OUTAGE PROBABILITY OF MULTI-CELLULAR MIMO SYSTEMS IN RAYLEIGH FADING

Yeliz Tokgoz and Bhaskar D. Rao

University of California, San Diego
akyildiz@ece.ucsd.edu, brao@ece.ucsd.edu

ABSTRACT

In this paper, we analyze the performance of multi cellular MIMO systems in Rayleigh fading. Consistent with practical scenarios, we assume two types of interference: intracell interference from users within the same cell as the desired user and intercell interference from outer cell users. We derive a compact closed form expression for the outage probability of such a system in the form of finite sums. The expression is easily computable and allows for simpler and faster study of various MIMO configurations. An interesting outcome of the analysis is that using antennas on the receiver side results in better performance since transmit diversity does not combat interference from same cell users.

1. INTRODUCTION

In wireless communication systems, system performance can be significantly improved by employing multiple antennas at the receiver and/or the transmitter, through combining the signals received from the multiple independent channels to mitigate fading and suppress interference. Although initial interest in smart antennas has mainly focused on receiver diversity, nowadays multiple input multiple output (MIMO) systems with both transmit and receive diversity are receiving a lot of attention.

Maximum Ratio Transmission (MRT) [1] is a popular and simple scheme that maximizes the system output signal-to-noise ratio (SNR) in MIMO systems. In [2], the performance of MRT is analyzed in Rayleigh fading and an average symbol error rate expression is provided. However, the impact of co-channel interference (CCI) is not considered. In the presence of CCI, the optimal strategy is to choose the transmission and receiver weights to maximize the system signal-to-interference plus noise ratio (SINR), thereby achieving interference suppression. However, this optimal technique does not provide significant performance improvement over MRT when the number of interferers is large, since diversity order is insufficient to cancel out all interferers. In such systems, MRT is preferred because of its implementation simplicity and near optimal performance.

The performance of MRT with equal power co-channel interferers is analyzed in [3], and an outage probability expression is provided. However in practical systems, unless the users are power controlled by the same station (BS), their received powers would not be the same. Besides, users choose their transmission weights according to the channel between them and their power controlling BS, which is the same as the desired user's BS for same cell users, but different for other cell users. Hence, for accurate characterization of CCI, one needs to consider a multi-cellular system, where the CCI from interferers within the same cell (intracell) and from other cells (intercell) are treated separately due to their different statistical characterizations.

In this paper, we extend the analysis of MIMO MRT systems to a multicellular environment, consistent with practical systems. We consider fixed number of equal power intracell interferers and intercell interferers with distinct powers. We obtain a simple closed form outage probability expression in the form of finite sums for arbitrary number of antennas. We then use these expressions to analyze the performance improvements achieved by increasing the number of antennas at the receiver and transmitter sides.

The paper is organized as follows: The system model is introduced in Section 2. In Section 3, the outage probability is derived. Section 4 contains the numerical results.

2. SYSTEM MODEL

We consider the uplink of a multi-cellular communication system with t transmit antennas at the mobile and r receive antennas at the BS. The system operates in the presence of thermal noise and multi-access interference, which consists of two components: intracell interference from users power controlled by the same BS as the desired user and intercell interference from users in other cells. The channel is spatially independent flat Rayleigh fading provided that the antenna spacing is sufficiently large. It is also assumed to be slowly-varying so that the fading coefficients remain unchanged over the frame, allowing a quasi-static analysis.

This work was supported by CoRe research grants Cor00-10074 and Com02-10119.

2.1. Received Signal

The $r \times 1$ received signal vector, $\mathbf{r}(t)$, consists of components from the desired user, N_{int} intracell users, N_{ext} intercell users and thermal noise.

$$\mathbf{r}(t) = \sqrt{P_0}\; H_{0,0}\, \mathbf{w}_{0,0}^t\, s_0(t) + \sum_{n=1}^{N_{int}} \sqrt{P_n}\; H_{n,0}\, \mathbf{w}_{n,0}^t\, s_n(t) + \sum_{k=1}^{N_{ext}} \sqrt{P_k}\; H_{k,0}\, \mathbf{w}_{k,m_k}^t\, s_k(t) + \mathbf{n}(t) \tag{1}$$

The user power levels are denoted by P_i, where i is the user index and index 0 corresponds to the desired user. Similarly, s_i corresponds to the information bits with zero-mean and unit variance. The $r \times 1$ noise vector $\mathbf{n}$ is complex white Gaussian with zero-mean and covariance $\sigma^2 I$. $\mathbf{w}_{i,j}^t$ represents the weight vector at the transmitter of the i-th mobile which is power controlled by the j-th BS, where $||\mathbf{w}_{i,j}^t|| = 1$. The $r \times t$ channel gain matrix between user i and BS j is denoted by $H_{i,j}$, and consists of independent complex Gaussian distributed elements with zero mean and unit variance, $\mathcal{CN}(0, 1)$. The received signals at the multiple antennas are combined with weight vector $\mathbf{w}_0^r$ to form the decision statistic $y(t) = (\mathbf{w}_{0,0}^r)^H\, \mathbf{r}(t)$, where $||\mathbf{w}_{0,0}^r|| = 1$.

2.2. Transmission and Receiver Weights

We choose the transmit and receive weight vectors to maximize the overall output SNR of the system (MRT) [1]. Implementing MRT is a practical choice for systems with large number of interferers, since using the optimal weights that maximize the output SINR provides very little performance gain over MRT due to insufficient diversity order, at the cost of increased complexity. Another advantage of MRT is that it does not require the mobiles to have full knowledge of the uplink channel to determine the transmitter weights. Only the largest right singular vector of the channel matrix is required. Since channel information is usually sent through a feedback channel, it means that less information has to be sent which can consequently be done more frequently or with more accuracy at the same feedback rate.

It has been shown that the MRT transmit and receive weight vectors are respectively the left and right singular vectors of the channel matrix H that corresponds to the largest singular value. The singular value decomposition of $H_{i,j}$ (channel matrix between user i and BS j) of rank m is given by $H_{i,j} = \sum_{k=1}^{m} \sigma_{i,j}^k\;\; \mathbf{u}_{i,j}^k\, (\mathbf{v}_{i,j}^k)^H$, where $\sigma_{i,j}^k$, $\mathbf{u}_{i,j}^k$, $\mathbf{v}_{i,j}^k$ are the singular values (in descending order) and the left and right singular vectors respectively. We note that the left and right singular vectors have the same distribution as normalized complex Gaussian random vectors [4, 5].

Users choose their transmit weight vectors according to the channel between them and the BS that they are power controlled by ($\mathbf{w}_{i,j}^t = \mathbf{v}_{i,j}^1$). The receive weight vector is chosen to coherently combine the signals from the desired user ($\mathbf{w}_{0,0}^r = \mathbf{u}_{0,0}^1$). Using these weights, the decision statistic ($y(t) = (\mathbf{w}_{0,0}^r)^H \cdot \mathbf{r}(t)$) can be expressed as

$$y(t) = \sqrt{P_0}\;\, \sigma_{0,0}\, s_0(t) + \sum_{n=1}^{N_{int}} \sqrt{P_n}\;\, \sigma_{n,0}\, (\mathbf{u}_{0,0}^1)^H\, \mathbf{u}_{n,0}^1\, s_n(t) + \sum_{k=1}^{N_{ext}} \sqrt{P_k}\;\, (\mathbf{u}_{0,0}^1)^H\, H_{k,0}\, \mathbf{v}_{k,m_k}^1\, s_k(t) + \mathbf{n}(t)$$

The output SINR is given as $\dfrac{P_0\, \lambda_0}{I_{int} + I_{ext} + \sigma^2}$, where

$$I_{int} = \sum_{n=1}^{N_{int}} P_n\, \lambda_n\, |(\mathbf{u}_{0,0}^1)^H\, \mathbf{u}_{n,0}^1|^2 \tag{2}$$

$$I_{ext} = \sum_{k=1}^{N_{ext}} P_k\, |(\mathbf{u}_{0,0}^1)^H\, H_{k,0}\, \mathbf{v}_{k,m_k}^1|^2 \tag{3}$$

$\lambda_i = |\sigma_{i,0}^1|^2$ denotes the largest eigenvalue of the complex Wishart matrix [4] $H_{i,0}^H\, H_{i,0}$ of user i.

3. OUTAGE PROBABILITY

Outage probability is one of the most common performance measures in wireless communication systems. It is defined as the probability that the system SINR falls below a certain quality of service threshold, ϵ. Since I_{int}, I_{ext} and λ_0 are statistically independent (discussed in more detail in Section 3.3), this probability can be written in integral form as

$$P_{out}(\epsilon) = \iint f_{I_{int}}(z)\, f_{I_{ext}}(y)\, F_{\lambda_0}\left(\frac{\epsilon\,(z + y + \sigma^2)}{P_0}\right) dy\, dz \tag{4}$$

where $f_{I_{int}}(z)$ and $f_{I_{ext}}(y)$ are the density functions of intracell and intercell interference respectively. $F_{\lambda_0}(x)$ denotes the cumulative distribution function (cdf) of the largest eigenvalue of the Wishart matrix $H_{0,0}^H\, H_{0,0}$. In order to compute the outage probability, we need to identify these distributions.

3.1. Largest Eigenvalue of a Wishart matrix

The distribution of the largest eigenvalue (λ) of a complex Wishart matrix is provided in [2] as a finite linear combination of elementary gamma densities. Hence, the cdf of λ can be expressed as [6]

$$F_\lambda(a) = \sum_{n=1}^{\min\{t,r\}} \sum_{m=|t-r|}^{(t+r)n-2n^2} \frac{d_{m,n}}{m!} \gamma(m+1, na) \tag{5}$$

The exact values of the coefficients of the summands, $d_{m,n}$, are computed and tabulated in [2] for most antenna configurations of interest. For $a > 0$, $\gamma(a, x)$ is the incomplete gamma function defined as

$$\gamma(a, x) = \int_0^x e^{-t}\, t^{a-1}\, dt = (a-1)! \left[1 - e^{-x} \sum_{i=0}^{a-1} \frac{x^i}{i!}\right]$$

The above finite sum representation of $\gamma(a, x)$ is valid when a is an integer. The k-th order moment of λ is given as

$$E[\lambda^k] = \sum_{n=1}^{\min\{t,r\}} \sum_{m=|t-r|}^{(t+r)n-2n^2} \frac{d_{m,n}(m+k)!}{n^k\, m!} \qquad (6)$$

3.2. Intracell Interference

We first analyze the distribution of the intracell interference term (2). Since all intracell interferers are power controlled by the desired user's BS, their average received power levels are assumed to be the same and equal to P_I. Since the vectors $\mathbf{u}_{i,0}^1$ (first left singular vectors of $H_{i,0}$) have the same distribution as normalized complex Gaussian random vectors of size $r \times 1$, the term $|(\mathbf{u}_{0,0}^1)^H \mathbf{u}_{n,0}^1|^2$ in (2) is identified as a normalized correlation coefficient and is denoted by ρ_n. For $r \geq 2$, ρ_n is known to be a beta random variable with the following density function.

$$f_{\rho_n}(x) = \begin{cases} (r-1)\,(1-x)^{(r-2)} & \text{if } 0 \leq x \leq 1, \\ 0 & \text{else} \end{cases}$$

where $E[\rho_n] = 1/r$ and $E[\rho_n^2] = 2/(r(r+1))$. Hence, intracell interference is the sum of the products of largest eigenvalues of Wishart matrices and beta random variables, all of which are independent from one another. A closed form expression is not available for the distribution of I_{int}. Therefore, we look for approximate distributions. Since each of the terms $\lambda_n\, \rho_n$ are positive, we make use of the central limit theorem for causal functions [7] and approximate the total intracell interference by a gamma distribution.

$$f_{I_{int}}(z) \approx \frac{z^{\alpha-1}\, e^{-z/\beta}}{\beta^\alpha\, \Gamma(\alpha)}, \quad z > 0,\ \alpha, \beta > 0 \qquad (7)$$

where $\alpha = \text{Var}(z)/E[z]$ and $\beta = (E[z])^2\,/\text{Var}(z)$.

As can be seen from the above expressions, we only need to know the first and second order moments of I_{int} to fully characterize the approximate distribution. These moments can be expressed as

$$E[I_{int}] = N_{int}\, P_I\, E[\lambda_n]\, E[\rho_n] \qquad (8)$$
$$\text{Var}(I_{int}) = N_{int}\, P_I^2\, \left[E[\lambda_n^2]\, E[\rho_n^2] - (E[\lambda_n]\, E[\rho_n])^2\right]$$

where the moments of λ_n (6) and ρ_n have already been provided in closed form.

This approximation is valid even for small number of intracell interferers because the distribution of the product $\lambda_n\, \rho_n$ is experimentally observed to be very similar to a gamma distribution, and sum of i.i.d. gamma variables is also a gamma random variable. In Figure 1, the intracell interference distribution is plotted through simulations for 2 and 6 interferer systems. In both cases, the gamma distribution provides a very close approximation to the actual distribution.

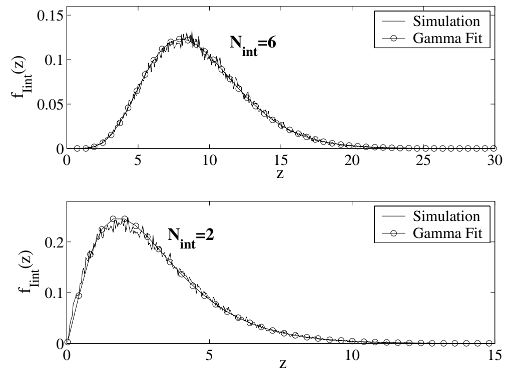

Fig. 1. Intracell interference distribution - $N_{int} = \{2, 6\}$

3.3. Intercell Interference

The intercell interference is given by (3). It has been shown in [8] that $d_k = (\mathbf{u}_{0,0}^1)^H\, H_{k,0}\, \mathbf{v}_{k,m_k}^1$ is complex Gaussian distributed with zero mean and unit variance. Hence, the intercell interference is a weighted sum of gamma random variables with a single parameter 1. Provided that these weights are distinct, a closed form expression for its distribution is known.

$$f_{I_{ext}}(y) = \sum_{k=1}^{N_{ext}} \frac{a_k}{P_k}\, e^{-y/P_k},\ a_k = \prod_{\substack{i=1\\ i\neq k}}^{N_{ext}} \frac{P_k}{P_k - P_i},\ y > 0 \quad (9)$$

The interference components I_{int} and I_{ext} are functions of independent variables with the exception of the common term $\mathbf{u}_{0,0}^1$ in both expressions. In [8] it is shown that I_{ext} is independent of the random vector $\mathbf{u}_{0,0}^1$, hence the intracell and intercell interference are statistically independent.

3.4. Outage Probability

For a MIMO system the outage probability can be expressed in closed form for an arbitrary number of transmit and receive antennas. The outage probability expression for systems with multiple antennas employed on only one side of the channel are provided in [8]. In this work we are interested in the case where $\min(t, r) \geq 2$. We first substitute the approximate distribution of the intracell interference (7) and the exact distributions of the intercell interference (9) and λ_0 (5) into the outage probability expression (4).

$$P_{out}(\epsilon) = \sum_{n=1}^{\min\{t,r\}} \sum_{m=|t-r|}^{(t+r)n-2n^2} \frac{d_{n,m}}{m!} \sum_{k=1}^{N_{ext}} a_k \int_0^\infty \frac{z^{\alpha-1}\, e^{-z/\beta}}{\beta^\alpha\, \Gamma(\alpha)}$$
$$\times \int_0^\infty \frac{e^{-y/P_k}}{P_k}\, \gamma\left(m+1, \frac{n\,\epsilon}{P_0}\, z + \frac{n\,\epsilon}{P_0}\, y + \frac{n\,\epsilon\,\sigma^2}{P_0}\right) dy\, dz$$

We first derive the following equality, making use of the integral definition of the incomplete gamma function.

$$\mu \int_0^\infty e^{-\mu y}\gamma(n, c\, y+d)\, dy = \gamma(n,d) + e^{\mu d/c}$$
$$\times \left(\frac{c}{c+\mu}\right)^n (\Gamma(n) - \gamma\,(n, d\,(1+\mu/c))) \quad (10)$$

Using the above equality and the finite sum representation of the incomplete gamma function, we can evaluate the outage probability integral and obtain a closed form expression.

$$P_{out}(\epsilon) = \sum_{n=1}^{\min\{t,r\}} \sum_{m=|t-r|}^{(t+r)n-2n^2} \frac{d_{n,m}}{m!} \sum_{k=1}^{N_{ext}} a_k$$
$$\times B\left(m+1, \frac{n\,\epsilon}{P_0}, \frac{n\,\epsilon\,\sigma^2}{P_0}, \frac{1}{P_k}, \alpha, \beta\right) \quad (11)$$

where $B(n, c, d, \mu, \alpha, \beta)$

$$= \int_0^\infty \frac{z^{\alpha-1}\, e^{-z/\beta}}{\beta^\alpha\, \Gamma(\alpha)} \int_0^\infty \mu\, e^{-\mu y}\, \gamma(n, cy+cz+d)\, dy\, dz$$
$$= (n-1)!\left\{1 - e^{-d}\sum_{i=0}^{n-1}\sum_{j=0}^{i}\binom{i}{j}\frac{\beta^j\, d^{i-j}\, c^j}{(1+c\,\beta)^{\alpha+j}\, i!}\right.$$
$$\left. \times \frac{\Gamma(\alpha+j)}{\Gamma(\alpha)}\left[1 - \left(\frac{c}{c+\mu}\right)^{n-i}\right]\right\}$$

In (11), ϵ denotes the SINR threshold for outage and σ^2 is the thermal noise variance. α and β are the gamma distribution parameters for the approximate intracell interference distribution (7) and a_k's are as given in (9). This outage probability expression is in the form of finite sums and is easily computable, allowing for simpler and faster analysis of multi cellular MIMO systems.

4. NUMERICAL RESULTS

In this section, we verify the outage probability expression we have derived (11) using simulations. We also analyze the effects of increasing the number of antennas on system performance. We consider a system with 10 equal power intracell interferers ($P_I = 1$) and 10 intercell interferers with power levels $\{3.3, 2.8, 2.4, 2.1, 1.8, 1.6, 1.5, 0.8, 0.5, 0.2\}$. P_0 is assumed to be 1 and the thermal noise variance σ^2 is 0.1. In Figure 2, we plot the outage probabilities computed using both the analytical expression and Monte Carlo simulations, for different antenna configurations.

As seen in the figure, the analytical and simulation results are in very close agrement. When we compare the curves with the same order of diversity, we observe that using multiple antennas at the receiver side results in better performance. The reason is that transmit diversity does not combat interference from intracell users, since they optimize their transmission weights according to the channel with the desired user's BS. Hence, they are benefiting from the transmit diversity that the system has to offer. Since intracell interference can only be combatted using receive diversity, the 2×8 system performs better than the 4×4 and 8×2 configurations.

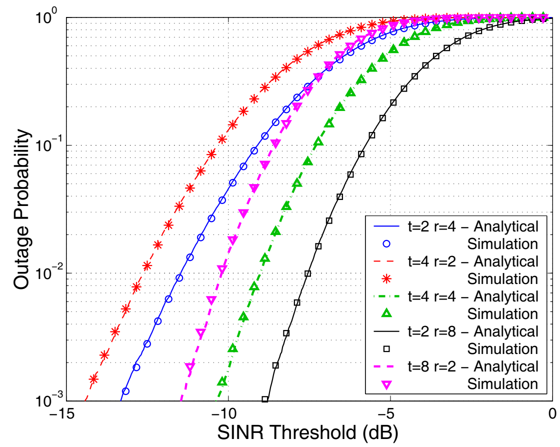

Fig. 2. Outage probability - $N_{int} = 6, N_{ext} = 10$

5. CONCLUSION

In this paper, we analyzed the performance of multi-cellular MIMO systems with co-channel interference from both same cell and other cell users, and derived a closed form, easily computable outage probability expression. We have shown that using antennas on the receiver side results in better performance since unlike transmit diversity, the receive diversity does combat interference from same cell users.

6. REFERENCES

[1] T. K. Y. Lo, "Maximum ratio transmission," *IEEE Trans. Commun.*, vol. 47, no. 10, Oct. 1999.

[2] P. A. Dighe, R. K. Mallik and S. S. Jamuar, "Analysis of transmit-receive diversity in rayleigh fading," *IEEE Trans. Commun.*, vol. 51, pp. 694–703, Apr. 2003.

[3] M. Kang and M.-S. Alouini, "A comparative study in the performance of MIMO MRC systems with and without co-channel interference," in *Proc. IEEE International Conf. on Commun.*, 2003, pp. 2154–2158.

[4] R. J. Muirhead, *Aspects of Multivariate Statistical Theory*, Wiley, 1982.

[5] N. R. Goodman, "Statistical analysis based on a certain multivariate complex gaussian distribution (an introduction)," *Ann. Math. Statist.*, vol. 34, 1963.

[6] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series, and Products*, Academic Press, Inc., 1980.

[7] A. Papoulis, *The Fourier Integral and Its Applications*, McGraw-Hill, New York, 1962.

[8] Y. Tokgoz and B. D. Rao, "Performance analysis of multi-cellular MIMO systems," *IEEE Trans. Wireless Commun.*, submitted for publication.