmit antennas in ISI channels," in *Proceedings of the 2003 IEEE International Conference on Acoustics, Speech, and Signal Processing (ICASSP'2003)*, Hong Kong, April 2003, vol. 4, pp. IV37–IV40.

[12] V. Tarokh, H. Jafarkhani, and A. R. Calderbank, "Space-time block codes from orthogonal designs," *IEEE Transactions on Information Theory*, vol. 45, no. 5, pp. 1456–1467, July 1999.