Nonlinear Equalization for Rician Multipath Fading Channel

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Abstract— **In this paper, we present a nonlinear equalization scheme for Rician multipath fading channel – a fuzzy logic-based approach. We show that each channel states of multipath Rician channel follows a Gaussian distribution, which means a Bayesian equalization can be implemented. The parameters of the Bayesian equalization are determined using an unsupervised clustering method – fuzzy c-means (FCM) method. An extremely small number of training symbols (about 1% of a burst) are used to determine the category of each channel state with the aid of data mining. Simulation results show that our Bayesian equalizer performs much better than the recently proposed nearest neighbor classifier-based equalizer at moderate to high signal-to-noise ratio (SNR).**

I. Introduction

There are two types of adaptive equalization: sequence estimation and symbol detection. Sequence estimation has very high computation complexity, because channel estimation is needed. Symbol detection is essentially a classification problem, in which the input baseband signal is mapped onto a feature space determined by the direct interpretation of a known training sequence. In [9], a nearest neighbor classifier equalizer is used to classify the distorted signal for GSM communications. In [8], a systematic feature space partitioning method is proposed to divide the entire feature space into two decision regions using a set of hyperplanes. In [6] [7], a type-2 fuzzy adaptive filter is proposed and applied to time-varying nonlinear channel equalization using both transversal and decision feedback structures. In all these classifier-based approach, channel estimation is unnecessary, which tremendously simplifies this approach, but all of them need a large number of training symbols (more than 10% of a whole burst). In this paper, we focus on the classifier approach to adaptive equalization, and show that each channel state of a satellite channel with multipath follows Gaussian distribution, and apply a Bayesian equalizer to such a fading channel, but use a very small number of training symbols (about 1%).

In Section II, we discuss the system model we used in this paper. In Section III, we show why each channel state of satellite channel with multipath follows Gaussian distribution, and apply a Bayesian equalizer to such a fading channel. In Section IV, we evaluated our Bayesian equalizer using simulations and compared it against the nearest neighbor classifier equalizer by [9]. Conclusions and future research directions are given in Section V.

II. System Model

Satellite channel is often modelled as a Rician fading channel. Rician fading occurs when there is a strong specular (direct path or line of sight component) signal in addition to the scatter (multipath) components. The channel gain,

$$g(t) = g_I(t) + jg_Q(t) \tag{1}$$

can be treated as a wide-sense stationary complex Gaussian random process, and $g_I(t)$ and $g_Q(t)$ are Gaussian random processes with non-zero means $m_I(t)$ and $m_Q(t)$, respectively; and they have same variance σ_g^2, then the magnitude of the received complex envelop has a Rician distribution [10],

$$p_\alpha(x) = \frac{x}{\sigma^2}\exp\{-\frac{x^2+s^2}{2\sigma^2}\}I_0(\frac{xs}{\sigma^2}) \quad x \geq 0 \tag{2}$$

where

$$s^2 = m_I^2(t) + m_Q^2(t) \tag{3}$$

and $I_0(\cdot)$ is the zero order modified Bessel function. This kind of channel is known as Rician fading channel. A Rician channel is characterized by two parameters, Rician factor K which is the ratio of the direct path power to that of the multipath, i.e., $K = s^2/2\sigma^2$ [10], and the Doppler spread (or single-sided fading bandwidth) f_d. We simulate the Rician fading using a direct path added by a Rayleigh fading generator. The Rayleigh fade generator is based on Jakes' model [4] in which an ensemble of sinusoidal waveforms are added together to simulate the coherent sum of scattered rays with Doppler spread f_d arriving from different directions to the receiver. The amplitude of the Rayleigh fade generator is controlled by the Rician factor K.

The number of oscillators to simulate the Rayleigh fading is 60.

In this paper, the system model we used in our simulations consists of random bits generator, burst (cell) builder, modulator, up-sampler by 16, pulse shaping filter (a square root raised cosine filter with roll off factor 0.35), Rician frequency selective fading channel, matched filter, down-sampler by 16, Bayesian equalizer, burst extractor, and bit error counter, as shown in Fig. 1. In Fig. 2, we summarize the burst format we used in this paper. Its length is 1013 QPSK symbols long, in which 980 symbols are payload. The random bits generator generates a binary data stream with equally likely zeros and ones, which are for the payload bits (1960 bits). The burst builder insert unique word (UW) and guard bits, and makes a complete burst with 2026 bits, and then the 2026 bits are modulated to 1013 QPSK symbols. The unique word (for training purposes) consists of 13 QPSK symbols, which only occupy 1.28% of a burst. In contrast, GSM uses 16.64% of a burst for unique word, and IS-54/136 uses 8.64% of a burst for unique word [10]. In our design, one burst takes $5ms$, which means the symbol rate is $202.6ks/s$, and payload bits rate is $392kb/s$ when it's uncoded.

III. Nonlinear Equalization for Rician Multipath Channel

A. Theoretical Basis

For the system we discussed in Section II, the matched filter output when sampled in time-synchronization can be modeled as

$$r(k) = \sum_{l=0}^{L-1} g(k,l)s(k-l) + n(k) \tag{4}$$

where L is the number of multipath, and

$$n(k) = n_I(k) + jn_Q(k) \tag{5}$$

is additive white Gaussian noise (AWGN) with mean 0 and variance σ_n^2 in the in-phase and quadrature components. $g(k,l)$ is the truncated channel gain. For QPSK modulation, $s(k) \in \{1, j, -1, -j\}$ are the signal points. Based on different values of $s(k), s(k-1), \cdots, s(k-L+1)$, there are $N_s = 4^L$ possible channel states. Assume there are 2 paths ($L = 2$), so there are $4^2 = 16$ channel states. If $s(k) = 1$ and $s(k-1) = 1$, then (4) can be expressed as

$$\begin{aligned} r(k) &= g(k,0) + g(k,1) + n(k) && (6) \\ &= [g_I(k,0) + g_I(k,1) + n_I(k)] \\ &+ j[g_Q(k,0) + g_Q(k,1) + n_Q(k)] && (7) \end{aligned}$$

Since $g_I(k,0)$, $g_I(k,1)$, and $n_I(k)$ are Gaussian distributions with mean $m_I(k,0)$, $m_I(k,1)$, and 0, and with variance σ_{g0}^2, σ_{g1}^2, and σ_n^2, respectively, so $r_I(k) \triangleq g_I(k,0)+g_I(k,1)+n_I(k)$ is a Gaussian distribution with mean $m_I(k,0)+m_I(k,1)$ and variance $\sigma_{g0}^2+\sigma_{g1}^2+\sigma_n^2$ [1]. Similarly, $r_Q(k) \triangleq g_Q(k,0)+g_Q(k,1)+n_Q(k)$ is a Gaussian distribution with mean $m_Q(k,0) + m_Q(k,1)$ and variance $\sigma_{g0}^2 + \sigma_{g1}^2 + \sigma_n^2$.

Similarly, it's easy to show that all the other channel states follow Gaussian distributions. So the received signals of one burst can be clustered to 4^L clusters using FCM method introduced in [2], and the signals associated with each cluster have Gaussian distribution. The mean (time average) and variance of each cluster can be computed, so a Bayesian equalizer can be implemented.

B. Designing the Bayesian Equalizer

B.1 Expanding the Unique Word Based on Data Mining

In this paper, we focus on a satellite channel with 2 paths. As shown in Fig. 2, there are 13 QPSK symbols (unique words) for training. But there are $4^2 = 16$ channel states for the channel with 2 paths, so it's obvious that the number of unique words (training symbols) is not enough. Besides, due to the intersymbol interference and AWGN, one channel state should have more than one symbol for reliable category determination (which will be discussed in III-B.2). We propose a method to expand the number of unique word based on data mining.

Multiplying j to both sides of (4), we obtain,

$$\begin{aligned} j \cdot r(k) &= j \sum_{l=0}^{L-1} g(k,l)s(k-l) + j \cdot n(k) && (8) \\ &= \sum_{l=0}^{L-1} g(k,l)[j \cdot s(k-l)] + j \cdot n(k) && (9) \end{aligned}$$

It's easy to prove that $j \cdot n(k)$ is an AWGN with same mean and variance as $n(k)$, so $r_j(k) \triangleq j \cdot r(k)$ is equivalent to the received signal if the input is $j \cdot [s(k), s(k-1), \cdots, s(k-L+1)]$. Similarly, we can artificially construct the received signal $r_{-1}(k) \triangleq -r(k)$ and $r_{-j}(k) \triangleq -j \cdot r(k)$ if the input patterns are $-1 \cdot [s(k), s(k-1), \cdots, s(k-L+1)]$ and $-j \cdot [s(k), s(k-1), \cdots, s(k-L+1)]$, respectively. In this paper, we assume there are two paths ($L = 2$), and we designed the 13 unique words (in QPSK) as $UW = [1, 1, -1, -j, 1, 1, -1, -j, 1, 1, -1, -j, 1]$. Based on (4), these 13 unique words can generate 12 outputs, $r(k)$ ($k = 1, 2 \cdots, 12$), (corresponding to 4 channel states with each repeated 3 times). Besides the 12 outputs generated from the unique words, we have artificially generated 36 virtual signals $r_j(k)$, $r_{-1}(k)$, and

$r_{-j}(k)$, where $(k = 1, 2, \cdots, 12)$, so we totally have $12 \times 4 = 48$ training prototypes (corresponding to 16 channel states with each repeated 3 times).

B.2 Determine the Mean, Variance, and Cluster Category

We use FCM method to cluster the $1013+36 = 1049$ symbols (in which 1013 symbols are the input to the equalizer, and 36 symbols are artificially-generated) into $c = 16$ (2 paths with QPSK modulation) clusters, and each cluster has the mean $\mathbf{v}_i$, $(i = 1, 2, \cdots, 16)$ obtained from FCM algorithm. The FCM method also generates $\mathbf{U}$, a 16×1049 matrix in this application. Every symbols r_k $(k = 1, 2, \cdots, 1049)$ has 16 membership grades $u_{ik} \in \mathbf{U}$ $(i = 1, 2, \cdots, 16)$ and $\sum_{i=1}^{16} u_{ik} = 1$ corresponding to the 16 clusters. Based on the 16 values of u_{ik} $(i = 1, 2, \cdots, 16)$ for each k, we can determine which cluster this symbol belongs to based on the maximum membership in u_{ik} $(i = 1, 2, \cdots, 16)$. So the 1049 symbols can be clustered to 16 cluster using this hard decision. We compute the variance of each cluster based on this decision. In Fig. 3, for illustration purpose, we scattered one received burst (1013 symbols) and 36 constructed symbols using dotted point, and 16 centers (corresponding to 16 channel states) using circles, when Rician fading $K = 12dB$, $f_d = 20Hz$ for both paths and $E_b/N_0 = 7dB$.

Based on the cluster to which the training symbols (48 symbols in total) have been assigned (based on the maximum membership), we can conclude the category (1, j, -1, or -j according to $s(k)$) of each cluster. Because of the channel fading, ISI, and AWGN, 3 training patterns for each channel state may be clustered to different clusters, we use majority logic to determine each cluster category. There are 4 clusters with category "1", 4 clusters with category "j", 4 clusters with category "-1", and 4 clusters with category "-j".

B.3 Computation Formula

We apply Bayesian equalization to every received symbol in one burst using the following rule: the signal $r(k)$ is a_i $(i = 1, 2, 3, 4)$ and $a_i \in \{1, j, -1, -j\}$ if

$$p(r(k)|s(k) = a_i) > p(r(k)|s(k) = a_l) \quad \forall a_l \neq a_i \tag{10}$$

where

$$p(r(k)|s(k) = a_i) = \sum_{j=1}^{4} p(r(k)|s(k) = a_i, s(k-1) = a_j) \tag{11}$$

To compute $p(r(k)|s(k) = a_i, s(k-1) = a_j)$, let $\mathbf{r} \triangleq [r_I(k), r_Q(k)]^T$,

$$p(r(k)|s(k) = a_i, s(k-1) = a_j) = p(\mathbf{r}|a_i, a_j)$$
$$= \frac{1}{(2\pi)|\Sigma_{\mathbf{ij}}|^{1/2}} \exp[-\frac{1}{2}(\mathbf{r} - \mathbf{m}_{ij})^T \Sigma_{\mathbf{ij}}^{-1} (\mathbf{r} - \mathbf{m}_{ij})] \tag{12}$$

where $\mathbf{m}_{ij} \triangleq [m_{ij}^I, m_{ij}^Q]^T$ and $\Sigma_{\mathbf{ij}} = diag\{\sigma_g^2 + \sigma_n^2, \sigma_g^2 + \sigma_n^2\}$ are the mean vector (2×1) and covariance matrix (2×2) of $[r_I(k), r_Q(k)]^T$ obtained via FCM clustering.

IV. Simulations

We compared our Bayesian equalizer with a nearest neighbor classifier (NNC) equalizer [9] for equalization of mobile satellite channel with 2 paths. The nearest-neighbor (NN) rule, and its extension, the K-NN algorithm [3] (if the number of training prototypes is N, then $K = \sqrt{N}$ is the optimal choice for K), are non-parameteric classification algorithms, that have been extensively applied to many pattern recognition problems. Recently, Savazzi, et al. [9] applied a NNC which used the K-NN algorithm to channel equalization for mobile radio communications and achieved good performance. In our example, we totally have $N = 48$ training symbols (12 symbols are from the provided unique words and 36 symbols are expanded using data mining), which means $K = \sqrt{48} \approx 7$. The NNC equalizer classify the category of $r(k)$ based on the categories of its 7 nearest neighbors from the 48 training symbols.

We studied two Rician fading channels: one with Rician factor $K = 12dB$, doppler shift $f_d = 20Hz$; and the other one with Rician factor $K = 9dB$, doppler shift $f_d = 10Hz$. For both channels, the symbol rate is $202.6ks/s$, i.e., the information (payload) bit rate is $392kb/s$.

For each channel, we ran our simulations for different E_b/N_0 values. At each E_b/N_0 value, we ran the simulations for 5000 bursts, and obtained the average bit error rate (BER) for the FCM-based Bayesian equalizer and NNC-based equalizer. The performances of the two equalizers in both channels are plotted in Figs. 4 and 5. Observe that our Bayesian equalizer performs much better than the NNC equalizer at moderate to high SNR ($E_b/N_0 > 7dB$) in both channels. At low SNR ($E_b/N_0 < 7dB$), NNC equalizer performs better than the Bayesian equalizer, because we only have 13 symbols unique words, and each channel state only has 3 training symbols after we expanded the unique words, and the training symbols could be clustered to a wrong cluster at low SNR.

V. Conclusions

We have proposed a nonlinear equalization scheme for Rician multipath fading channel – a fuzzy logic-based approach. We show that each channel state of multipath satellite channel follows a Gaussian distribution, which means a Bayesian equalizer can be implemented. The parameters of the Bayesian equalizer are determined using an unsupervised clustering method – fuzzy c-means (FCM) method. An extremely small number of training symbols (about 1% of a burst) are used to determine the category of each channel state with the aid of data mining. Simulation results show that our Bayesian equalizer performs much better than the recently proposed nearest classifier-based equalizer at moderate to high SNR.

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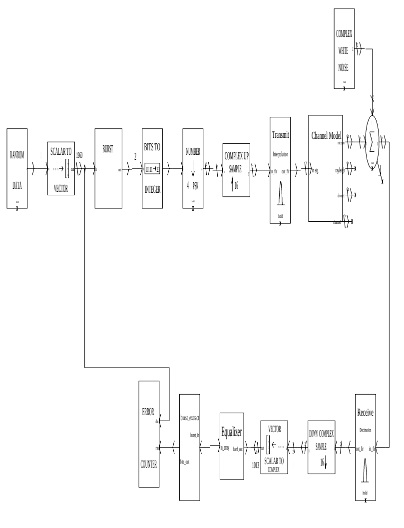

Fig. 1. System model we used in our simulation.

G	Payload	UW	Payload	G
10	490	13	490	10

Fig. 2. Burst format we used in this paper.

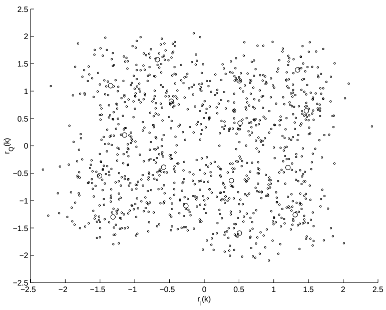

Fig. 3. The centers of 16 channel states (denoted by circles) obtained via FCM clustering when $E_b/N_0 = 7dB$, $K = 12dB$, and $f_d = 20Hz$.

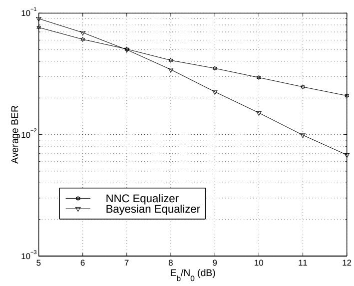

Fig. 4. The performances of nearest neighbor classifier (NNC) equalizer and our Bayesian equalizer for satellite channel with 2 paths and Rician factor $K = 12dB$ and $f_d = 20Hz$.

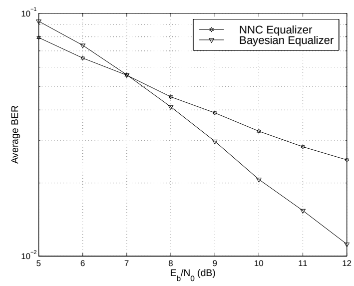

Fig. 5. The performances of nearest neighbor classifier (NNC) equalizer and our Bayesian equalizer for satellite channel with 2 paths and Rician factor $K = 9dB$ and $f_d = 10Hz$.