ITERATIVE ALGORITHMS FOR LCMP AUXILIARY-VECTOR FILTER

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ABSTRACT

The iterative algorithm for minimum variance distortion-less response filter based on auxiliary-vector (IMVDR-AV) is generalized in this paper from different perspective. By extending the optimization criteria on filter design, it is generalized to an iterative algorithm of linear constrained minimum power filter (ILCMP-AV). Starting from this algorithm, we present an iterative algorithm of LCMP filter based on local optimization criterion (ILCMP-LOC) which converges rapidly by further generalizing the conditional optimization criterion on weighting coefficient computation. For any positive definite input auto-correlation matrix and any linear constraint, the ILCMP-AV algorithm recursively generates a sequence of auxiliary vectors by maximizing the magnitude cross correlation under some constraint conditions and its corresponding weighting coefficients by minimizing the filter output variance. Theoretical analysis illustrates that the combination of the updated filters and the weighted auxiliary vectors forms a sequence of filters that converges to the LCMP solution.

1. INTRODUCTION

It's well known that LCMP (LCMV) filter minimizes the variance of its output for stationary input signal meanwhile satisfies a linear constraint. The earlier LCMP filter is developed in array signal processing field [1], which is often implemented by matrix inversion operation or by some recursive algorithms such as RLS and LMS. In order to reduce the computational complexity and speed up the convergence of adaptive algorithm without performance deterioration, the study of MVDR filter, which is known as a special case of LCMV filter, or minimum mean square error (MMSE) filter has made a great progress recently. The research interest of MMSE filter now focuses on the multistage Wiener filter (MSWF) [2,3] which concerns more about the reduced-rank interference suppression in direct-sequence code-division multiple access (DS-CDMA) wireless communication systems and array processing, and the development of iterative MVDR (IMVDR) filter concentrates primarily on the auxiliary vector method [4,5]. Although the IMVDR filter and MSWF are derived from different perspective, the spanned subspace in the two filters is equivalent when the auxiliary vectors are orthogonal [6].

In this paper, we extend the IMVDR-AV method [5] into the ILCMP-AV algorithm, in which the auxiliary vector is orthogonal to the space spanned by the constraint vectors rather than the matched filter in the IMVDR-AV algorithm. Start from any initial filter under the linear constraint, the ILCMP-AV algorithm generates a series of auxiliary vectors with maximum magnitude cross correlation criterion under certain constraint conditions and its corresponding coefficients with MMSE criterion. It is proved that the sequence of filters recursively obtained converges to the LCMP filter. A similar idea of a special case of LCMP filter has ever been applied to suppress interference in DS-CDMA communication systems [7], but the convergence of the algorithms hasn't been theoretically analyzed. We also provide a generalized initial vector instead of the matched filter that can speed up the convergence.

In order to further accelerate the convergence of the presented ILCMP-AV algorithm, we generalize the IMVDR-LOC algorithm [8] by applying a local optimization criterion for computing the weighting coefficients that leads to ILCMP-LOC algorithm. By effectively suppressing the interference contribution in the plane spanned by two auxiliary vectors rather than canceling interference only in the direction of one auxiliary vector in each step, less auxiliary vectors are required to achieve the steady status for the ILCMP-LOC algorithm than for the ILCMP-AV method.

In Section 2, the generalized ILCMP-AV algorithm is introduced, and its convergence analysis is provided in Section 3. The ILCMP-LOC algorithm is presented in Section 4. Finally, the concluding remarks are given in Section 5.

2. AN ITERATIVE LCMP-AV ALGORITHM

In this section, an ILCMP-AV algorithm for implementing LCMP filter without matrix inversion is presented by generalizing the IMVDR-AV method.

Without loss of generality, assume that $\mathbf{r}$ is a random, zero mean, complex input vector of dimension N, $\mathbf{r} \in C^N$. Its input autocorrelation matrix $\mathbf{R} = E\{\mathbf{r}^H \mathbf{r}\} \in C^{N \times N}$ is processed by an N-tap filter $\mathbf{w} \in C^N$. The LCMP filter minimizes its output power $\mathbf{w}^H \mathbf{R} \mathbf{w}$, subject to

$$\mathbf{V}^H \mathbf{w} = \mathbf{f} \quad , \qquad (2.1)$$

where $\mathbf{V} = [\mathbf{v}_1, \cdots, \mathbf{v}_M]$, $\mathbf{f} = [f_1, \cdots, f_M]^T$ ($\mathbf{v}_i \in C^N$, $f_i \in C, (i = 1, \cdots, M)$) are the constraint matrix with uncorrelated vectors and corresponding vector of constraint values, M is the number of constraint vectors. $E\{\cdot\}$ denotes the statistical expectation operation, $\mathbf{x}^H$ and $\mathbf{x}^T$ denote the Hermitian transpose and transpose of $\mathbf{x}$, respectively.

The solution of LCMP filter [1] is

$$\mathbf{w}_{LCMP} = \mathbf{R}^{-1}\mathbf{V}\left[\mathbf{V}^H \mathbf{R}^{-1} \mathbf{V}\right]^{-1} \mathbf{f} . \qquad (2.2)$$

It is obvious that there are multiple solutions that meet the constraint equation (2.1) except $\mathbf{w}_{LCMP}$. However, the analysis in next section reveals that the generated sequence of filters will converge to LCMP solution by initiating from any vector satisfying equation (2.1). Motivated by the idea of GSC, we arbitrarily choose an initial vector $\mathbf{w}_0$ satisfying equation (2.1), then by carrying out orthogonal decomposition on $\mathbf{w}_{LCMP}$, we have

$$\mathbf{w}_{LCMP} = \mathbf{w}_0 + \mathbf{u} \quad , \qquad (2.3)$$

where $\mathbf{u} \in C^N$ orthogonal to $\mathbf{w}_0$. From (2.1) and (2.3), we have

$$\mathbf{V}^H \mathbf{u} = \mathbf{V}^H (\mathbf{w}_{LCMP} - \mathbf{w}_0) = \mathbf{V}^H \mathbf{w}_{LCMP} - \mathbf{V}^H \mathbf{w}_0 = \mathbf{0} , \qquad (2.4)$$

which means that $\mathbf{u}$ is orthogonal to the space spanned by the constraint vectors. The remainder of this section will give the method to obtain $\mathbf{u}$ corresponding to $\mathbf{w}_{LCMP}$, which is

$$\mathbf{u} = -\sum_{n=1}^{\infty} \mu_n \mathbf{g}_n \quad , \qquad (2.5)$$

where $\mathbf{g}_n$ is the so-called auxiliary vector and μ_n is its corresponding coefficient, with which the sequence $\{\mathbf{w}_n = \mathbf{w}_{n-1} - \mu_n \mathbf{g}_n (n > 1)\}$ converges to $\mathbf{w}_{LCMP}$.

The auxiliary vector is orthogonal to the matched filter in IMVDR-AV algorithm, while (2.4) indicates that the auxiliary vector must be orthogonal to the space spanned by the constrain vectors in the ILCMP-AV algorithm.

The selection criterion for the auxiliary vector $\mathbf{g}_n$ is to maximize the magnitude of the cross correlation between $\mathbf{w}_{n-1}^H \mathbf{r}$ and $\mathbf{g}_n^H \mathbf{r}$ under some constraints.

***Proposition* 1** The auxiliary vector

$$\mathbf{g}_n = \frac{(\mathbf{I} - \mathbf{V}(\mathbf{V}^H \mathbf{V})^{-1} \mathbf{V}^H) \mathbf{R} \mathbf{w}_{n-1}}{\| (\mathbf{I} - \mathbf{V}(\mathbf{V}^H \mathbf{V})^{-1} \mathbf{V}^H) \mathbf{R} \mathbf{w}_{n-1} \|} ,$$

which maximizes $\mathbf{w}_{n-1}^H \mathbf{R} \mathbf{g}_n$, subject to the constraints

$$\mathbf{V}^H \mathbf{g}_n = \mathbf{0} \text{ and } \mathbf{g}_n^H \mathbf{g}_n = 1 . \qquad (2.6)$$

(The proof is given in the Appendix.)

The value of complex scalar μ_n that minimizes the function $E\left\{ |\mathbf{w}_n^H \mathbf{r}|^2 \right\} = E\left\{ |(\mathbf{w}_{n-1} - \mu_n \mathbf{g}_n)^H \mathbf{r}|^2 \right\}$ is

$$\mu_n = \mathbf{g}_n^H \mathbf{R} \mathbf{w}_{n-1} (\mathbf{g}_n^H \mathbf{R} \mathbf{g}_n)^{-1} \quad . \qquad (2.7)$$

It is clear that the product of μ_n and $\mathbf{g}_n$ is immune to the normalization constraint for $\mathbf{g}_n$, thus we can omit the constraint $\mathbf{g}_n^H \mathbf{g}_n = 1$ when the iterative filter $\mathbf{w}_n = \mathbf{w}_{n-1} - \mu_n \mathbf{g}_n$ is computed.

Now, we consider the selection of initial vector $\mathbf{w}_0$. If input signal $\mathbf{r}$ is complex white Gaussian noise, then the signal autocorrelation $\mathbf{R}$ is an identical matrix, so the LCMP filter in this situation is $\hat{\mathbf{w}}_{LCMP} = \mathbf{V}\left[\mathbf{V}^H \mathbf{V}\right]^{-1} \mathbf{f}$.

In order to accelerate the convergence procedure, we select $\hat{\mathbf{w}}_{LCMP}$ as initial vector $\mathbf{w}_0$, where it is worth clarifying that the initial vector doesn't affect the convergence property of the ILCMP-AV algorithm. This claim will be shown in the next section.

The ILCMP-AV algorithm can be summarized as follows.

$$\mathbf{w}_0 = \hat{\mathbf{w}}_{LCMP} ,$$

$$\mathbf{g}_n = (\mathbf{I} - \mathbf{V}(\mathbf{V}^H \mathbf{V})^{-1} \mathbf{V}^H) \mathbf{R} \mathbf{w}_{n-1} ,$$

$$\mu_n = \mathbf{g}_n{}^H \mathbf{R} \mathbf{w}_{n-1} (\mathbf{g}_n{}^H \mathbf{R} \mathbf{g}_n)^{-1} ,$$

$$\mathbf{w}_n = \mathbf{w}_{n-1} - \mu_n \mathbf{g}_n \quad . \qquad (2.8)$$

If $\mathbf{g}_n$ equals zero vector, then it demonstrates that filter $\mathbf{w}_n$ has already converges to $\mathbf{w}_{LCMP}$. We will prove that $\|\mathbf{g}_n\| \xrightarrow[n\to\infty]{} \mathbf{0}$ in the next section.

3. THE CONVERGENCE ANALYSIS OF THE ILCMP-AV ALGORITHM

In this section, the properties of the auxiliary vector and its weighting coefficient of ILCMP-AV algorithm are generalized from IMVDR-AV method as proposition 2, where the proof is given in the Appendix.

***Proposition* 2** For the Hermitian positive definite matrix $\mathbf{R}$, we have

(1) the generated sequence of weighting coefficients $\{\mu_n\}, n = 1,2,\cdots,$ is real-valued, positive, and bounded as

$$0 < \frac{1}{\lambda_{\max}} \le \mu_n \le \frac{1}{\lambda_{\min}}, \qquad n = 1,2,\cdots \tag{3.1}$$

where $\lambda_{\min}$ and $\lambda_{\max}$ are the minimum and maximum eigenvalues of $\mathbf{R}$, respectively.

(2) the sequence of auxiliary vectors $\{\mathbf{g}_n\}, n = 1,2,\cdots,$ converges to the zero vector, *i.e.*,

$$\lim_{n\to\infty} \mathbf{g}_n = \mathbf{0}. \tag{3.2}$$

With the proposition 2, now we show that $\mathbf{w}_n \to \mathbf{w}_{LCMP}$.

The proposition 2 and (2.8) imply

$$\mathbf{g}_{n+1} = (\mathbf{I} - \mathbf{V}(\mathbf{V}^H\mathbf{V})^{-1}\mathbf{V}^H)\mathbf{R}\mathbf{w}_n \xrightarrow[n\to\infty]{} \mathbf{0}. \tag{3.3}$$

Multiply both sides of (3.3) by $\mathbf{R}^{-1}$, we obtain

$$\mathbf{w}_n - \mathbf{R}^{-1}\mathbf{V}(\mathbf{V}^H\mathbf{V})^{-1}\mathbf{V}^H\mathbf{R}\mathbf{w}_n \xrightarrow[n\to\infty]{} \mathbf{0}. \tag{3.4}$$

For simplicity of notation and convenience of proof, we define a vector sequence $\{\boldsymbol{\alpha}_n = (\mathbf{V}^H\mathbf{V})^{-1}\mathbf{V}^H\mathbf{R}\mathbf{w}_n, n = 0,1,2,\cdots\}$.

By describing (3.4) with the basic definition of limit in a matrix norm space, (3.4) means that for each given $\varepsilon > 0$, there exists some $N > 0$ such that for every $n > N$,

$$\|\mathbf{w}_n - \mathbf{R}^{-1}\mathbf{V}\boldsymbol{\alpha}_n\| < \varepsilon \cdot \|\mathbf{V}^H\|^{-1}, \tag{3.5}$$

i.e.,
$$\|\mathbf{V}^H(\mathbf{w}_n - \mathbf{R}^{-1}\mathbf{V}\boldsymbol{\alpha}_n)\| < \varepsilon. \tag{3.6}$$

Due to $\mathbf{V}^H\mathbf{w}_n = \mathbf{f}, n = 0,1,2,\cdots$, (3.6) becomes $\|\mathbf{f} - \mathbf{V}^H\mathbf{R}^{-1}\mathbf{V}\boldsymbol{\alpha}_n\| < \varepsilon$. Multiply both sides of it by $\|(\mathbf{V}^H\mathbf{R}^{-1}\mathbf{V})^{-1}\|$, then we have

$$\|(\mathbf{V}^H\mathbf{R}^{-1}\mathbf{V})^{-1}\| \cdot \|\mathbf{f} - \mathbf{V}^H\mathbf{R}^{-1}\mathbf{V}\boldsymbol{\alpha}_n\| < \varepsilon \|(\mathbf{V}^H\mathbf{R}^{-1}\mathbf{V})^{-1}\|$$

$$\Rightarrow \|(\mathbf{V}^H\mathbf{R}^{-1}\mathbf{V})^{-1}\mathbf{f} - \boldsymbol{\alpha}_n\| < \varepsilon \|(\mathbf{V}^H\mathbf{R}^{-1}\mathbf{V})^{-1}\|. \tag{3.7}$$

It is evident that the vector sequence $\boldsymbol{\alpha}_n$ converges to $(\mathbf{V}^H\mathbf{R}^{-1}\mathbf{V})^{-1}\mathbf{f}$. Hence we know from (3.4) that

$$\mathbf{w}_n \xrightarrow[n\to\infty]{} \mathbf{R}^{-1}\mathbf{V}(\mathbf{V}^H\mathbf{R}^{-1}\mathbf{V})^{-1}\mathbf{f}. \tag{3.8}$$

It's obvious from the derivation that the initial vector $\mathbf{w}_0$ is not limited to any form. It's only required to satisfy the linear constraints (2.1).

4. AN ITERATIVE LCMP-LOC ALGORITHM

As we have known, the auxiliary vector $\mathbf{g}_n$ in ILCMP-AV algorithm can capture most of the interference contribution in the maximum magnitude cross correlation sense. By estimating the corresponding weight coefficient μ_n via minimizing $E\{|\mathbf{w}_n^H\mathbf{r}|^2\}$, the interference contribution in the direction of $\mathbf{g}_n$ is suppressed in the MSE sense.

Note that the ILCMP-AV algorithm accomplishes a conditional optimization in the direction of only one auxiliary vector in each step, which is unnecessary. Therefore, we present a more generalized algorithm named ILCMP-LOC algorithm which aims at speeding up the convergence of the iterative algorithm by introducing local optimization in a space spanned by two auxiliary vectors $\mathbf{g}_n$ and $\mathbf{g}_{n+1}$.

Compute the auxiliary vectors $\mathbf{g}_n$ and $\mathbf{g}_{n+1}$ with ILCMP-AV algorithm, then estimate μ_n and μ_{n+1} for available $\mathbf{g}_n$ with $\mathbf{g}_{n+1}$ through minimizing

$$E\{|\mathbf{w}_{n+1}^H\mathbf{r}|^2\} = E\{|(\mathbf{w}_{n-1} - \mu_n\mathbf{g}_n - \mu_{n+1}\mathbf{g}_{n+1})^H\mathbf{r}|^2\}, \tag{4.1}$$

we have the ILCMP-LOC algorithm. In following formulas of the algorithm, we denote the auxiliary vector, the coefficient and the detector as $\tilde{\mathbf{g}}_n$, $\tilde{\mu}_n$ and $\tilde{\mathbf{w}}_n$, respectively, where $\tilde{\mathbf{w}}_0 = \mathbf{w}_0$.

P*roposition*3 The scalar $\tilde{\mu}_{n-1}, \tilde{\mu}_n$ that minimizes the variance $E\{|(\tilde{\mathbf{w}}_{n-2} - \tilde{\mu}_{n-1}\tilde{\mathbf{g}}_{n-1} - \tilde{\mu}_n\tilde{\mathbf{g}}_n)^H\mathbf{r}|^2\}$ at the output of $\tilde{\mathbf{w}}_n$ is,

$$\tilde{\mu}_{n-1} = \frac{(\tilde{\mathbf{g}}_{n-1}^H\mathbf{R}\tilde{\mathbf{w}}_{n-2})}{(\tilde{\mathbf{g}}_{n-1}^H\mathbf{R}\tilde{\mathbf{g}}_{n-1}) - \dfrac{|\tilde{\mathbf{g}}_n^H\mathbf{R}\tilde{\mathbf{g}}_{n-1}|^2}{(\tilde{\mathbf{g}}_n^H\mathbf{R}\tilde{\mathbf{g}}_n)}},$$

$$\tilde{\mu}_n = -\frac{\tilde{\mathbf{g}}_n^H\mathbf{R}\tilde{\mathbf{g}}_{n-1}}{\tilde{\mathbf{g}}_n^H\mathbf{R}\tilde{\mathbf{g}}_n} \cdot \tilde{\mu}_{n-1}. \tag{4.2}$$

The proof of the proposition is omitted because of the length limitation in this paper, where $\tilde{\mathbf{g}}_n^H \mathbf{R}\tilde{\mathbf{w}}_{n-2} = 0$ is applied in the derivation, which is derived in Appendix (A.6).

The auxiliary vector, corresponding weighting coefficient and LCMP filter are computed according to (2.8) when n is odd number. When n is even, the sequence of filters $\tilde{\mathbf{w}}_n$ can be calculated as

$$\tilde{\mathbf{w}}_n = \tilde{\mathbf{w}}_{n-2} - \tilde{\mu}_{n-1}\tilde{\mathbf{g}}_{n-1} - \tilde{\mu}_n\tilde{\mathbf{g}}_n ,$$

where $\tilde{\mathbf{g}}_n = (1 - \mathbf{V}(\mathbf{V}^H\mathbf{V})^{-1}\mathbf{V}^H)\mathbf{R}\tilde{\mathbf{w}}_{n-1}$, $\tilde{\mu}_{n-1}$ and $\tilde{\mu}_n$ can be calculated from (4.2).

We know from (2.6) that the auxiliary vectors of both ILCMP-AV and ILCMP-LOC algorithms lie in the blocking matrix space, where ILCMP-AV filter suppresses the interference in one dimension of the space spanned by the auxiliary vectors, and ILCMP-LOC filter can optimally cancel the interference in two dimension of the space simultaneously. It's not hard to understand intuitively that ILCMP-LOC algorithm may have more rapid convergence, which has been illustrated for IMVDR-LOC by simulation in [8].

5.CONCLUSION

In this paper, an iterative algorithm of LCMP filter is presented by generalizing the iterative MVDR method and the theoretical analysis of its convergence is provided. At the same time, based on the local optimization criteria an ILCMP-LOC algorithm is presented which can accelerate the convergence procedure compared to the ILCMP-AV method by extending the result in [8].

APPENDIX

A. Proof of Proposition 1

Consider the Lagrarian function

$$L(\mathbf{g}) = \mathbf{g}^H\mathbf{R}\mathbf{w} - \mathbf{g}^H\mathbf{V}\cdot\boldsymbol{\lambda} - \beta(\mathbf{g}^H\mathbf{g} - 1) , \qquad \text{(A.1)}$$

where $\boldsymbol{\lambda} = [\lambda_1 \ \cdots \ \lambda_M]^T, \lambda_i \in C \quad i = 1,\cdots,M$ and $\beta \in C$.

Let its conjugate gradient equal to the null vector

$$\nabla_{\mathbf{g}^*} L(\mathbf{g}) = \mathbf{R}\mathbf{w} - \mathbf{V}\cdot\boldsymbol{\lambda} - \beta\cdot\mathbf{g} = \mathbf{0} \Rightarrow \mathbf{g} = \frac{1}{\beta}(\mathbf{R}\mathbf{w} - \mathbf{V}\cdot\boldsymbol{\lambda}) . \qquad \text{(A.2)}$$

Consider the constraints on $\mathbf{g}$ in (2.6), we have

$$\mathbf{V}^H\mathbf{g} = \frac{1}{\beta}(\mathbf{V}^H\mathbf{R}\mathbf{w} - \mathbf{V}^H\mathbf{V}\cdot\boldsymbol{\lambda}) = \mathbf{0} \Rightarrow \boldsymbol{\lambda} = (\mathbf{V}^H\mathbf{V})^{-1}\mathbf{V}^H\mathbf{R}\mathbf{w} , \qquad \text{(A.3)}$$

where $\mathbf{g}$ can be computed with normalization in (A.2).

B. Proof of Proposition 2

(1) Consider two successively generated auxiliary vectors $\mathbf{g}_n$ and $\mathbf{g}_{n+1}, n \geq 1$. Since $\mathbf{V}^H\mathbf{g}_n = \mathbf{0}$, we can obtain

$$\mathbf{g}_n = (\mathbf{I} - \mathbf{V}(\mathbf{V}^H\mathbf{V})^{-1}\mathbf{V}^H)\mathbf{g}_n . \qquad \text{(A.4)}$$

The property of $\mathbf{g}_n$ orthogonal to $\mathbf{g}_{n+1}$ can be derived as

$$\mathbf{g}_n^H\mathbf{g}_{n+1} = \mathbf{g}_n^H(\mathbf{I} - \mathbf{V}(\mathbf{V}^H\mathbf{V})^{-1}\mathbf{V}^H)\mathbf{R}\mathbf{w}_n = \mathbf{g}_n^H\mathbf{R}\mathbf{w}_n \overset{(2.8)}{=} 0 . \qquad \text{(A.5)}$$

From (A.4) and (2.8), we have

$$\mathbf{g}_n^H\mathbf{R}\mathbf{w}_{n-2} = \mathbf{g}_n^H\mathbf{g}_{n-1} = 0 , \qquad \text{(A.6)}$$

$$\mu_n = \frac{\mathbf{g}_n^H\mathbf{R}\mathbf{w}_{n-1}}{\mathbf{g}_n^H\mathbf{R}\mathbf{g}_n} = \frac{\mathbf{g}_n^H(\mathbf{I} - \mathbf{V}(\mathbf{V}^H\mathbf{V})^{-1}\mathbf{V}^H)\mathbf{R}\mathbf{w}_{n-1}}{\mathbf{g}_n^H\mathbf{R}\mathbf{g}_n} = \frac{\mathbf{g}_n^H\mathbf{g}_n}{\mathbf{g}_n^H\mathbf{R}\mathbf{g}_n} ,$$

Hence we can obtain the bound of μ_n [5].

(2) It is easy to prove that $\mathbf{g}_n$ converges to the null vector using the similar method mentioned in [5].

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