AN LMI–BASED DECENTRALIZED $\mathcal{H}_\infty$ FILTERING FOR INTERCONNECTED LINEAR SYSTEMS

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ABSTRACT

This paper focuses on decentralized $\mathcal{H}_\infty$ filtering problem for interconnected linear systems. The problem we address is to find a decentralized filter where each local filter is based only on local available information on its own subsystem and the overall filtering error is totally asymptotically stable and the $\mathcal{L}_2$-gain from the exogenous noise input to the filtering error less than a prespecified level. This paper shows that the decentralized $\mathcal{H}_\infty$ filtering problem can be solved by using linear matrix inequality (LMI) techniques, which are numerically efficient due to recent advances in convex optimization.

1. INTRODUCTION

A number of large-scale systems founded in the real world are composed of a set of small interconnected subsystems, such as power systems, digital communication networks, economic systems and urban traffic networks. The filtering for processing of data in large-scale interconnected dynamic systems often involves the measurement and manipulation of a great numbers of variables. It is generally impossible to design a fully centralized filter where all measurements from all the sensors are gathered in one vector and are processed centrally, and it is also too costly and numerically unreliable even if they can be implemented. These difficulties motivate the development of decentralized filtering theory where each local filter is constructed independently on the basis of its own performance criterion and locally available information, thus processes its own data of each subsystem. Therefore the computational requirements can be significantly reduced by the decentralized technique.

During the past few years, the Kalman filtering method has been extended to the decentralized filtering of large-scale systems. In [2, 6, 12] and references therein, A decentralized hierarchical or two-stage Kalman filtering structure was proposed, that is, the local filters (processor) process their own data in parallel to yield the best possible local estimates in the first stage (low-level) and their solutions are then sent and fused by the central or master filter (processor) to make a best possible global estimate in the second stage (high-level). In [8, 13, 11, 14] and references therein, the decentralized filtering approach has no central filter (processor), i.e., the decentralized filter with an non-hierarchical structure, but employs the principle of information exchange among the local Kalman filters. That is, in the dynamical model of the each local estimator, there is an interconnected term related to the estimated states of other local estimators. This strategy yields in general a stable and globally suboptimal estimate for the overall system.

Note that the filtering methods mentioned above do not yield fully decentralized, and have still large communication and computational loads, and the estimate results are difficult to be applied to decentralized control of large-scale systems [10]. An fully decentralized filter is that the data of each sensor is processed locally and there is no central processing filter and information exchange among the local filters. It is well known that if standard filtering theory is used to design local filter where the subsystem are treated as if they were decoupled, then this passive filter design without any regard for the interaction terms may be overly conservative and result in unsatisfactory performance of the system when the local filters are used in the composite system with interconnections. For example, it can easily be shown that the filtering error for each subsystem will be governed by the unaccounted interconnection term and there will in general be no guarantee for the convergence of the estimate. It is therefore necessary to account for the interaction term in designing the fully decentralized filter. This paper belongs to this case.

On the other hand, the existing centralized and decentralized filters are derived under the assumption that the noise sources are 'white' processes with known statistics. In many applications, this would be an excessive prior knowledge of the nature of noise. This motivated the study of filtering in an $\mathcal{H}_\infty$ framework, which reflects the worst-case "gain" of the system. In the $\mathcal{H}_\infty$ setting, the exogenous noise sources are assumed to be energy bounded rather than Gaussian. An $\mathcal{H}_\infty$ filter is designed to guarantee a prescribed $\mathcal{H}_\infty$ performance for the filtering error for all admissible noises. Although many results in centralized $\mathcal{H}_\infty$ filtering have been obtained, see [9, 15, 16] and references therein, however, to my best knowledge, the $\mathcal{H}_\infty$ filtering method is not applied to large-scale systems yet. In this paper, we will design a fully decentralized $\mathcal{H}_\infty$ filter which such that the overall filtering error is totally asymptotically stable and the $\mathcal{L}_2$-gain from the exogenous noise input to the filtering error less than a prescribed level. In this paper, we also show that the decentralized $\mathcal{H}_\infty$ filtering problem can be solved in terms of linear matrix inequalities (LMI's). An efficient algorithm exists for solving LMI's [5].

2. PROBLEM STATEMENT AND PRELIMINARIES

Consider the following interconnected linear systems which consists of N subsystems:

$$\begin{aligned} \dot{x}_i &= A_i x_i + B_i \omega_i + \sum_{j=1, j\neq i}^{N} A_{ij} x_j \\ y_i &= C_i x_i + D_i \omega_i \\ z_i &= L_i x_i + E_i \omega_i, \qquad i = 1, 2, \cdots, N \end{aligned} \tag{2.1}$$

where $x_i \in \Re^{n_i}$ is the state of the ith subsystem, $\omega_i \in \Re^{q_i}$ is the exogenous noise input belonging to $\mathcal{L}_2[0\ \infty)$, $y_i \in \Re^{m_i}$ is the measured output of the ith subsystem, $z_i \in \Re^{r_i}$ is the signal to be estimated, A_i, B_i, C_i, D_i, L_i, E_i and A_{ij} are known real constant matrices with appropriate dimensions. Furthermore, the A_{ij} represents the interaction effect between the ith subsystem and the jth subsystem.

In this paper, we will consider decentralized filter of the form:

$$\begin{aligned} \dot{x}_{if} &= A_{if} x_{if} + B_{if} y_i \\ z_{if} &= C_{if} x_{if} + D_{if} y_i \end{aligned} \tag{2.2}$$

where $x_{if} \in \Re^{n_i}$ and $z_{if} \in \Re^{r_i}$ are the estimated state (filter state) and the estimate of output signal z_i, respectively, A_{if}, B_{if}, C_{if} and D_{if} are constant matrices with appropriate dimensions to be designed latter.

Define the filtering error as

$$e_i = z_i - z_{if}. \tag{2.3}$$

Then, the filtering error dynamic model is given by

$$\begin{aligned} \dot{x}_{ie} &= A_{ie} x_{ie} + B_{ie} \omega_i + \sum_{j=1, j\neq i}^{N} \tilde{A}_{ij} x_j \\ e_i &= L_{ie} x_{ie} + E_{ie} \omega_i \end{aligned} \tag{2.4}$$

where $x_{ie} = [x_i^T\ \ x_{if}^T]^T$ and

$$A_{ie} = \begin{bmatrix} A_i & 0 \\ B_{if} C_i & A_{if} \end{bmatrix},\ B_{ie} = \begin{bmatrix} B_i \\ B_{if} D_i \end{bmatrix},\ \tilde{A}_{ij} = \begin{bmatrix} A_{ij} \\ 0 \end{bmatrix},$$
$$L_{ie} = [L_i - D_{if} C_i \quad -C_{if}], \qquad E_{ie} = E_i - D_{if} D_i$$

In this paper, we deal with the *decentralized $\mathcal{H}_\infty$ filtering* problem for the interconnected system (2.1). More precisely, *given scalars $\gamma_i > 0$, $i = 1, 2, \cdots, N$, we are concerned with the design of decentralized filter (2.2) for each subsystem such that the overall error dynamics (2.4) is totally asymptotically stable and the $\mathcal{L}_2$-gain from the exogenous noise input $\omega = [\omega_1^T\ \cdots\ \omega_N^T]^T$ to the filtering error $e = [e_i^T\ \cdots\ e_N^T]^T$ less than $\gamma = [\gamma_1\ \cdots\ \gamma_N]^T$ in the sense that*

$$\sum_{i=1}^{N} \int_0^N e_i^T e_i dt < \sum_{i=1}^{N} \gamma_i^2 \int_0^N \omega_i^T \omega_i dt + \delta(x_{e0}) \tag{2.5}$$

for all $\omega_i \in \mathcal{L}_2[0\ \infty)$, where $\delta(x_{e0})$ with $\delta(0) = 0$ is a real-valued function of the initial state $x_{e0} = [x_{1e}^T(0)\ \cdots\ x_{Ne}^T(0)]^T$.

Remark 2.1 *In the above problem statement, γ_i, $i = 1, 2, \cdots, N$ can be regarded as the prespecified level of disturbance attenuation for each individual subsystem. When $\gamma_i = \gamma_0$, $i = 1, 2, \cdots N$, (2.5) becomes*

$$\|e\|_2^2 = \gamma_0^2 \|\omega\|_2^2 + \delta(x_{e0}) \tag{2.6}$$

for all $\omega_i \in \mathcal{L}_2[0\ \infty)$. This case is called as standard decentralized $\mathcal{H}_\infty$ filtering *problem.*

Remark 2.2 *Note that the decentralized state estimation problem is a special case of the above decentralized $\mathcal{H}_\infty$ filtering problem with $L_i = I$, $E_i = 0$, $C_{if} = I$ and $D_{if} = 0$.*

Remark 2.3 *The decentralized filter (2.2) is fully decentralized form where the local filter is based only on the local available information of its own subsystem, and there is not hierarchical structure and information exchange among other local filter.*

3. ANALYSIS OF DECENTRALIZED $\mathcal{H}_\infty$ FILTER

The decentralized $\mathcal{H}_\infty$ filter analysis problem associated with the interconnected system (2.1) is as follows: Given $\gamma_i > 0$, $i = 1, 2, \cdots, N$, and a decentralized filter of the form (2.2), we will determine conditions under which the error dynamics (2.4) is totally asymptotically stable and satisfies (2.5) for all $\omega_i \in \mathcal{L}_2[0\ \infty)$.

The following theorem give the main result of the $\mathcal{H}_\infty$ filter analysis.

Theorem 3.1 *The following conditions, all guaranteeing the solution to the decentralized $\mathcal{H}_\infty$ filter analysis problem associated with the interconnected system (2.1) and the filter (2.2), are equivalent.*

(i) There exist matrices $P_i = P_i > 0$ and $Q_{ij} = Q_{ij} > 0$, $i, j = 1, 2, \cdots, N$, such that

$$\mathcal{L}_1 = \begin{bmatrix} \Xi_i & P_i B_{ie} + L_{ie}^T E_{ie} \\ B_{ie}^T P_i + E_{ie}^T L_{ie} & E_{ie}^T E_{ie} - \gamma_i^2 I \end{bmatrix} < 0 \tag{3.1}$$

where

$$\Xi_i = P_i A_{ie} + A_{ie}^T P_i + P_i R_i^* P_i + H_{ie}^T Q_{ie}^{-1} H_{ie} + L_{ie}^T L_{ie}$$

$$R_i = \sum_{j=1, j\neq i}^{N} Q_{ij}, \qquad R_i^* = \begin{bmatrix} R_i & 0 \\ 0 & 0 \end{bmatrix}, \tag{3.2}$$

$$H_i = [A_{1i}^T\ \ A_{2i}^T \cdots A_{i-1,i}^T\ \ A_{i+1,i}^T \cdots A_{Ni}^T]^T, \tag{3.3}$$

$$H_{ie} = [H_i\ \ 0] \tag{3.4}$$

$$Q_{ie} = diag\{Q_{1i}, \cdots, Q_{i-1,i}, Q_{i+1,i}, \cdots, Q_{Ni}\} \tag{3.5}$$

(ii)There exist matrices $P_i = P_i > 0$ and $Q_{ij} = Q_{ij} > 0$, $i, j = 1, 2, \cdots, N$, such that

$$\mathcal{L}_2 = \begin{bmatrix} P_i A_{ie} + A_{ie}^T P_i & P_i B_{ie} & L_{ie}^T & H_{ie}^T & P_i \begin{bmatrix} I \\ 0 \end{bmatrix} R_i \\ B_{ie}^T P_i & -\gamma_i^2 I & E_{ie}^T & 0 & 0 \\ L_{ie} & E_{ie} & -I & 0 & 0 \\ H_{ie} & 0 & 0 & -Q_{ie} & 0 \\ R_i [I\ \ 0] P_i & 0 & 0 & 0 & -R_i \end{bmatrix} < 0 \tag{3.6}$$

where R_i, H_{ie} and Q_{ie} are given in (3.2)–(3.5).

Proof: Define $V_i = x_{ie}^T P_i x_{ie}$, the time derivative of V_i along the state trajectory of the system (2.1) is

$$\begin{aligned}\dot{V}_i =& x_{ie}^T(P_i A_{ie} + A_{ie}^T P_i)x_{ie} + 2x_{ie}^T P_i B_{ie}\omega_i \\ &+2x_{ie}^T P_i \sum_{j=1, j\neq i}^{N} \tilde{A}_{ij} x_j \end{aligned} \quad (3.7)$$

Notice that for any vectors x, y and any matrix $Q = Q^T > 0$ of appropriate dimensions

$$2x^T y \leq x^T Q x + y^T Q^{-1} y$$

Hence, we have that for any some symmetric positive matrices $Q_{ij} > 0$, $i, j = 1, 2, \cdots, N$, the following inequalities hold

$$\begin{aligned}2x_{ie}^T P_i \sum_{j=1,j\neq i}^{N} \tilde{A}_{ij} x_j =& \sum_{j=1,j\neq i}^{N} 2x_{ie}^T P_i \begin{bmatrix} I \\ 0 \end{bmatrix} A_{ij} x_j \\ \leq& x_{ie}^T P_i \sum_{j=1,j\neq i}^{N} \begin{bmatrix} I \\ 0 \end{bmatrix} Q_{ij} P_i [I \;\; 0] x_{ie} \\ &+ \sum_{j=1,j\neq i}^{N} x_j^T A_{ij}^T Q_{ij}^{-1} A_{ij} x_j \\ =& x_{ie}^T P_i R_i^* P_i x_{ie} + \Delta_{ij} \end{aligned} \quad (3.8)$$

where R_i^* is as in (3.2) and

$$\Delta_{ij} \triangleq \sum_{j=1,j\neq i}^{N} x_j^T A_{ij}^T Q_{ij}^{-1} A_{ij} x_j$$

Also define

$$\Delta_{ji} \triangleq \sum_{j=1,j\neq i}^{N} x_i^T A_{ji}^T Q_{ji}^{-1} A_{ji} x_i$$

Then, we have that

$$\sum_{i=1}^{N} \Delta_{ij} = \sum_{i=1}^{N} \Delta_{ji} \quad (3.9)$$

and

$$\Delta_{ji} = \sum_{j=1,j\neq i}^{N} x_{ie}^T \begin{bmatrix} A_{ji}^T \\ 0 \end{bmatrix} Q_{ji}^{-1} [A_{ji} \;\; 0] x_{ie} = x_{ie}^T H_{ie}^T Q_{ie}^{-1} H_{ie} x_{ie} \quad (3.10)$$

It follows from (3.7), (3.8) and (3.10) that

$$\begin{aligned}&\dot{V}_i + e_i^T e_i - \gamma_i^2 \omega_i^T \omega_i \\ &\leq x_{ie}^T(P_i A_{ie} + A_{ie}^T P_i + P_i R_i^* P_i)x_{ie} + 2x_{ie}^T P_i B_{ie}\omega_i + \Delta_{ij} \\ &\quad + x_{ie}^T L_{ie}^T L_{ie} x_{ie} + 2x_{ie}^T L_{ie}^T E_{ie}\omega_i + \omega_i^T E_{ie}^T E_{ie}\omega_i - \gamma_i^2 \omega_i^T \omega_i \\ &= [x_{ie}^T \;\; \omega_i^T]\mathcal{L}_1 \begin{bmatrix} x_{ie} \\ \omega_i \end{bmatrix} + \Delta_{ij} - \Delta_{ji} \end{aligned} \quad (3.11)$$

Thus, in view of (3.9) and (3.11), we have

$$\sum_{i=1}^{N} \left[\dot{V}_i + e_i^T e_i - \gamma_i^2 \omega_i^T \omega_i\right] \leq \sum_{i=1}^{N} [x_{ie}^T \;\; \omega_i^T]\mathcal{L}_1 \begin{bmatrix} x_{ie} \\ \omega_i \end{bmatrix} \quad (3.12)$$

Hence, if $\mathcal{L}_1 < 0$, then it follows from (3.12) that when $\omega_i = 0$, the error dynamics (2.4) is totally asymptotically stable.

Integrating the above inequality (3.12) over $[0 \;\; \infty)$, and noting that $V_i(x_{ie}) \geq 0$, we have

$$\sum_{i=1}^{N} \int_0^N e_i^T e_i dt < \sum_{i=1}^{N} \gamma_i^2 \int_0^N \omega_i^T \omega_i dt + \delta(x_{e0}) \quad (3.13)$$

where $\delta(x_{e0}) = \sum_{i=1}^{N} V_i(x_{ie}(0))$. Therefore, the decentralized $\mathcal{H}_\infty$ filter analysis problem associated with the interconnected system (2.1) and the filter (2.2) is solved.

(ii) Using Schur complements, we obtain that the inequalities (3.1) and (3.6) are equivalent. The proof of this theorem is completed. □

4. SYNTHESIS OF DECENTRALIZED $\mathcal{H}_\infty$ FILTER

In this section, we will deal with the decentralized $\mathcal{H}_\infty$ filter design problem defined in Section 2. To this end, we need the following assumption:

Assumption 4.1
(a) A_i is asymptotically stable
(b) $(A_i \;\; C_i)$ is detectable, $i = 1, 2, \cdots, N$.

Note that in the matrix inequalities (3.1) and (3.6), the decentralized $\mathcal{H}_\infty$ filter parameters A_{if}, B_{if}, C_{if} and D_{if} are unknown and occur in nonlinear fashion, therefore, (3.1) and (3.6) can not be considered as an LMI problem. In the sequel, we shall use a method of changing variables [3, 4] such that (3.6) is reduced to two LMIs for given positive matrices Q_{ij}, $i, j = 1, 2, \cdots, N$. Therefore, the decentralized $\mathcal{H}_\infty$ filter parameters can be designed based on LMI technique.

First, partition P_i and its inverse as

$$P_i = \begin{bmatrix} S_i & N_i \\ N_i^T & V_i \end{bmatrix}, \qquad P_i^{-1} = \begin{bmatrix} T_i & M_i \\ M_i^T & U_i \end{bmatrix} \quad (4.1)$$

where $S_i,\ T_i,\ M_i,\ N_i \in \Re^{n_i \times n_i}$.

Note that the identity $P_i P_i^{-1} = I$ gives

$$M_i N_i^T = I - T_i S_i \quad (4.2)$$

Define

$$\phi_{i1} = \begin{bmatrix} T_i & I \\ M_i^T & 0 \end{bmatrix}, \qquad \phi_{i2} = \begin{bmatrix} I & S_i \\ 0 & N_i^T \end{bmatrix} \quad (4.3)$$

Then the following identity holds

$$P_i \phi_{i1} = \phi_{i2}$$

It also follows that

$$\phi_{i1}^T P_i \phi_{i1} = \phi_{i1}^T \phi_{i2} = \begin{bmatrix} T_i & I \\ I & S_i \end{bmatrix} \quad (4.4)$$

Now, define the new controller variables as

$$\begin{aligned}\mathcal{C}_{if} &\triangleq C_{if} M_i^T + D_{if} C_i T_i && (4.5) \\ \mathcal{B}_{if} &\triangleq N_i B_{if} && (4.6) \\ \mathcal{A}_{if} &\triangleq S_i A_i T_i + N_i B_{if} C_i T_i + N_i A_{if} M_i^T && (4.7)\end{aligned}$$

Therefore, given positive definite matrices T_i, S_i and invertible matrices D_{if}, M_i and N_i, the controller matrices A_{if}, B_{if} and C_{if} can be uniquely determined by $\mathcal{A}_{if}$, $\mathcal{B}_{if}$ and $\mathcal{C}_{if}$.

We are now in the position to state our main result on decentralized $\mathcal{H}_\infty$ filter design based on an LMI approach.

Theorem 4.1 *Given $\gamma_i > 0$, $i = 1, 2 \cdots, N$, consider the interconnected system (2.1) satisfying Assumption 4.1. Then there exists a decentralized $\mathcal{H}_\infty$ filter of the form (2.2) such that the overall error dynamics (2.4) is totally asymptotically stable and the $\mathcal{L}_2$-gain satisfies (2.5), if there exist some matrices $T_i = T_i^T > 0$, $S_i = S_i^T > 0$, $\mathcal{A}_{if}$, $\mathcal{B}_{if}$, $\mathcal{C}_{if}$ and D_{if}, $i, j = 1, 2, \cdots, N$, satisfying the following set of LMIs*

$$\begin{bmatrix} T_i & I \\ I & S_i \end{bmatrix} > 0, \quad (4.8)$$

$$J_i = \begin{bmatrix} J_{i11} & J_{i12} \\ J_{i12}^T & J_{i22} \end{bmatrix} < 0 \quad (4.9)$$

where

$$J_{i11} = \begin{bmatrix} A_iT_i + T_iA_i^T & A_i + \mathcal{A}_{if}^T \\ A_i^T + \mathcal{A}_{if} & A_i^TS_i + S_iA_i + C_i^T\mathcal{B}_{if}^T + \mathcal{B}_{if}C_i \end{bmatrix},$$

$$J_{i12} = \begin{bmatrix} B_i & T_iL_i^T - \mathcal{C}_{if}^T & T_iH_i^T & R_i \\ S_iB_i + \mathcal{B}_{if}D_i & L_i^T - C_i^TD_{if}^T & H_i^T & S_iR_i \end{bmatrix},$$

$$J_{i22} = \begin{bmatrix} -\gamma_i^2 I & (E_i - D_{if}D_i)^T & 0 & 0 \\ E_i - D_{if}D_i & -I & 0 & 0 \\ 0 & 0 & -Q_{ie} & 0 \\ 0 & 0 & 0 & -R_i \end{bmatrix}$$

and R_i, H_i and Q_{ie} are as in (3.2)–(3.5). If the above LMIs are feasible, the decentralized $\mathcal{H}_\infty$ filter can be computed using (4.5)-(4.7).

Proof: Premultiplying and postmultiplying matrix inequality (3.6) by the block-diagonal matrices $diag\{\phi_{i1}^T, \ I, \ \cdots, \ I\}$ and $diag\{\phi_{i1}, \ I, \ \cdots, \ I\}$, respectively, and considering the change of controller variables (4.5)–(4.7), then (4.9) can be obtained. This completes the proof of this theorem. □

Remark 4.1 *Note that given the symmetric positive matrices Q_{ij}, $i, j = 1, 2, \cdots, N$, (4.8) and (4.9) are linear in $T_i, S_i, \mathcal{A}_{if}, \mathcal{B}_{if}$, $\mathcal{C}_{if}$ and D_{if}. Therefore, the existing LMI tool [5] can be applied to find a feasible solution if exists.*

Remark 4.2 *Given any feasible solutions to the LMIs (4.8) and (4.9) in Theorem 4.1, a decentralized $\mathcal{H}_\infty$ filter and the positive definite matrix P_i can be constructed as follows*
1. Compute invertible matrices M_i and N_i by using the singular value decomposition of $M_iN_i^T = I - T_iS_i$ (M_i and N_i are square invertible by (4.8)). Then define the matrices ϕ_{i1} and ϕ_{i2} as in (4.3). Thus, $P_i = \phi_{i2}\phi_{i1}^{-1}$.
2. Obtain the decentralized $\mathcal{H}_\infty$ filter parameters C_{if}, B_{if} and A_{if} by solving (4.5)–(4.7).

5. CONCLUSION

In this paper, we have provided an LMI approach to the decentralized $\mathcal{H}_\infty$ problem for interconnected linear systems. Our approach has some advantages: the local filter is based only on its own local available information and there is no any information exchange among subsystems; The communication and computational loads are reduced efficiently; The filtering error is totally asymptotically stable and the $\mathcal{L}_2$ gain from the exogenous noise to the filtering error is less than a prespecified level; The LMI approach is computationally efficient owing to recent advances in convex optimization [1, 5].

6. REFERENCES

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