INDICATORS OF CYCLOSTATIONARITY: PROPOSAL, STATISTICAL EVALUATION AND APPLICATION TO DIAGNOSTICS

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ABSTRACT

This paper proposes new and simple indicators of cyclostationarity for the characterization of stochastic processes whose statistics are periodically varying with respect to some generic variables. Indicators of cyclostationarity from first to fourth order are introduced and their statistical properties are derived. In order to illustrate the use of these indicators, an application to the diagnosis of spalling in a gearbox is described. Results demonstrate their effectiveness for discriminating between different cyclostationary states.

1. STATEMENT OF THE PROBLEM

In signal processing, most of the established methods often rest on fundamental assumptions of stationarity and ergodicity of the involved processes. However, this excludes many real-life non stationary signals. More particularly, there is a subclass of non stationary signals called cyclostationary signals. These signals are characterized by periodic variations of their statistical parameters. The estimation theory of periodically correlated processes, i.e. second order cyclostationary, was first introduced in [1]. For higher orders, the general theory of cyclic statistics has been developed in both the stochastic and fraction of time probability frameworks [2],[3] and has found many applications in communications. Surprisingly, very few applications have been reported in related areas such as in mechanical engineering until recently when it was recognized that cyclostationary processes fit the properties of rotating machinery [4]. Some precursory works [4],[5] have recognized that cyclostationarity can characterize faults in mechanical systems that would produce repetitive non-linearities and non-stationarities. A relevant statistical parameter for studying such properties is the *n*th-order cyclic polyspectrum [2],[3]. However, it is not conceivable to estimate higher order polyspectra ($n > 2$) in the case of on-line monitoring because of the high cost of calculation they involve. In order to overcome this problem, concise and global indicators that measure the cyclostationarity from order 1 to 4 are proposed in this paper. The idea is based on the precursory works of [7] and [8].

The paper is organized as follows. After presenting the basic principles of cyclostationarity in section 2, the theory and the statistical properties of the indicators are proposed in section 3. To illustrate these methods, an application to industrial vibration signals is described. These signals were recorded on a gearbox. Results are presented and discussed in section 4. Conclusions are drawn in section 5.

2. MOMENTS OR CUMULANTS TO DEFINE INDICATORS?

A process $x(t)$ is said to be *n*th-order cyclostationary with period T if its *n*th-order moments exist and are periodic with period T. In order to define indicators of cyclostationarity, there are two possible methods based either on the use of moments or cumulants. The first approach uses the periodicity of the *n*th-order moment, while the second uses the periodicity of the *n*th order cumulant.

The use of *n*th-order cumulants is more advantageous than *n*th-order moments for the following reason: it is often the case that an *n*th-order moment is impure in the sense that it is partly or wholly made up of products of lower order moments. To purify *n*th-order moments, all the impure terms induced from lower orders must be extracted. This is exactly what is achieved by using cumulants rather than moments. For $n = 2$ and 3, the purification is easy because extracting the first–order moment is enough to obtain the cumulants. For higher orders, Leonov's type formulae must be used [2].

3. PROPOSAL OF INDICATORS OF CYCLOSTATIONARITY

3.1 General theory

Consider the first moment, second, third and fourth order cumulants of signal $x(t)$ respectively defined by: $M_{1x}(t), C_{2x}(t,\tau),\ C_{3x}(t,\tau_1,\tau_2)$ and $C_{4x}(t,\tau_1,\tau_2,\tau_3)$. The cyclic moment and the cyclic cumulants at zero lags are used here to measure the cyclostationarity because they summarize all the spectral information of a cyclostationary signal according to the projections:

$$\begin{aligned} c_{2x}^{\alpha}(0) &= \int S_{2x}^{\alpha}(\nu)d\nu \\ c_{3x}^{\alpha}(0,0) &= \int S_{3x}^{\alpha}(\nu_1,\nu_2)d\nu_1 d\nu_2 \\ c_{4x}^{\alpha}(0,0,0) &= \int S_{4x}^{\alpha}(\nu_1,\nu_2,\nu_3)d\nu_1 d\nu_2 d\nu_3 \end{aligned} \qquad (1)$$

where $c_{nx}^{\alpha}(\underline{\tau})$ represents the Fourier coefficients of $C_{nx}(t,\underline{\tau})$ with respect to t and $S_{nx}^{\alpha}(\underline{\nu})$ is the nth order cyclic polyspectrum of $x(t)$, i.e. the Fourier transform of $c_{nx}^{\alpha}(\underline{\tau})$ with respect to the lag variables $\underline{\tau}$.

The problem of defining a measure of second-order cyclostationarity processes was first addressed in [2] and [7]. In [7], W.A.Gardner proposed a degree of cyclostationarity for each cyclic frequency α defined by:

$$DCS^{\alpha} = \int \left|c_{2x}^{\alpha}(\tau)\right|^2 d\tau \Big/ \int \left|c_{2x}^{0}(\tau)\right|^2 d\tau \qquad (2)$$

In this paper, we propose new simplified ($\tau = 0$) indicators of cyclostationarity from order one to four defined as follows:

$$I_{ix}^{n} = \left|c_{2x}^{0}(0)\right|^{-i} \sum_{\alpha\neq 0} \left|P_{ix}^{\alpha}\right|^2 \qquad (3)$$

where $i = 1..4$; $P_{ix}^{\alpha} = M_{1x}^{\alpha}$ for i=1 and $c_{2x}^{\alpha}(0), c_{3x}^{\alpha}(0,0), c_{4x}^{\alpha}(0,0,0)$ respectively for i=2, 3 and 4.

These indicators are motivated by the fact that:

1. They are monotonic and increasing functions of the degree of nth-order cyclostationarity.
2. They are theoretically zero if the process is stationary.
3. They are normalized by the energy of the signal to give a dimensionless ratio.
4. They generalize the well-known standardized cumulants, i.e. the classical RMS value, the skewness and the kurtosis by giving them a 'cyclic' counterpart.

3.2 Estimation techniques

Consistent estimators, for a discrete signal $x(n)$, are defined as below [2]:

$$\begin{aligned} \hat{c}_{2x}^{\alpha}(0) &\overset{N\to\infty}{=} \frac{1}{N}\sum_{n=0}^{N-1} x_c^{\,2}(n)\exp(-j2\pi\, n\alpha/N) \\ \hat{c}_{3x}^{\alpha}(0,0) &\overset{N\to\infty}{=} \frac{1}{N}\sum_{n=0}^{N-1} x_c^{\,3}(n)\exp(-j2\pi\, n\alpha/N) \\ \hat{c}_{4x}^{\alpha}(0,0,0) &\overset{N\to\infty}{=} \frac{1}{N}\sum_{n=0}^{N-1} x_c^{\,4}(n)\exp(-j2\pi n\alpha/N) - 3\sum_{\beta} c_{2x}^{\alpha-\beta}(0)c_{2x}^{\beta}(0) \end{aligned} \qquad (4)$$

where $x_c(n) = x(n) - \hat{M}_{1x}(n)$ is the signal obtained after extracting the synchronous average $\hat{M}_{1x}(n)$.

3.3. Statistical properties of indicators

The objective of this paragraph is to evaluate the statistical properties (bias and variance) of the proposed indicators in order to be able to establish thresholds on their estimated values. However, the properties of I_{4x}^{n} will not be investigated in this paper. This is due to the complexity of the relation that relates $\hat{I}_{4x}^{n}$ to the cyclic sample moments. Therefore, the exact computation of $E\{\hat{I}_{4x}^{n}\}$ and $\mathrm{var}\{\hat{I}_{4x}^{n}\}$ leads to intricate formula which are of no practical value. So, it was decided to estimate these latter quantities by using the bootstrap technique. This statistical technique involves generating subsets of the data on the basis of random sampling with replacements as the data are sampled. The positive motive for bootstrap resampling is the general relative ease of devising an appropriate resampling when the experimental design is complex.
Since we are testing for cyclostationarity, all the calculations for order 1 to 3 are made under the null hypothesis H_0 which supposes that the signal is stationary.

- Bias of the indicators

$$E\{\hat{I}_{ix}^{n}\} = E\left\{\frac{\sum_{\alpha}\left|P_{ix}^{\alpha}\right|^2}{\left|\hat{c}_{2x}^{0}(0)\right|^{i}}\right\} \approx \frac{1}{\left|c_{2x}^{0}(0)\right|^{i}}\sum_{\alpha} E\left\{\left|P_{ix}^{\alpha}\right|^2\right\} \qquad (5)$$

This approximation is valid if $c_{2x}^{0}(0) \succ\succ \left[\operatorname{var}\left\{\hat{c}_{2x}^{0}(0)\right\}\right]^{1/2}$ which is asymptotically true. After some algebra, one can then obtain that:

$$E\left\{\hat{I}_{ix}^{n}\right\} \approx \frac{1}{N\left|c_{2x}^{0}(0)\right|^{i}} \sum_{\alpha} S_{2x^{i}}(\alpha) \tag{6}$$

where $S_{2x^{i}}$ is the moment spectrum of the signal $\left[x(t)\right]^{i}$, $i = 1,2,3$.

- Variance of the indicators

$$Var\left\{\hat{I}_{ix}^{n}\right\} = E\left\{\left|\hat{I}_{ix}^{n}\right|^{2}\right\} - \left|E\left\{\hat{I}_{2x}^{n}\right\}\right|^{2} \tag{7}$$

Initially, the calculation of $E\left\{\left|\hat{I}_{ix}^{n}\right|^{2}\right\}$ is considered.

$$E\left\{\left|\hat{I}_{ix}^{n}\right|^{2}\right\} \approx \frac{1}{\left|c_{2x}^{0}(0)\right|^{2i}} \sum_{\alpha}\sum_{\beta} E\left\{\left|\hat{P}_{ix}^{\alpha}\hat{P}_{ix}^{\beta}\right|^{2}\right\} \tag{8}$$

Assuming that the estimates $\hat{P}$ are asymptotically gaussian and using [8], one can obtain :

$$E\left\{\left|\hat{I}_{ix}^{n}\right|^{2}\right\} = \frac{1}{\mathrm{N}^{2}\left|c_{ix}^{0}(0)\right|^{2i}}\Big(\sum_{\alpha}\left|S_{2x^{i}}(\alpha)\right|^{2} + \sum_{\alpha}\sum_{\beta}\left|S_{2x^{i}}^{\alpha-\beta}(-\beta)\right|^{2} + \sum_{\alpha}\sum_{\beta}\left|S_{2x^{i}}^{\alpha+\beta}(\beta)\right|^{2}\Big) \tag{9}$$

Finally, the expression of the variance is found as:

$$Var\left\{\hat{I}_{ix}^{n}\right\} = \frac{2}{\mathrm{N}^{2}\left|c_{ix}^{0}(0)\right|^{2i}} \sum_{\alpha}\sum_{\beta}\left|S_{2x^{i}}^{\alpha+\beta}(\beta)\right|^{2} \tag{10}$$

3.5 Application to diagnostics

The objective of this paragraph is to propose an application to the diagnostic of any system composed of two sets of different cyclic frequency ϖ_1 and ϖ_2. The protocol is resumed in table 1:

	Set 1	**Set 2**
CS1	$I_{1x}^{n}(\varpi_1 - \varpi_1 \cap \varpi_2)$	$I_{1x}^{n}(\varpi_2 - \varpi_1 \cap \varpi_2)$
CS2	$I_{2x}^{n}(\varpi_1 - \varpi_1 \cap \varpi_2)$	$I_{2x}^{n}(\varpi_2 - \varpi_1 \cap \varpi_2)$
CS3	$I_{3x}^{n}(\varpi_1 - \varpi_1 \cap \varpi_2)$	$I_{3x}^{n}(\varpi_2 - \varpi_1 \cap \varpi_2)$
CS4	$I_{4x}^{n}(\varpi_1 - \varpi_1 \cap \varpi_2)$	$I_{4x}^{n}(\varpi_2 - \varpi_1 \cap \varpi_2)$

Tab.1: Monitoring protocol

where CSi characterizes the *i*th-order cyclostationarity. To follow the evolution of each set of cyclic frequencies, one has to be careful to compute the indicators of cyclostationarity only for the specific set without including the common cyclic frequencies between the two sets. The evolution of one of these indicators corresponding to a specific set is an effective way of detecting any abnormal change in the system.

For example, consider $\varpi_1 = \left\{k / T_{1c}\right\}$ and $\varpi_2 = \left\{k / T_{2c}\right\}$, $k = -K,...,K$ two sets of harmonically related cyclic frequencies with K the number of harmonics in the frequency band of interest and T_{ic} the periods of cyclostationarity. Further, suppose that $\varpi_1 \cap \varpi_2 = \left\{N_1 k / T_{1c}\right\}$. Then, expressions (6) and (10) $i = 1\ldots3$ are given by:

$$E\left\{\hat{I}_{ix}^{n}\right\} \approx \frac{C_{2x^{2}}(0)}{\left|c_{2x}^{0}(0)\right|^{i}} \frac{M_1(N_1-1)}{NN_1}$$

$$\text{and } Var\left\{\hat{I}_{ix}^{n}\right\} \approx \frac{2\mathrm{M}_1(N_1-1)}{N^{2}N_1\left|c_{2x}^{0}(0)\right|^{2i}} \sum_{p} C_{2x^{i}}^{2}(p) \tag{11}$$

where M_1 is the number of samples corresponding to T_{1c}.

4. APPLICATION TO THE DIAGNOSIS OF A GEARBOX.

The system examined is a power circulating gear-testing machine. It is composed of two single-stage gear units mounted back to back. Both units contain a pair of spur gears. The first pair has 20 teeth each. Data was measured with an accelerometer at a sampling rate of 80 kHz. 256 states were measured during the experiments, and angular re-sampled by using top reference and interpolation techniques. Inspection of the spectrum of a vibration signal showed that it was essentially discrete below 16 kHz and continuous above 16 kHz.

Fig.1 presents the four indicators I_{ix}^{n} when all signals are low-pass filtered (the cut off frequency is equal to 16 kHz). The indicator of CS1 clearly increases at the end of the campaign, this being a classical result similar to those obtained in [4]. For the other three indicators, a small increase in cyclostationarity can be observed from the state 232 onward which corresponds to the last day of acquisition. It was also observed that the amplitudes of the indicators increase with order. Fig.2 shows the evolution of the four indicators when the signals are high-pass filtered. As can be predicted from inspection of the spectrum, the amplitude of the synchronous mean decreases in comparison with Fig.1. In contrast,

I_{2x}^n, I_{3x}^n and I_{4x}^n increase in a spectacular way. This result is very net on the 174th state, which corresponds to the 8th day, i.e. 3 days before the end of measurement. Furthermore, the cyclostationarity is visible at all orders. To evaluate the properties of these indicators, the bias and the variance of these indicators have been computed using equations (6) and (10). The null hypothesis H_0 (stationarity) was rejected with a level of significance of 0.13%.

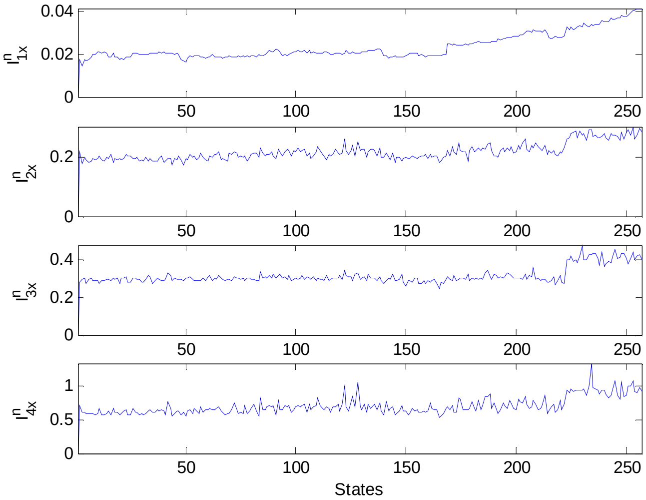

Fig.1: Evolution of cyclostationary indicators with different states of the gearbox system when all signals are low pass filtered

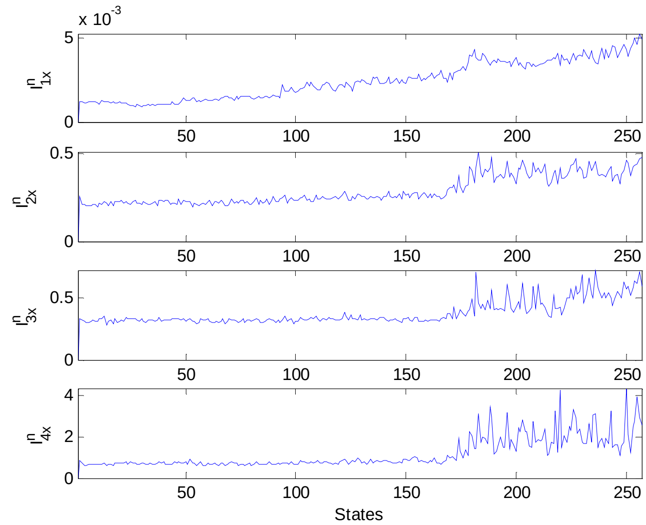

Fig.2: Evolution of cyclostationary indicators with different states of the gearbox system when all signals are high pass filtered

5. CONCLUSION

This paper deals with some new indicators of cyclostationarity. The main idea in the development of these indicators was to exploit cyclostationarity at different orders without using *n*th-order polyspectra which are inappropriate for real-time computation. The proposed indicators are expressed in terms of cumulants and are normalized variance. They generalize the degrees of cyclostationarity introduced by [7]. Their statistical properties are also briefly investigated in this paper and may be useful for testing cyclostationarity order by order. One application detailed in this paper is for diagnostics. Indeed, an industrial case is described where the proposed indicators are used in the monitoring of a gearbox. The obtained results are very promising and motivate further research in this area.

6. REFERENCES

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