TRANSMIT/RECEIVE MIMO ANTENNA SUBSET SELECTION

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ABSTRACT

This paper discusses antenna subset selection in MIMO wireless systems. The subsets of transmit and receive antennas are selected so as to maximise the channel capacity. First of all, we establish the relationship between the multiplexing gain and the diversity gain achievable with adaptive antenna subset selection. Second, we indicate a selection rule that allows to achieve a full diversity advantage with a reduced computational effort, by decoupling the combined transmit/receive selection into the separate selection of transmit/receive subsets. Finally, we study the performance of practical systems with antenna selection in the context of high throughput MIMO-OFDM WLAN.

1. INTRODUCTION

Multi-input multi-output (MIMO) antennas improve the performance of wireless links by increasing reliability via space-time coding or by increasing data rate through spatial multiplexing techniques. A major impediment in deploying multiple antennas is the cost of the hardware associated with each antenna (power amplifiers, A/D converters etc). Antenna subset selection with transmission and/or reception performed through a selection of the total available antennas is a powerful solution that reduces the need for many analogue chains yet retains most of the diversity benefits.

Recently, there has been increasing interest in applying antenna selection to MIMO links. Several algorithms for antenna subset selection have been designed [1, 2, 3, 4] based on various performance criteria such as Shannon capacity of the MIMO channel, error rate performance etc. Some results on performance analysis of MIMO antenna subset selection are available [5, 4, 6, 7].

The main focus of this contribution is the analysis of antenna subset selection on both sides of the link. Although mentioned by a number of contributions cited above, the problem of combined transmit/receive MIMO antenna selection has received little attention. Particularly, there has been not enough insight in the statistical properties (*e.g.*, diversity) of MIMO systems with combined selection as well as in the complexity issues.

In the following sections, we explain the fundamental relationship between the multiplexing gain and the diversity gain of MIMO systems with combined antenna selection. Next we indicate a suboptimal combined selection rule which decouples the combined selection into separate transmit and receive selection. This allows us to reduce the computational effort for moderate and big number of antennas, without giving up diversity gain. Finally, we show simulation results for a practical MIMO-OFDM system devised for high throughput WLAN (IEEE 802.11n) which makes use of antenna selection.

Notation. All vectors and matrices are in bold font. $\boldsymbol{X}^T$ and $\boldsymbol{X}^H$ stand for the transpose and the Hermitian conjugate of $\boldsymbol{X}$, respectively. Also $\boldsymbol{X}_{a,c}$ stands for the element of $\boldsymbol{X}$ in the a-th row and the c-th column, $\boldsymbol{X}_{a:b,c:d}$ stands for a block spanned by rows a through b and columns c through d, and (:) alone stands for the whole scope of indices (*e.g.*, $\boldsymbol{X}_{:,c}$ is the c-th column of $\boldsymbol{X}$).

2. DATA MODEL

We assume a MIMO point-to-point system with M_T transmit and M_R receive antennas. The channel is assumed to be frequency flat. The $M_R \times M_T$ channel matrix is denoted by $\boldsymbol{H}$. The signal model is

$$\boldsymbol{x}[k] = \sqrt{\rho}\,\boldsymbol{H}\boldsymbol{s}[k] + \boldsymbol{n}[k], \qquad (1)$$

where the $M_R \times 1$ vector $\boldsymbol{x}[k] = [\boldsymbol{x}_1[k], \ldots, \boldsymbol{x}_{M_R}[k]]^T$ represents the k-th sample of signals collected at the outputs of M_R receive antennas and sampled at the symbol rate, $\boldsymbol{s}[k] = [\boldsymbol{s}_1[k], \ldots, \boldsymbol{s}_{M_T}[k]]^T$ is the vector of M_T symbols transmitted by the transmit antennas, ρ is the average signal-to-noise ratio (SNR) per transmit/receive antenna and per channel use and $\boldsymbol{n}[k] = [\boldsymbol{n}_1[k], \ldots, \boldsymbol{n}_{M_R}[k]]^T$ is additive white Gaussian noise (AWGN) with energy (1/2) per real dimension. In the future analysis, we will assume perfect channel state information (CSI) at the receiver and no CSI at the transmitter. We also assume that symbols of different transmit antennas are uncorrelated and have unit power: $\mathbb{E}\{\boldsymbol{s}[k]\boldsymbol{s}[k]^H\} = \boldsymbol{I}_{M_T}$. With these assumptions, the capacity $C(\boldsymbol{H}, \rho)$ of a deterministic MIMO channel (1) in bits per channel use equals the mutual information between $\{\boldsymbol{s}[k]\}$ and $\{\boldsymbol{x}[k]\}$ maximised over the distribution of $\{\boldsymbol{s}[k]\}$:

$$C(\boldsymbol{H}, \rho) \;=\; \log_2 \det(\; \boldsymbol{I}_{M_T} \;+\; \rho\, \boldsymbol{H}^H \boldsymbol{H} \;), \tag{2}$$

where $\det(\;\cdot\;)$ stands for determinant and $\boldsymbol{I}_m$ is the $m \times m$ identity matrix. Later in this paper, we will consider $\boldsymbol{H}$ to be a realisation of a MIMO uncorrelated Rayleigh fading such that the entries of $\boldsymbol{H}$ are *i.i.d.* complex circular Gaussian with zero mean and variance $(1/2)$ per real dimension.

3. DIVERSITY VERSUS MULTIPLEXING

In this section, we recall the fundamental relationship between multiplexing gain and diversity gain in MIMO systems [8]. Next, we extend this result to the case with transmit/receive MIMO subset selection.

3.1. No antenna selection: known results

The notion of tradeoff between multiplexing gain and diversity gain in MIMO systems has been introduced in [8] to express the relationship between the rate achieved by a MIMO transmission scheme and its associated reliability. We will consider a family of MIMO transmission schemes (codes) corresponding to different SNR levels, that can sustain a rate $R(\rho)$ with a probability $(1 - P_{out}(\rho))$, where $P_{out}(\rho)$ is the *outage* probability. The respective multiplexing gain r and diversity gain d are defined as follows:

$$r \;=\; \lim_{\rho\to\infty} \frac{R(\rho)}{\log_2 \rho}, \qquad d \;=\; -\lim_{\rho\to\infty} \frac{\log P_{out}(\rho)}{\log \rho}. \tag{3}$$

Intuitively, a well designed MIMO transmission scheme will compromise between the achievable rate and the reliability (outage). Hence a natural subject of interest is the achievable region of the pairs (r, d) or equivalently the maximum diversity gain $d(r)$ for a given multiplexing gain r. Since the maximum achievable data rate is given by the capacity in (1), the maximum achievable diversity gain $d_\bullet(r)$ subject to a fixed multiplexing gain satisfies

$$d_\bullet(r) \;=\; -\lim_{\rho\to\infty} \frac{\log \mathbb{P}\left\{C(\boldsymbol{H}, \rho) < r \log_2 \rho\right\}}{\log \rho}, \tag{4}$$

where $\mathbb{P}\{\cdot\}$ stands for the probability of the event $(\cdot)$. The authors of [8] showed that for uncorrelated MIMO flat Rayleigh fading channel, the maximum achievable diversity gain $d_\bullet(r)$ is the piecewise linear function of r connecting the points $(m,\, d_\bullet(m))$, $0 \le m \le \min\{M_T, M_R\}$, where

$$d_\bullet(m) \;=\; (M_R - m)\,(M_T - m). \tag{5}$$

3.2. Transmit/receive antenna selection

We will assume a MIMO system equipped with M_T transmit and M_R receive antennas whereas the numbers of actual transmit/receive chains is N_T and N_R respectively. By successively sounding all the transmit/receive antennas with the available transmit/receive chains, the receiver can estimate the channel matrix $\boldsymbol{H}$ of the full system. However, only N_T transmit and N_R receive antennas may be used at once. The preferred set of N_T transmit antennas may be communicated via a low-rate feedback link. As we aim at the maximum channel capacity, we search for the $N_R \times N_T$ block of $\boldsymbol{H}$ that maximises the capacity:

$$C(\underline{\boldsymbol{H}}, \rho) \;=\; \log_2 \det(\; \boldsymbol{I}_{N_T} \;+\; \rho\, \underline{\boldsymbol{H}}^H \underline{\boldsymbol{H}} \;), \tag{6}$$

over the set $\underline{\boldsymbol{H}} \in \mathbb{H}_{N_R,N_T}(\boldsymbol{H})$ of all $N_R \times N_T$ blocks of $\boldsymbol{H}$. This maximum capacity may be written as

$$C_\star(\boldsymbol{H}, \rho) \;=\; \sup\{\, C(\underline{\boldsymbol{H}}, \rho) :\; \underline{\boldsymbol{H}} \in \mathbb{H}_{N_R,N_T}(\boldsymbol{H}) \,\}. \tag{7}$$

We will extend (5) to the described model of antenna selection. According to (7), we have

$$d_\star(r) \;=\; -\lim_{\rho\to\infty} \frac{\log \mathbb{P}\left\{C_\star(\boldsymbol{H}, \rho) < r \log_2 \rho\right\}}{\log \rho}. \tag{8}$$

Note that $C_\star(\boldsymbol{H}, \rho)$ is the maximum of $C(\underline{\boldsymbol{H}}, \rho)$ over all $\binom{M_R}{N_R}\binom{M_T}{N_T}$ blocks $\underline{\boldsymbol{H}}$ of $\boldsymbol{H}$.

Let us specify a simplified selection rule which is based on two steps. At the first step, N_T transmit antennas are selected while keeping all M_R receive antennas. At the second step, N_R receive antennas are selected, with the fixed subset of N_T transmit antennas. The selection at both steps is based on the maximum capacity criterion. The selection algorithm is stated below.

ALGORITHM I
DECOUPLED TX/RX SELECTION

Find a set $\mathcal{I}_T$ of N_T transmit antennas through the maximisation over all possible subsets $\mathcal{I} \subset \{1, \ldots, M_T\}$ of size N_T:

$$\mathcal{I}_T \;=\; \arg\max_{\mathcal{I}} \det(\; \boldsymbol{I}_{N_T} \;+\; \rho\, \boldsymbol{H}^H_{:,\mathcal{I}} \boldsymbol{H}_{:,\mathcal{I}} \;).$$

Find a set $\mathcal{I}_R$ of N_R receive antennas through the maximisation over all possible subsets $\mathcal{I} \subset \{1, \ldots, M_R\}$ of size N_R:

$$\mathcal{I}_R \;=\; \arg\max_{\mathcal{I}} \det(\; \boldsymbol{I}_{N_T} \;+\; \rho\, \boldsymbol{H}^H_{\mathcal{I},\mathcal{I}_T} \boldsymbol{H}_{\mathcal{I},\mathcal{I}_T} \;).$$

Note that, due to the symmetry of the capacity criterion w.r.t. rows and columns of $\boldsymbol{H}$, the order of transmit/receive antenna selection may be exchanged.

One can notice a substantial reduction in the computation effort w.r.t. the exhaustive search. Indeed, Algorithm I requires computing $\binom{M_R}{N_R} + \binom{M_T}{N_T}$ determinants of size $N_T \times N_T$ matrices, instead of $\binom{M_R}{N_R}\binom{M_T}{N_T}$ determinants of the same size for the exhaustive search. Further simplifications may be achieved by replacing the exhaustive search of transmit or/and receive selection by a recursive maximisation of the determinant as explained in [7]. Moreover, extensions to the frequency selective fading scenario may be obtained as indicated in [3].

The analysis of the diversity gain achievable with Algorithm I is much similar to the analysis presented in [9]. In this paper, we briefly sketch the main steps of the proof. First of all, we make use of similar arguments to those in [9], to establish the following result.

Lemma 1 *Under the assumption presented in section 2, the capacity $C_\star(\boldsymbol{H}, \rho)$ achieved with optimal antenna selection satisfies*

$$C_\star(\boldsymbol{H}, \rho) \geq C_o(\boldsymbol{\Lambda}, \rho) \, - \log_2 \binom{M_T}{N_0} \, - \log_2 \binom{M_R}{N_0},$$

$$C_o(\boldsymbol{\Lambda}, \rho) = \sum_{k=1}^{N_0} \log_2 \left(1 + \rho \, \boldsymbol{\Lambda}_k^2 \right). \tag{9}$$

The result of this Lemma is established by the arguments used to prove Lemma 1 and Lemma 2 in [9]. The difference lies in the fact that we assume Algorithm I as a basis for the lower bound (9) in this paper.

Similarly to Lemma 3 in [9], one can show that (8) and (9) lead to a lower bound $d_\circ(r)$ on the diversity gain $d_\star(r)$:

$$d_\star(r) \geq d_\circ(r) = - \lim_{\rho \to \infty} \frac{\log \mathbb{P}\left\{ C_o(\boldsymbol{\Lambda}, \rho) < r \log_2 \rho \right\}}{\log \rho}.$$

The remainder of the proof consists of applying the Laplace principle to the p.d.f. of $C_o(\boldsymbol{\Lambda}, \rho)$ at high SNR limit. By reiterating the arguments from [8], one can show that $d_\circ(r) = d_\bullet(r)$. Furthermore, $d_\circ(r) \leq d_\star(r) \leq d_\bullet(r)$. Hence

Theorem 1 *Under the assumptions presented in section 2, $d_\star(r)$ is the piecewise linear function of r connecting the points $(m,\, d_\star(m))$, $0 \leq m \leq \min\{M_T, M_R\}$, where*

$$d_\star(m) \, = \, (M_R - m)\,(M_T - m). \tag{10}$$

In other words, a MIMO system which selects $N_T \leq M_T$ out of M_T transmit antennas and $N_R \leq M_R$ out of M_R receive antennas, achieves the same diversity gains as a system that makes use of all M_T and M_R receive antennas.

It is worthwhile reiterating that the lower bound from Lemma 1 is based on the suboptimal antenna selection rule stated in Algorithm I. Hence the conclusion of the Theorem 1 also carries over to the results of Algorithm I:

Corollary 1 *Under the assumption presented in section 2, the suboptimal antenna subset selection rule stated in Algorithm I achieves the same diversity gain $d_\star(r)$ as the optimal subset selection.*

Clearly, this fundamental result assumes optimal encoding and decoding procedures that may be not suitable for practical systems. In the following section, we will see how this analysis may be used to evaluate the diversity gain of practical MIMO systems.

4. APPLICATION TO MIMO WLAN SYSTEMS

The discussed MIMO-OFDM transmitter and receiver are shown in Fig. 1. Such a transceiver performs spatial multiplexing over N_T transmit antennas in order to increase the data rate by a factor of N_T compared to the standard 802.11a/g systems. At the receiver, the original data stream is reconstructed from N_R received signals. When antenna subset selection is used, these N_T transmit (N_R receive) antennas are selected from the total M_T transmit (M_R receive) available antennas. Antenna selection algorithms will be applied to the main tap of the (frequency selective) $M_R \times M_T$ MIMO channel. This approach is justified in typical WLAN environments since the RMS delay spread is often less than 10ns, for the total signal bandwidth of 20MHz. At the transmitter, user bits are encoded by the standard (133_8,171_8) convolutional FEC code with coding rate $3/4$ achieved through puncturing. The coded bits are distributed in a round Robin fashion between the N_T transmit streams. Next, the standard frequency interleaving scheme is applied to every stream. In Fig. 1, these operations are carried out by the space-frequency interleaver. The sequences of interleaved bits are mapped into N_T sequences of 64QAM symbols and further transmitted via N_T antennas after the OFDM modulation as per 802.11a/g.

At the receiver, the captured signals are sampled and, after frequency and timing recovery, mapped to the frequency domain. In a MIMO-OFDM system, the N_R received signals at each subcarrier are instantaneous mixtures of the N_T symbols transmitted at this subcarrier. In this paper, we assume that the N_T symbols are retrieved from their N_R noisy mixtures by the optimal linear (MMSE) filter at every subcarrier. The set of N_T signal-to-interference-and-noise ratio (SINR) values at the respective outputs of the filter are also computed and subsequently used in the soft demapper along with the output signals of the filter. The details on the computing of MMSE filters and output SINR values can be found in *e.g.*, [3].

In the remainder of this paper, we set the number of transmit/receive data streams to $N_T = N_R = 2$. We simulate the system in Fig. 1 with the optimal combined transmit receive antenna selection specified in (7) as well as the suboptimal selection defined by Algorithm I.

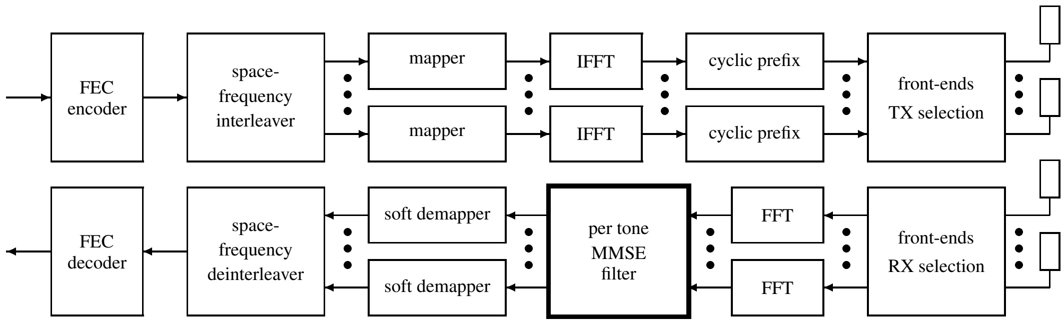

Fig. 1. MIMO-OFDM transceiver: block diagram.

In Fig.1, we plot packet error rate (PER) versus the average SNR per receive antenna for optimal and suboptimal antenna selection, for different values of M_T and M_R. Note that PER will approximate the outage rate in a high SNR mode and/or with a powerful FEC. According to the definition (3), the slope of the PER curve in the log-scale will approach the diversity gain d in high SNR region. It is possible to show that for a MIMO transceiver with MMSE multiplexing and $N_T = 2$, the multiplexing gain satisfies $1 \leq r \leq 2$, with the lower limit ($r = 1$) attained at high SNR and vise versa. This observation is due to the fact that the output SINR of the MMSE filter is dominated by the worst eigenvalue of the channel matrix (see *e.g.*, [7]) which is the N_T-th ordered eigenvalue (non-increasing order) corresponding to the multiplexing gain $r = (N_T - 1)$ (note that $N_T = 1$ corresponds to $r = 0$ for a fixed rate and high SNR limit). Hence we can apply (10) with $m = 1$ to compute the diversity gains, which gains should resemble slopes of the respective PER curves at high SNR. We find $d(1) = 1$ for $M_T = M_R = 2$, $d(1) = 2$ for $M_T = 2$ and $M_R = 3$, $d(1) = 3$ for $M_T = 2$ and $M_R = 4$, $d(1) = 4$ for $M_T = M_R = 3$ and finally $d(1) = 9$ for $M_T = M_R = 4$. Check that these numbers correspond to the slopes in Fig.1. Also note that the suboptimal selection produces the same slopes as the optimal selection, with a slight loss in SNR.

5. REFERENCES

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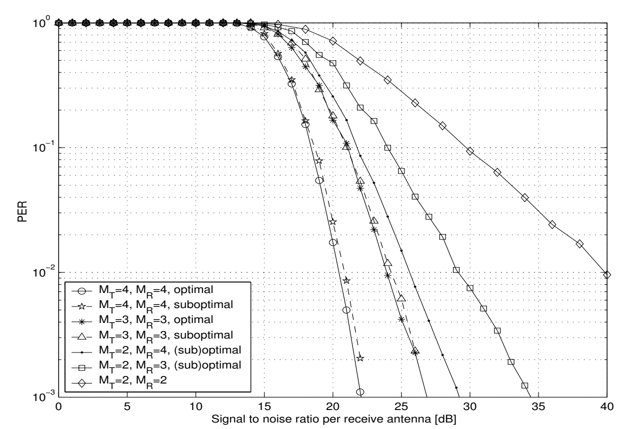

Fig.1. Packet error rates versus SNR per TX/RX antenna: 108Mbps, 1000 bytes/packet, $N_T = N_R = 2$.