HIGHER ORDER EVOLUTIONARY SPECTRAL ANALYSIS

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ABSTRACT

Power Spectral Density of a signal is calculated from the second order statistics and provides valuable information for the characterization of stationary signals. This information is only sufficient for Gaussian and linear processes. Whereas, most real-life signals, such as biomedical, speech, and seismic signals may have non-Gaussian, non-linear and non-stationary properties. Higher Order Statistics (HOS) are useful for the analysis of such signals. Time-Frequency (TF) analysis methods have been developed to analyze the time-varying properties of non-stationary signals. In this work, we combine the HOS and the TF approaches, and present a method for the calculation of a Time-Dependent Bispectrum based on the positive distributed Evolutionary Spectrum.

1. INTRODUCTION

Power Spectral Density of signals give valuable information for the characterization of deterministic and random stationary signals.Power spectrum of a signal shows the distribution of power among signal frequency components. This information is only sufficient for Gaussian and linear processes and it does not show any phase relations between frequency components. However, there are non-Gaussian and non-linear processes in practical situations, such as biomedicine, oceanography, sonar, radio astronomy and sunspot data where power spectrum may not give enough information. In such cases, higher than second order statistics of the signal are used for detection of non-Gaussian and non-linear properties of the signal. Higher Order Spectra (HOS), also known as Polyspectra, is defined [7, 8, 9] as the Fourier transform of higher order statistics of a stationary signal. HOS of a signal can be defined in terms of its moments and cumulants. Moments can be very useful in the analysis of deterministic signals whereas cumulants are of great importance in the analysis of random signals.

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There are several general motivations behind the use of higher-order spectra in signal processing [7, 8, 9]: 1) to extract information due to deviations from Gaussianness 2)to suppress Gaussian noise process and to suppress non-Gassian noise with symmetric probability density function 3) to estimate the phase of non-Gaussian signals 4) to detect and characterize non-linearities of signals.

If $x(n)$ is a real-valued stationary random process and its moments up to the order k exist, then k^{th} moment of $x(n)$ can be written as

$$m_k^x(\tau_1, \tau_2, ..., \tau_{k-1})) = E\{x(n)x(n+\tau_1)\dots x(n+\tau_{k-1})\} \quad (1)$$

The k^{th} order moment spectrum is defined as the $(k-1)$ dimensional Fourier transform of the k^{th}-order moment [7]. Special cases of HOS are the third order spectrum, called the Bispectrum, and the fourth order spectrum, called the Trispectrum. If the process is zero mean, the second and third order cumulants are identical to the second and third-order moments, respectively. The Bispectrum of a stationary signal x(n) can be written as,

$$B(\omega_1, \omega_2) = X(\omega_1)X(\omega_2)X^*(\omega_1+\omega_2) \quad (2)$$

where

$$|\omega_i| \leq \pi, \;\; i = 1, 2 \qquad\qquad |\omega_1 + \omega_2| \leq \pi$$

$X(\omega)$ is the Fourier transform of $x(n)$. The physical significance of Bispectrum becomes apparent when we express $x(n)$ as Cramer Spectral representation [3].

$$x(n) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{j\omega n}\, dZ(\omega) and \quad (3)$$

for all n where $Z(\omega)$ is a zero mean, complex valued process with orthogonal increments, i.e., $E\{dZ(\omega)\} = 0$,

$$E\{dZ(\omega_1)dZ^*(\omega_2)\} = \begin{cases} 0, & \omega_1 \neq \omega_2; \\ 2\pi P(\omega), & \omega_1 = \omega_2 = \omega. \end{cases} \quad (4)$$

and

$$E\{dZ(\omega_1)dZ(\omega_2)dZ^*(\omega_3)\} = \begin{cases} B(\omega_1,\omega_2)d\omega_1 d\omega_2, & \omega_1+\omega_2=\omega_3; \\ 0, & \omega_1+\omega_2\neq\omega_3. \end{cases} \quad (5)$$

where $P(\omega)$ is the power spectrum of the signal. It is therefore apparent that the power spectrum represents the contribution to the mean product of two Fourier components whose frequencies are the same, whereas the bispectrum $B(w_1, w_2)$ represents the contribution to the mean product of three Fourier components where one frequency equals the sum of the other two [9].

Higher order spectral methods explained above are useful for the analysis of stationary signals. Whereas real-world signals, such as biomedical, speech, seismic signals have time-varying characteristics. Time-frequency (TF) analysis methods has been developed to analyze the time-varying properties of such signals. The Short-Time Fourier Transform (STFT), Cohen's Class of TF Distributions (TFDs), Positive TFDs, Affine Class of TFDs and Evolutionary Spectral (ES) analysis are employed to analyze the time-varying properties of a signal [1, 2]. It has been agreed that TF analysis is a very useful tool for the characterization of non-stationary signals [1]. In [11, 13, 12], Higher order spectrum based on Wigner distribution is defined. However, WD does not guarantee the positive spectral density. It is explained in [16] that, a positive TF distribution is needed for introducing a Time-Dependent Bispectrum. This work, we present a method where we combine the HOS and the TF approaches for the higher order time-frequency analysis of non-stationary signals.We present a method for calculation the Time-Dependent Bispectrum using the positive distributed Evolutionary Spectrum.

2. EVOLUTIONARY SPECTRUM AND ITS ESTIMATION

A zero-mean non-stationary process x(n) may be represented as Wold-Cramer decomposition[3]:

$$x(n) = \int_{-\pi}^{\pi} H(n,\omega)\, e^{j\omega n}\, dZ(\omega), \qquad 0 \le n \le N-1,$$

$$E[dZ(\omega_1)dZ^*(\omega_2)] = \frac{1}{2\pi}\delta(\omega_1-\omega_2)\, d\omega$$

where $H(n,\omega)$ is a slowly varying amplitude function with a Fourier transform that has maximum value at $\omega = 0$. Since the instantaneous variance of $x(n)$ is given by

$$E\{|x(n)|^2\} = \frac{1}{2\pi}\int_{-\pi}^{\pi} |H(n,\omega)|^2 d\omega, \quad (6)$$

the Wold-Cramer Evolutionary Spectrum is defined as:

$$S(n,\omega) = |H(n,\omega)|^2 \quad (7)$$

In [4], we consider the following discrete-time and discrete-frequency model that is analogous to the Wold-Cramer representation for the non-stationary processes.

$$x(n) = \sum_{k=0}^{K-1} A(n,\omega_k)e^{j\omega_k n}, \qquad 0 \le n \le N-1, \quad (8)$$

where $\omega_k = 2\pi k/K$, K is the number of frequency samples, and $A(n,\omega_k)$ is an evolutionary kernel. We have previously showed that $A(n,\omega_k)$ can be obtained by using conventional signal representations such as the Gabor and the Malvar transforms [4, 6]. For example, the multi-window Gabor Expansion for a finite-support signal $x(n)$ is given as

$$x(n) = \frac{1}{I}\sum_{k=0}^{K-1}\sum_{i=0}^{I-1}\sum_{m=0}^{M-1} a_{i,m,k}\, h_{i,m,k}(n) \quad (9)$$

M and K are the number of time and frequency samples and I is the number of scales used to analyze the signal [4]. $\{a_{i,m,k}\}$ are the Gabor coefficients, $\{h_{i,m,k}\}$ are the Gabor basis functions that are obtained by shifting a single window function in time and frequency:

$$h_{i,m,k}(n) = h_i(n-mL)\, e^{j\omega_k n} \quad (10)$$

where L is the time step. The synthesis window $h_i(n)$ is obtained by scaling a unit-energy mother window $g(n)$ as

$$h_i(n) = 2^{i/2}\, g(2^i n),\ i = 0, 1, \cdots, I-1.$$

The multi-window Gabor coefficients are evaluated by

$$a_{i,m,k} = \sum_{n=0}^{N-1} x(n)\, \gamma_i^*(n-mL)\, e^{-j\omega_k n} \quad (11)$$

where the analysis window $\gamma_i(n)$ is solved from the bi-orthogonality condition between $h_i(n)$ and $\gamma_i(n)$ [4]. The evolutionary kernel is obtained by comparing the spectral and the Gabor representations of the signal:

$$A(n,\omega_k) = \frac{1}{I}\sum_{i=0}^{I-1}\sum_{m=0}^{M-1} a_{i,m,k}\, h_i(n-mL) \quad (12)$$

$$= \frac{1}{I}\sum_{i=0}^{I-1} A_i(n,\omega_k) \quad (13)$$

Replacing for the coefficients $\{a_{i,m,k}\}$, one can also write

$$A(n,\omega_k) = \sum_{\ell=0}^{N-1} x(\ell)\, \mathrm{w}(n,\ell)\, e^{-j\omega_k \ell}, \quad (14)$$

where the time-varying window is defined as

$$\mathrm{w}(n,\ell)=\frac{1}{I}\sum_{i=0}^{I-1}\sum_{m=0}^{M-1}h_i(n-mL)\,\gamma_i^*(\ell-mL).$$

Then the evolutionary spectrum of $x(n)$ is

$$S(n,\omega_k)=\frac{1}{K}\,|A(n,\omega_k)|^2,$$

where the factor $1/K$ is used for proper energy normalization. We should also mention that normalizing the $\mathrm{w}(n,\ell)$ to unit energy, the total energy of the signal is preserved thus justifying the use of $S(n,\omega_k)$ as a TF representation for $x(n)$. Furthermore, $S(n,\omega_k)$ is always positive and hence, in contrast to many TFDs, can be used to obtain a time-dependent higher order spectra.

3. EVOLUTIONARY BISPECTRUM FOR NONSTATIONARY SIGNALS

The time-dependent Bispectrum of a zero mean non- stationary signal is defined as below [14, 15, 16]. A time-dependent third order moment of $x(n)$ can be written as,

$$\begin{aligned}R(n,\tau_1,\tau_2)=E\{x(n)x(n+\tau_1)x(n+\tau_2)\}=\\ \int_{-\pi}^{\pi}\int_{-\pi}^{\pi}\int_{-\pi}^{\pi}H(n,\omega_1)H(n+\tau_1,\omega_2)H(n+\tau_2,\omega_3)\\ \times e^{j(\omega_2\tau_1+\omega_3\tau_2)}\;e^{jn(\omega_1+\omega_2+\omega_3)}\\ \times E\{dZ(\omega_1)dZ(\omega_2)dZ(\omega_3)\}\end{aligned}\quad(15)$$

and for $\omega_1+\omega_2+\omega_3=0$ and $\tau_1=\tau_2=0$ it reduces to

$$\begin{aligned}R(n,0,0)=E\{x(n)^3\}=\\ \int_{-\pi}^{\pi}\int_{-\pi}^{\pi}H(n,\omega_1)H(n,\omega_2)H^*(n,\omega_1+\omega_2)d\omega_1 d\omega_2\end{aligned}\quad(16)$$

Hence, an Evolutionary Bispectrum is defined as:

$$S(n,\omega_1,\omega_2)=H(n,\omega_1)H(n,\omega_2)H^*(n,\omega_1+\omega_2)\quad(17)$$

In this paper we represent an estimation method of Evolutionary Bispectrum for discrete-time non-stationary signals. Using the multi-window Gabor expansion, the evolutionary kernel of $x(n)$ is obtained from the signal by

$$A(n,\omega_k)=\sum_{\ell=0}^{N-1}x(\ell)\,\mathrm{w}(n,\ell)\,e^{-j\omega_k\ell}\quad(18)$$

Hence the Evolutionary Bispectrum can be estimated by

$$\hat{S}(n,\omega_1,\omega_2)=A(n,\omega_1)A(n,\omega_2)A^*(n,\omega_1+\omega_2)\quad(19)$$

Similarly, higher than third order evolutionary spectra can be obtained using $A(n,\omega_k)$.

4. EXPERIMENTAL RESULTS

We consider a signal, composed of sinusoidal and a linear chirp components. Evolutionary spectrum is obtained using the multi-window Gabor expansion with parameters $N=128$, $L=2$, $K=128$, and $I=3$ and given in Fig. 1. A time-dependent Bispectrum for this signal is calculated and shown for two time instants: Fig. 2 at $n=50$ and Fig. 3 at $n=80$. As shown, different information can be obtained from the time slices of this evolutionary bispectrum for practical signals such as EEG, ECG, and speech.

5. CONCLUSIONS

In this paper, we present a method for the calculation of a Time-Dependent Bispectrum for discrete-time, non-stationary signals. This method combines the advantages of higher order statistics and time-frequency methods for the investigation of non-Gaussian, non-linear signal properties. The proposed time-varying bispectrum is obtained based on the evolutionary spectrum which is previously connected with atomic signal decompositions such as multi-window Gabor expansion and the Malvar expansion. Examples show that evolutionary bispectrum provides higher order statistical relations in the signal as a function of time.

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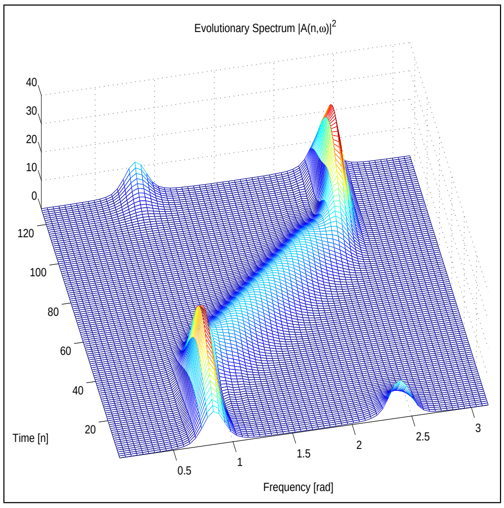

Fig. 1. Evolutionary spectrum of the signal.

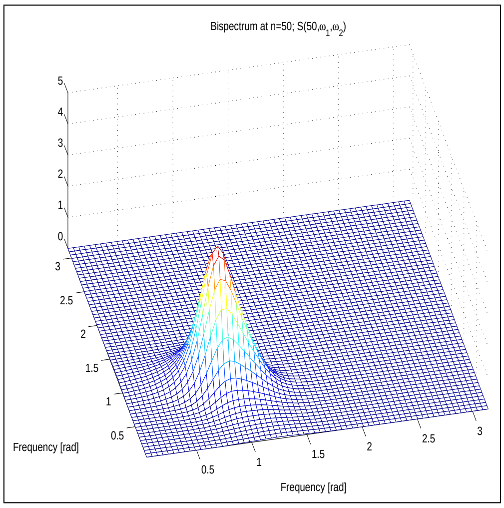

Fig. 2. Evolutionary bispectrum of the signal at $n = 50$.

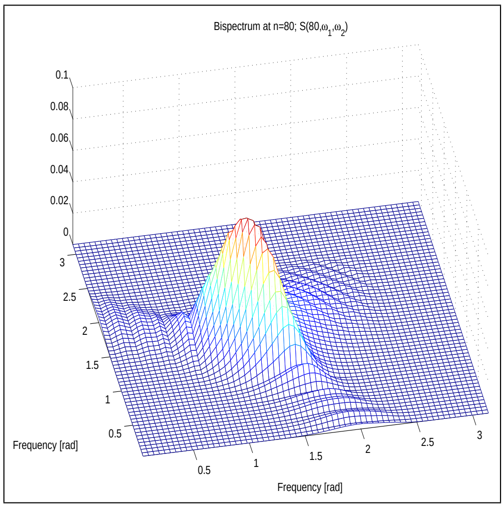

Fig. 3. Evolutionary bispectrum at $n = 80$.