SEQUENTIAL MONTE CARLO METHOD FOR BLIND EQUALIZATION OF A NONLINEAR SATELLITE COMMUNICATION CHANNEL

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ABSTRACT

This paper proposes a sequential Monte Carlo simulation method for equalizing a satellite communication channel. The main difficulties encountered are the nonlinear distorsions due to the amplifier stage in the satellite. The aim of the method is to blindly restore the emitted message by considering a Bayesian approach. Thus, *prior* knowledge on the modeling of the nonlinearity is taken into account in the *posterior* distribution of the input sequence. Such a distribution is very difficult to study and thus motivates the implementation of Monte Carlo techniques. This approach makes it possible to solve the problem for a simplified model. The simulation scheme dealing with the complete transmission chain uses the method developed for the simplified model. Performance of the equalization algorithm is evaluated using Bit Error Rate versus Signal-to-Noise Ratio curves.

1. INTRODUCTION

The importance of telecommunication since the last decades leads to consider satellite systems for transmitting information. The main drawback of this approach is the attenuation of the signal due to its trip through the atmosphere. Therefore, one aim of the satellite is to "re-amplify" the signal before sending it back to the Earth. Because of the lack of space and energy available on the satellite, Traveling Wave Tube (TWT) amplifiers are often used for this purpose [1]. Unfortunately, such devices have strong nonlinear behaviors and thus imply complex processing methods. Neural-networks algorithms have been successfully implemented [2, 3, 4] but require a learning/training input sequence for adapting the parameters of the equalizer. The knowledge of such sequences is sometimes impossible: for low Signal-to-Noise Ratios, SNR, or in non-cooperative communication contexts for instance. Blind equalization methods have thus to be considered. Many approaches suppose precise hypothesis on the signals: Gaussianity and circularity [5], and perform identification or equalization for nonlinear channels if they admit a Volterra filter representation [6, 7]. Although such a representation can be succesfully processed with Viterbi algorithms for identification purposes [8], a Volterra modelization is not adequate for equalizing the channel studied hereinafter, the resulting filter being unstable. Moreover, classical methods do not take fully into account *prior* knowledge available on some parametric expressions for TWT amplifiers [1]. This motivates the approach proposed in [9] and in this paper where a Bayesian framework is considered leading to the estimation of the *posterior* distribution of the transmitted symbol sequence. This distribution is difficult to compute due to the nonlinearity of the model but Monte Carlo simulation methods make it possible to build a formal blind equalization algorithm. The aim of this paper is thus to present such an algorithm by extending the one proposed in [9] to a more robust estimation method also enabling in-line processing.

The paper is organized as follows: a brief description of the model is given in §2. Considering a simpler model leads to a first Monte Carlo estimation method of the input sequence, described in §3. In §4, the complete model is considered and a particle filtering based equalization method is proposed. The simulation scheme uses the algorithm specifically designed for the simpler model. Paragraph §5 is devoted to illustrate performance of the approach on simulated data. The paper is concluded by discussing some advantages and drawbacks of the algorithm, and some perspectives of the method.

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2. MODELING THE CHANNEL

The complete transmission chain studied is commonly considered in the field of satellite communication [4] and is depicted in figure 1. The information signal is a digital sequence of symbols $(e_k)_{1\leq k\leq K}$ generated at a rate T. In the following, symbols are generated from 4-QAM modulations: $e_k = \exp(\imath\phi_k)$ where ϕ_k is independently, identically (i.i.d.) and uniformly distributed on the set $\{\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}\}$. The emission stage is modeled as a 4-pole Chebychev filter F0 whose 3dB bandwitdh equals to $\frac{1.66}{T}$. The emitted signal is distorted by its trip through the atmosphere. This is modeled by an additive *i.i.d.* circular Gaussian noise signal $n_e(t)$. The variance σ_e^2 of the latter signal practically provides an uplink SNR around 15dB. The amplitude of the information signal, denoted as parameter A, is set at the transmitter stage on the Earth to reach at least such a noise level. The signal is then amplified by the satellite and sent back to the Earth. This stage is performed by a TWT amplifier which is modeled by the following amplitude gain and phase wrapping [1]:

$$A(r) = \frac{\alpha_a r}{1+\beta_a r^2} \qquad \Phi(r) = \frac{\alpha_p r^2}{1+\beta_p r^2} \tag{1}$$

where r denotes the input signal amplitude and $(\alpha_a, \beta_a, \alpha_p, \beta_p)$

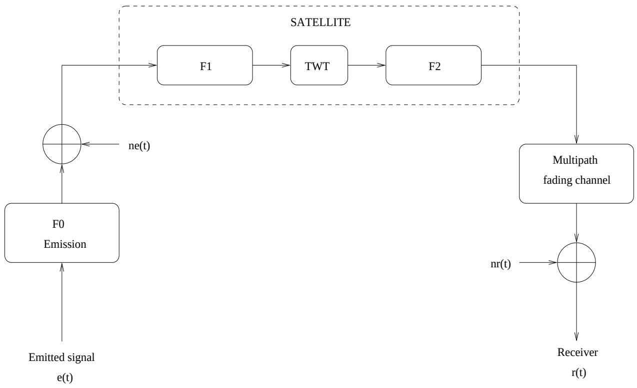

Fig. 1. Satellite communication channel

are the coefficients of the TWTA model [1, table 1]. Such a system may not be inversible but only the amplificative part of the modulus characteristic will be considered [9]. The TWT amplifier lies between devices F1 and F2 performing the task of multiplexing and modeled by 4-pole Chebychev filters, whose 3dB bandwidths equal respectively to $\frac{2}{T}$ and $\frac{3.3}{T}$. The transmission of the signal back to the Earth is much less powerful than the previous one because of straight technical constraints of the satellite. Thus, the influence of the atmospheric propagation medium is usually modeled by a linear multipath fading channel [4]. Finally, the signal is additively corrupted by an *i.i.d.* circular Gaussian noise signal $n_r(t)$ whose variance is denoted as σ_r^2. The goal is then to recover the emitted symbol sequence from the only knowledge of the received signal, denoted as $r(t)$, and the type of constellation. Since this problem is difficult, a simpler model, depicted in figure 2, is firstly studied. This model focuses on the nonlinearity by omitting the linear filters and the multipath fading channel. The only perturbations considered are the uplink and downlink noises, and of course the effect of the TWTA. The equalization of this simple model is tackled in the following section.

3. MONTE CARLO ESTIMATION TECHNIQUES

In this section, the received signal is supposed to be sampled at symbol rate T. The problem is then to estimate a 4-QAM symbol ϕ from a sample r. A Bayesian approach is proposed by considering the *posterior* distribution $p(\phi|r)$ and its *maximum a posteriori* (MAP) associated estimator. A Monte Carlo method is developped to compute this estimator by using parametric expressions (1) for the amplifier. If the parameters of the model depicted in figure 2 are known, it is possible to consider the conditional *posterior* distribution

$$p(\phi|A, \sigma_e, TWT, \sigma_r, r) \tag{2}$$

for the estimation task, TWT denoting the parameters of the amplifier, see §2. Applying Bayes *formula* for (2), the difficulty is then to estimate the likelihood $p(r|A, \phi, \sigma_e, TWT, \sigma_r)$. A solution, developped in [9], consists in writing this expression as the expectation

$$\mathrm{E}\left\{\exp\left(-\frac{1}{\sigma_r^2}\left|r - TWT(x)\right|^2\right)\right\} \tag{3}$$

where

$$x \sim \mathcal{N}_{CC}\left(A\exp(\imath\phi), \sigma_e^2\right) \tag{4}$$

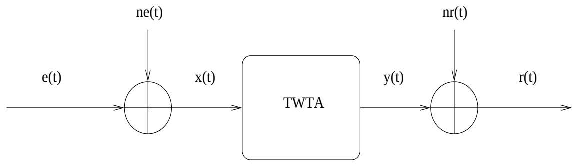

Fig. 2. Simple TWTA channel

It is possible to compute Monte Carlo estimates [10] of expression (3) from sequences of i.i.d. samples $(x_\ell)_{1\leq\ell\leq N}$ simulated from distribution (4):

$$\frac{1}{N}\sum_{\ell=1}^{N}\exp\left(-\frac{1}{\sigma_r^2}\left|r - TWT(x_\ell)\right|^2\right) \tag{5}$$

For instance, approximation (5) is computed for an emitted symbol $\phi = \frac{\pi}{4}$ and TWT parameters given by $(\alpha_a, \beta_a, \alpha_p, \beta_p) = (2, 1, 4, 9.1)$, [1, table 1]. The amplitude A is set to 0.5 and variances of noises are such that $\mathrm{SNR}_e = 10$dB and $\mathrm{SNR}_r = 3$dB. 100 realizations are simulated for sequences (4) composed of 100 samples each. Mean values and standard deviations of estimates (5) are depicted in table 1 (left column). Even with a few number of samples, say 10, it is possible to estimate efficiently the MAP of distribution (2). However, the approach above requires the values of the channel parameters. These ones may be not known or easily estimated for a nonstationary channel or in case of non-cooperative communication contexts, like passive listening for instance. When parameters $(A, \sigma_e, TWT, \sigma_r)$ are unknown, a solution consists in considering the full *posterior* distribution $p(\phi, A, \sigma_e, TWT, \sigma_r|r)$. Such an approach can be implemented with a Gibbs sampling simulation scheme [9] but do not lead to significant results in practice for the estimation of the parameters. The approach developped in the following is to consider the marginal *posterior* distribution:

$$p(\phi|r) = \int p(\phi, A, \sigma_e, TWT, \sigma_r|r)d(A, \sigma_e, TWT, \sigma_r) \tag{6}$$

From the Bayes *formula*, the integral above can be written as the expectation

$$\mathrm{E}_{p(A,\sigma_e,TWT,\sigma_r)}\left\{p(r|\phi, A, \sigma_e, TWT, \sigma_r)\right\} \tag{7}$$

whose computation can be processed thanks to Monte Carlo techniques as exposed previously. Expression (7) is thus approximated by

$$\frac{1}{N_p}\sum_{k=1}^{N_p} p(r|\phi, A_k, \sigma_e(k), TWT_k, \sigma_r(k)) \tag{8}$$

where sequences of samples $(A_k, \sigma_e(k), TWT_k, \sigma_r(k))$ are i.i.d. and sampled from the *prior* distribution

$$p(A, \sigma_e, TWT, \sigma_r) \tag{9}$$

The simulation algorithm for the full *posterior* distribution presented in [9] also requires the setting of *prior* distribution (9). Some *prior* information is generally available for the parameters of the channel. The TWT amplifier is supposed to work in its amplificative regime, an uniform *prior* distribution is then chosen for the amplitude of the input signal $A \sim \mathcal{U}_{[0,1]}$. To reach a SNR roughly equal to 15dB at the emission stage, it is possible to infer $\sigma_e \sim \mathcal{U}_{[0.01,0.5]}$. Coefficients of expressions (1)

ϕ	$\widehat{p}(\phi\|r, A, \sigma_e, TWT, \sigma_r)$	$\widehat{p}(\phi\|r)$
$\frac{\pi}{4}$	0.69 ± 0.28	0.61 ± 0.21
$\frac{3\pi}{4}$	0.13 ± 0.21	0.23 ± 0.20
$\frac{5\pi}{4}$	0.02 ± 0.05	0.05 ± 0.03
$\frac{7\pi}{4}$	0.16 ± 0.24	0.12 ± 0.14

Table 1. Monte Carlo estimates

are supposed to be independent from the other parameters and also one with each other. Considering values experimentally measured and presented in [1], a possible *prior* distribution is given by $(\alpha_a, \beta_a, \alpha_p, \beta_p) \sim \mathcal{U}_{[1,3]} \times \mathcal{U}_{[0,2]} \times \mathcal{U}_{[1,5]} \times \mathcal{U}_{[2,10]}$. The extremal values of standard deviation σ_r of the noise signal at the receiver can be estimated from amplitudes of the other parameters. In the following, a distribution $\mathcal{U}_{[0.1,1.1]}$ is considered. Once *prior* distribution (9) is defined, it is possible to implement a Monte Carlo estimation method for (7) thanks to approximations (8) and (5). For instance, expression (8) is considered with the same parameter values as in the numerical experiments run for known values. 100 realizations are simulated where sequences

$$(A_k, \sigma_e(k), TWT_k, \sigma_r(k))_{1 \le k \le N_p} \tag{10}$$

composed of 100 samples generated from (9) are considered. For each sample, expressions (5) are computed with a sequence (10) of 10 samples each. Mean values and standard deviations of estimates (8) are depicted in table 1 (right column). Even with a few number of samples, say 10, it is possible to estimate robustly the MAP of (6). Numerical values are of the same order than the ones obtained in case of known parameters. Using the marginalized *posterior* distribution (6) seems thus a good strategy. A simulation method based on the Monte Carlo techniques developped above is proposed in the following section for equalizing the complete transmission chain.

4. PARTICLE FILTERING EQUALIZATION METHOD

The equalization of the satellite communication channel depicted in figure 1 is a difficult task as several phenomenons have to be dealt with :

1. The unknown parameters of filters F0, F1 and F2
2. The multipath fading channel for the downlink transmission
3. The correlation induced by the filters and fading model for the received signal

The approach proposed still consists in considering the *posterior* distribution of the samples of the emitted signal conditionally to the samples of the received signal

$$p((e(jT_{ech}))_{1 \le j \le M} | (r(jT_{ech}))_{1 \le j \le M}) \tag{11}$$

A Bayesian estimation procedure is implemented by computing the MAP estimator of distribution (11). Monte Carlo estimation techniques developped in §3 are modified in order to take into account the parameters of the complete transmission chain, cf. points 1. and 2. above. In fact, the main problem is to estimate the number of samples per symbol duration $p = \frac{T}{T_{ech}}$, as parameters of filters F0, F1 and F2 depend on this parameter. This can be done by computing the correlation of the received signal [9]. This correlation can also be used explicitely in a recursive equalization algorithm as shown below. Many applications in telecommunication require in-line processing methods [11]. Sequential Monte Carlo simulation algorithms are thus investigated. A simulation scheme based on particle filtering techniques is considered for simulating distribution (11). The main idea is to generate sequences of particles

$$(e_i(0), e_i(T_{ech}), \ldots, e_i(jT_{ech}))_{1 \le i \le N} \tag{12}$$

iteratively which are sampled from desired distribution (11). In the present case, the phase samples $\phi(jT_{ech}) = \phi(j)$ of the emitted signal are directly simulated. The sampling scheme is now described:

Equalization algorithm

1. `initialization: sample` $\phi_i(0)$ `as i.i.d. 4-QAM symbols for` $i = 1, \ldots, N$; `set` $j = 1$
2. `importance sampling: simulate`
$$\widetilde{\phi}_i(j) \sim p(\phi(j) | \phi_i(j-1)) \tag{13}$$
`for` $i = 1, \ldots, N$; `actualize the sample path`
$$\left[\widetilde{\phi}_i(0), \ldots, \widetilde{\phi}_i(j)\right] = \left[\phi_i(0), \ldots, \phi_i(j-1), \widetilde{\phi}_i(j)\right]$$
3. `Compute the weights`
$$\widetilde{w}_i(j) = p(r(j) | \widetilde{\phi}_i(j)) \times w_i(j-1) \tag{14}$$
`and normalize them`
4. `selection of the particles: resample the` N `particles` $(\phi_i(0), \ldots, \phi_i(j))$ `from` $(\widetilde{\phi}_i(0), \ldots, \widetilde{\phi}_i(j))$ `according to their weights.`
5. $j \leftarrow j + 1$ `and go to (2)`

It is possible to take into account parameter p in the simulation method by sampling candidates from *prior* distribution (13) defined as follows:

- `prob` $1 - \frac{1}{p}$: `set` $\widetilde{\phi}_i(j) = \widetilde{\phi}_i(j-1)$
- `prob` $\frac{1}{p}$: `sample` $\widetilde{\phi}_i(j)$ `as a 4-QAM symbol`

The computation of weights (14) is processed thanks to analogeous Monte Carlo techniques as the ones developped previously, including now the parameters of the filters in expressions (3) and (6). As the variable of interest is discrete, a few numbers of particles are required, say some dozens. On the other hand, this property imply a strong degeneracy phenomenon for the weights [11] but this is not really a drawback in the present case as the simulation scheme is used for MAP estimation purposes. The algorithm is now tested on simulated data.

5. NUMERICAL EXPERIMENTS

The equalization method is runned for 100 realizations of sequences composed of 1000 samples each considering various downlink SNR_r for a fixed uplink SNR_e=15db. The number of samples received per symbol duration is set to $p = 8$. For each realization, the estimated symbol sequence is taken as the MAP sample path

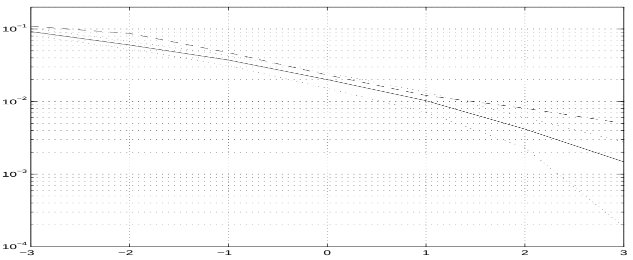

Fig. 3. BER of estimated signals (mean and $\pm$ std. deviation of the MAP, MLP estimates dotted) versus SNR_r in dB, SNR_e=15dB

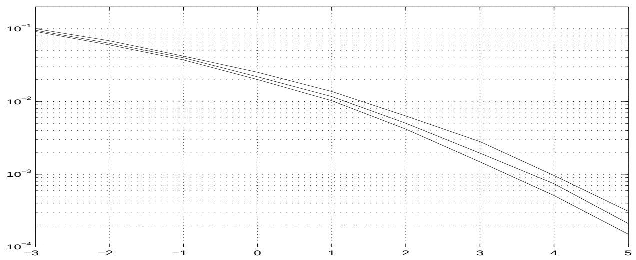

Fig. 4. BER of estimated signals (mean of the MAP) versus SNR_r in dB, SNR_e=10, 12 and 15dB

computed from a distribution (12) of 50 particles. The weights (14) are approximated by Monte Carlo techniques considering sequences of 10 samples each. Mean values (straight line) and its standard deviations (dots) of the estimated Bit-Error-Rates (BER) are depicted in figure 3. Simulation results runned for a neural network equalization method (2-10-4 MLP with hyperbolic activation functions) are also depicted in dashed lines in figure 3. The BER computed with this method are roughly of the same order as the ones obtained with the particle filtering approach but MLP algorithms requires at least the use of learning sequences of 100 samples length in this case. The algorithm is also tested for other values of uplink SNR : 10dB and 12dB. The mean values of BER computed from the estimated phase samples are depicted in figure 4 for these values of SNR_e. The curves from the bottom to the top of the figure are associated to a decreasing SNR . A characteristic of the proposed equalization method is to be robust with respect to nonstationarities of the channel. This property comes from the consideration of the marginalized *posterior* distribution (6). Numerical experiments runned with perturbations of the channel parameters lead to similar results as those presented in figures 3 and 4. On the contrary, in case of dysfunction of the amplifier or sudden change of the intensity of noises, the algorithm remains insensitive to these variations. The approach developped thus cannot be used for diagnostic purposes like neural networks methods for instance [3]. However, a significant advantage of the proposed method is not to require any learning input sequence. The algorithm is then efficient for blind communication tasks. A calibration step is however necessary in order to estimate parameter p; this tuning can be processed by computing the correlation of the received signal.

6. CONCLUSION

The equalization method presented in this paper makes it possible to estimate symbol sequences transmitted through a satellite communication channel and to take into account explicitely the nonlinear distorsions induced by its amplification stage. The algorithm studied is based on sequential Monte Carlo methods, precisely on a particle filtering scheme. This approach enables a recursive estimation procedure of the emitted signal. A Bayesian point of view is adopted by considering the *posterior* distribution of the sampled symbol sequence, marginalized with respect to the parameters of the channel, and its MAP estimator. The method is thus robust for facing nonstationarities of the channel but cannot be used to detect the latter. As many practical implementations of particle filtering simulation methods, the algorithm is more computing time demanding as classical approaches. However, the method does not require the knowledge of training/learning input sequences for the equalization of the channel, which is a very interesting property for blind communication.

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