USING CARRIER FREQUENCY ESTIMATORS DEVELOPED FOR AWGN CHANNEL IN MULTIPATH FADING CHANNEL

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ABSTRACT

A general approach that allows the extension of an CRB-achieving unbiased frequency estimator developed for the AWGN channel to be used for the multipath fading channel by using a periodic transmit sequence is given. The optimality of its performance is guaranteed in a multipath fading channel under certain conditions. The proposed method has low complexity but a reduced frequency operating range. The exploitation of the multipath diversity for fading channel is also investigated. It is illustrated through a correlation based frequency estimator how the general approach can be used, leading to a known estimator being used in a novel way.

1. INTRODUCTION

High performance frequency estimators in AWGN channel has been well explored. Estimators such as based on the correlation of the received signal [1] and maximum likelihood estimators (MLE) [2], are able to achieve the Cramer-Rao bound (CRB) when the signal-to-noise ratio (SNR) exceeds their associated threshold SNR.

In multipath fading channels, the problem of performing frequency offset estimation is two-fold: multipaths introduces inter-symbol-interference (ISI) hence introducing memory into the system and leading to increase complexity, while fading channels causes the signal to fade in certain time instants such that estimation is unreliable. Previous work in multipath channel develop the estimators by considering the problem of frequency estimation [3, 4] from scratch. The estimators have relatively high complexity and their applicability in fading channels are not discussed.

For multipath channels, an alternative approach is to design a periodic transmit sequence (possibly in communication systems) such that complexity is reduced without significant performance degradation [3]. Motivated by this, we give a general method to extend any estimator $\mathcal{E}$ developed in AWGN channel to be used in a multipath fading channel and call it estimator $\mathcal{M}$. When $\mathcal{E}$ achieves the CRB, we state the sufficient conditions when $\mathcal{M}$ achieves the CRB in a multipath fading channel. This result implies that any estimators developed for AWGN can be applied optimally for the multipath case. We also calculate the degradation when the channel is multipath fading compared to a multipath non-fading case, leading to the observation that multipath diversity is exploited so that the performance in a fading channel is maintained. We further illustrate how to develop estimator $\mathcal{M}$ with $\mathcal{E}$ as a low complexity correlation based estimator [1] , and show that it indeed achieve the relevant CRB at high SNR.

The following notations are adhered to. Bold lower case letters are used to denote column vectors. Bold upper case letters are used to denote matrices. The superscripts $*$, T and H are used to denote complex conjugate, transpose and Hermitian, respectively. The $(i,j)^{th}$ element of $\mathbf{X}$ is denoted as $[\mathbf{X}]_{ij}$; the i^{th} element of $\mathbf{x}$ as $[\mathbf{x}]_i$.

2. PROPOSED METHOD

We first state the CRB for different channel conditions and detailed the steps to develop the estimator $\mathcal{M}$.

2.1. CRB for Different Channels

Consider MN samples of a complex sinusoid $x_k, k = 0, 1, \cdots, MN-1$, arranged as a vector $\mathbf{x}$

$$\mathbf{x} = \mathbf{\Gamma}(\omega)[s_0 \cdots s_{MN-1}]^T + \mathbf{v} \qquad (1)$$

received in independent zero mean complex AWGN $\mathbf{v}$, with each element v_k of variance σ^2. $\mathbf{\Gamma}(\omega)$ is a square diagonal matrix with the k^{th} diagonal element being $\exp(j\omega k)$, and s_k and ω are deterministic values of the complex transmit sequence and frequency of the sinusoid respectively. With $s_k = s$, independent of k, and $\text{SNR} = |s|^2/\sigma^2$, the estimation of ω has a CRB given as [2]

$$\text{CRB}_{awgn}(\omega) = \frac{6}{MN[(MN)^2-1]\text{SNR}}. \qquad (2)$$

For multipath channels, the received signal becomes

$$\mathbf{x} = \mathbf{\Gamma}(\omega)\mathbf{S}\mathbf{h} + \mathbf{v} \qquad (3)$$

where $\mathbf{S}$ is a $MN \times M$ data Toeplitz matrix with elements $[\mathbf{S}]_{ij} = s_{i-j}, 0 \le i \le MN-1, 0 \le j \le M-1$ and $\mathbf{h} = [h_0 \cdots h_{M-1}]^T$ is the channel vector. The CRB given a multipath channel $\mathbf{h}$ is [3]

$$\text{CRB}_{multi}(\omega|\mathbf{h}) = \frac{\sigma^2}{2\mathbf{y}^H(\mathbf{I}-\mathbf{B})\mathbf{y}} \qquad (4)$$

where $\mathbf{y} = \mathbf{KSh}, \mathbf{B} = \mathbf{S}\left(\mathbf{S}^H\mathbf{S}\right)^{-1}\mathbf{S}^H$ and $\mathbf{K}$ is a diagonal matrix with the k^{th} diagonal element being k.

Now, we restrict the class of preamble as a periodic sequence with periodicity matched to the length of the channel, M. Denoting $k = nM + m$, we thus have $s_k = s_m$, where m can be alternatively obtained as k modulo M. Defining the $M \times M$ matrix $\widetilde{\mathbf{S}}$ with the $(i,j)^{th}$ element being s indexed with $i-j$ modulo M, $\mathbf{S}$ becomes

$$\mathbf{S} = [\widetilde{\mathbf{S}}^T \cdots \widetilde{\mathbf{S}}^T]^T. \qquad (5)$$

The CRB for the multipath channel with periodic sequence (a subscript p is added) can be obtained by substituting (5) into (4):

$$\text{CRB}_{multi,p}(\omega|\mathbf{h}) = \frac{6\sigma^2}{M^2N(N^2-1)\mathbf{h}^H\widetilde{\mathbf{S}}^H\widetilde{\mathbf{S}}\mathbf{h}}. \qquad (6)$$

The signal power is defined $\sigma_s^2(\mathbf{h}) = \|\widetilde{\mathbf{S}}\mathbf{h}\|^2/M$, and the SNR

$$\mathrm{SNR}(\mathbf{h}) = \sigma_s^2(\mathbf{h})/\sigma^2 \qquad (7)$$

so that the they are consistent with the definitions for the AWGN channel. With this, (6) becomes

$$\mathrm{CRB}_{multi,p}(\omega|\mathbf{h}) = \frac{6}{MN[(MN)^2 - M^2]\mathrm{SNR}(\mathbf{h})}. \qquad (8)$$

Comparing this with a AWGN channel using (2), the CRB is increased by a factor of $\frac{M^2-1/N^2}{M^2(1-1/N^2)} \geq 1$ which approaches 1 when N is large. Thus, using a periodic sequence reduces the CRB slightly, but is well motivated due to the savings in complexity as detailed subsequently.

Several methods for deriving the CRB are possible when the channel is multipath fading. We make use of the *modified CRB* [5] which is found to give the tightest bound for our purpose here (see [7] for discussion on other methods). Assuming that an unbiased estimator exists such that the bound $\mathrm{CRB}_{multi,p}(\omega|\mathbf{h})$ is attained for any probable $\mathbf{h}$, the modified CRB for a multipath fading channel is obtained by taking the expectation of (8):

$$\begin{aligned} &\mathrm{CRB}_{fade,p}(\omega) = E_{\mathbf{h}}\left[\mathrm{CRB}_{multi,p}(\omega|\mathbf{h})\right] \\ = \ &\frac{6}{MN[(MN)^2 - M^2]} E_{\mathbf{h}}\left[\mathrm{SNR}^{-1}(\mathbf{h})\right]. \end{aligned} \qquad (9)$$

2.2. Proposed Method and its Properties

We consider the specific case of performing frequency offset estimation based on a transmitted periodic sequence, such as in the standards of IEEE 802.11a [6]. Each periodic segment with length M is referred to as a short preamble, and the total number of short preamble is $N+1$. In [6], we have $M = 16$ and $N = 9$.

To provide a tractable solution for frequency offset estimation, we discard the first M received samples which is corrupted by ISI, and hence obtain (3). Substituting (5) into (3), the received signal is represented as N vectors each of size $M \times 1$:

$$\mathbf{x}_n = \mathbf{\Gamma}_n(\omega)\widetilde{\mathbf{r}} + \mathbf{v}_n, \ \ n = 0, \cdots, N-1, \ \ \widetilde{\mathbf{r}} \triangleq \widetilde{\mathbf{S}}\mathbf{h} \qquad (10)$$

where with $k = nM + m$, $[\mathbf{x}_n]_m = x_k$, $[\mathbf{v}_n]_m = v_k$ and $\mathbf{\Gamma}_n(\omega)$ is diagonal with the m^{th} element as $\exp(j\omega k)$. Using (10) , we define

$$\begin{aligned} \mathbf{Y} \ &\triangleq \ [\mathbf{x}_0 \, \mathbf{x}_1 \cdots \mathbf{x}_{N-1}]^T \\ &= \ [\mathbf{\Gamma}_0(\omega)\widetilde{\mathbf{r}} \cdots \mathbf{\Gamma}_{N-1}(\omega)\widetilde{\mathbf{r}}]^T + [\mathbf{v}_0 \cdots \mathbf{v}_{N-1}]^T \end{aligned} \qquad (11)$$

Define $\widetilde{r}_m = [\widetilde{\mathbf{r}}]_m$. Let the m^{th} column of $\mathbf{Y}$ be

$$\mathbf{y}(m) \triangleq \mathbf{y}^s(m) + \mathbf{u}(m) \qquad (12)$$

where $\mathbf{y}^s(m) = \widetilde{r}_m e^{jw(m-1)} \left[1 \; e^{j\omega M} \; \cdots \; e^{j\omega(N-1)M}\right]$ is the signal part with and $\mathbf{u}(m)$ is still AWGN of variance σ^2. Note that $\mathbf{y}(m)$ corresponds to transmitting $\widetilde{r}_m e^{jw(m-1)}$ with carrier frequency ωM in an AWGN channel. Using data $\{\mathbf{y}(m)\}_{m=0}^{M-1}$, we propose estimator $\mathcal{M}$ for ω according to the following steps:

- $\mathcal{S}1$: pick an unbiased estimator $\mathcal{E}$ consisting of, without loss of generality, cascaded filters f_1 and f_2
- $\mathcal{S}2$: perform maximal ratio combining (MRC) after filter f_1 so that the SNR after MRC is maximized (if and only if the sum of individual SNR is equal to the SNR after MRC)

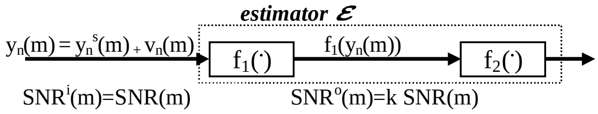

Fig. 1. Model of estimator used for Properties 1 and 2.

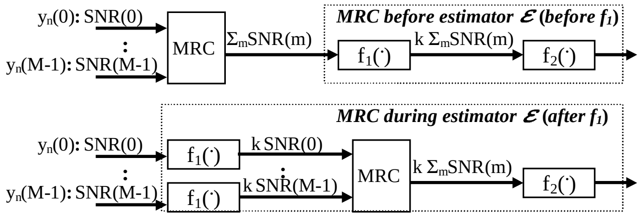

Fig. 2. MRC before or after f_1 results in a statistically equivalent data input to f_2, based on model in Fig. 1.

- $\mathcal{S}3$: divide the combined estimate by M to obtain the final estimate for ω

We next remark on some of the properties of estimator $\mathcal{M}$.

Property 1: Step $\mathcal{S}2$ is equivalent to performing MRC at the input, given estimator model in Fig. 1 where the ratio of the output and the input SNR of the first filter is independent of m:

$$\frac{\mathrm{SNR}^o}{\mathrm{SNR}^i} = k, \text{ a constant.} \qquad (13)$$

This property is illustrated through Fig. 2 where the estimator is modelled with Fig. 1. Note that filter f_2 cannot distinguish (statistically) whether the MRC, a linear operation across m, is carried out before or after f_1. Thus, performing MRC before or after filter f_1 causes the estimator $\mathcal{E}$ to achieve the same performance.

Property 2: If the estimator $\mathcal{E}$ achieves $\mathrm{CRB}_{awgn}(\omega)$, then estimator $\mathcal{M}$ achieves $\mathrm{CRB}_{multi,p}(\omega|\mathbf{h})$ in a multipath channel, and $\mathrm{CRB}_{fade,p}(\omega)$ in a multipath fading channel if $\mathcal{E}$ is unbiased for any probable channel $\mathbf{h}$.

Proof: To show that the specified bounds are achieved, instead of using MRC after f_1 in step $\mathcal{S}2$, we use MRC at the input. Both approaches gives the same performance by Property 1.

For step $\mathcal{S}1$, we choose a CRB-achieving estimator $\mathcal{E}$. If we apply it on each data vector $\mathbf{y}_m$, the estimator variance is

$$\sigma_{\mathcal{E}}^2(N\omega) = \frac{6}{N(N^2-1)\mathrm{SNR}_m}, \ \ m = 0, \cdots, M-1, \qquad (14)$$

where $\mathrm{SNR}_m = |r_m|^2/\sigma^2$, as given in (2). Clearly, an arbitrary combining of the data is sub-optimal. Using MRC on $\mathbf{y}_m$, before f_1, the SNR is $\mathrm{SNR}_{MRC} = \sum_{m=1}^{M} \mathrm{SNR}_m$. Using $\widetilde{\mathbf{r}} = \widetilde{\mathbf{S}}\mathbf{h}$ and (7), we note that $\mathrm{SNR}_{MRC} = \|\widetilde{\mathbf{r}}\|^2/\sigma^2 = M\mathrm{SNR}(\mathbf{h})$. Replacing SNR_m as SNR_{MRC} in (14), the variance of the estimator after step $\mathcal{S}2$ is equivalently

$$\sigma_{\mathcal{S}2}^2(M\omega) = \frac{6}{MN(N^2-1)\mathrm{SNR}(\mathbf{h})}. \qquad (15)$$

Dividing the combined estimate by M as required by step $\mathcal{S}3$ reduces the variance by M^2, which is equivalent to (8), thus achieving the CRB in a multipath channel.

Furthermore, we note from (9) that implies that since $\mathcal{M}$ achieves $\mathrm{CRB}_{multi,p}(\omega|\mathbf{h})$ in a multipath channel, if $\mathcal{E}$ is unbiased for any probable channel $\mathbf{h}$, $\mathcal{M}$ also achieves $\mathrm{CRB}_{fade,p}(\omega)$. ■

Property 3: Assume a complexity of $\mathcal{C}_1(N)$ and $\mathcal{C}_2(N)$ multiplications using estimator $\mathcal{E}$ operating on N data points, corresponding to complexity of filters f_1 and f_2, respectively. From Fig. 2, using estimator $\mathcal{M}$ with MRC at the input incurs a complexity of $M + \mathcal{C}_1(N) + \mathcal{C}_2(N)$, since MRC requires M multiplications. However, this requires knowledge of $\widetilde{r}_m e^{j\omega(m-1)}$ which is not known and is therefore not practical. Instead, we need to find a suitable filter f_1 where performing MRC after f_1 requires only known information; the complexity is increased to $M + M\mathcal{C}_1(N) + \mathcal{C}_2(N)$. The complexity may still be smaller when an arbitrary transmit sequence is used and the complexity incurred is (usually at least) $\mathcal{C}_1(MN) + \mathcal{C}_2(MN)$, especially so when the rate of increase of the filters' complexity with data length is faster then linear.

Property 4: Using $\mathcal{M}$ would imply using a lower sampling rate of $1/M$ times the baud rate f_s. This implies that the maximum frequency that can be detected is $\pm f_s/(2M)$ (instead of $\pm f_s/2$ if the baud rate sequence is used).

3. EFFECT OF MULTIPATH DIVERSITY ON FREQUENCY ESTIMATION

We first develop a robust transmit sequence that minimizes the maximum CRB for any arbitrary multipath channel. Based on this, we explore the degradation of the CRB in a multipath fading channel as compared to a non-fading one which is found to be small if the multipath diversity is sufficiently rich.

3.1. Training Sequence Selection

Denote the eigendecomposition $\widetilde{\mathbf{S}} = \mathbf{F}^H \mathbf{\Lambda} \mathbf{F}$, where $\mathbf{F}$ is an unitary matrix whose columns are the eigenvectors and $\mathbf{\Lambda}$ is a diagonal matrix with eigenvalue λ_m as its m^{th} element on the diagonal. Next, defining $\mathbf{h}_F = \mathbf{F}\mathbf{h}$, then $\mathbf{h}^H \widetilde{\mathbf{S}}^H \widetilde{\mathbf{S}} \mathbf{h}$ becomes $\mathbf{h}_F^H |\mathbf{\Lambda}|^2 \mathbf{h}_F$. To select an appropriate transmit sequence independent of the channel statistics, we choose the criteria that minimizes the worse CRB for an arbitrary $\mathbf{h}$:

$$\min_{\mathbf{\Lambda}} \left\{ \max_{\mathbf{h}} \mathrm{CRB}_{multi,p}(\omega|\mathbf{h}) \right\} = \min_{\{\lambda_m^2\}} \left\{ \max_{\mathbf{h}_F} \frac{1}{\sum_m \lambda_m^2 |h_{F,m}|^2} \right\} \quad (16)$$

where $[\mathbf{h}_F]_m = h_{F,m}$. We subject (16) to the transmit power constraint that tr$(\widetilde{\mathbf{S}}^H \widetilde{\mathbf{S}}) = \sum_{m=1}^{M} \lambda_m^2 = M\sigma_s^2$, and that channel is non-fading (but arbitrary) such that $\mathbf{h}^H \mathbf{h} = \mathbf{h}_F^H \mathbf{h}_F$ is constant.

Without loss of generality, re-indexed λ_m^2 such that $\lambda_0^2 \le \lambda_1^2 \le \cdots \le \lambda_{M-1}^2$. With that, $\left(\sum_m \lambda_m^2 |h_{F,m}|^2\right)^{-1}$ is maximized when $|h_{m,F}|^2$ takes the value $\mathbf{h}^H \mathbf{h}$ for $m = 0$, and the value 0 otherwise. Therefore (16) becomes

$$\min_{\{\lambda_m^2\}} \left\{ (\lambda_0^2 \mathbf{h}^H \mathbf{h})^{-1} \right\} = \min_{\{\lambda_m^2\}} \left\{ (\lambda_0^2)^{-1} \right\} = \max_{\{\lambda_0^2\}} \left\{ \lambda_0^2 \right\}.$$

The largest possible value of λ_0^2 is $\sum_m \lambda_m^2 / M = \sigma_s^2$ when $\lambda_m^2 = \sigma_s^2$, independent of m. This choice of transmit sequence, though not optimal for $\mathrm{CRB}_{fade,p}(\omega)$ with some channel distribution, is robust and leads to ease of developing analytical solutions.

3.2. Multipath Diversity

In wireless systems, the Rayleigh fading channel is modelled by assigning h_m's as independent zero mean circular complex random variables. The variance of each channel taps, $E[h_m h_m^*], m = 0, \cdots, M-1$, depends on the power delay profile. For a uniform power delay profile, $E[h_m h_m^*] = 1/M$ while for an exponential power delay, $E[h_m h_m^*] = \exp(-m/\tau_{rms})/P$, where P is defined such that $\sum_{m=0}^{M-1} E[h_m h_m^*] = 1$, and τ_{rms} is a parameter describing the slope of the decay of the power delay profile.

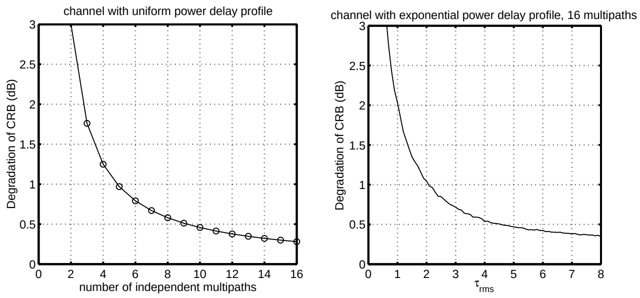

Fig. 3. Degradation of CRB for multipath fading channel when compared to a multipath non-fading channel.

Consider the value $\alpha \triangleq E_{\mathbf{h}} \left(\left[\mathbf{h}^H \widetilde{\mathbf{S}}^H \widetilde{\mathbf{S}} \mathbf{h} \right]^{-1} \right)$ which reduces to $E_{\mathbf{h}} \left(\left[\sigma_s^2 \mathbf{h}^H \mathbf{h} \right]^{-1} \right)$ when chosen according to Section 3.1. When the channel has a uniform delay profile with $E[h_m h_m^*] = \sigma_h^2$, then it can be shown that $\alpha = [(M-1)\sigma_h^2 \sigma_s^2]^{-1}$. Substituting this result into (9), we get

$$\mathrm{CRB}_{fade,p}(\omega) = \frac{6}{M^2 N (N^2-1)(M-1)\mathrm{SNR}_f} \quad (17)$$

where the SNR for the multipath fading channel is $\mathrm{SNR}_f = \frac{\sigma_h^2 \sigma_s^2}{\sigma^2}$. Comparing (17) with (8), and setting $\mathrm{SNR}_f = \mathrm{SNR}(\mathbf{h})$, we note that a degradation of $10 \log_{10} M/(M-1)$ is incurred when compared to the non-fading channel. Thus, if the channel has sufficient multipath diversity by having large M, the performance degradation is small, as shown on the left graph of Fig. 3.

When the channel has an arbitrary (such as exponential) power delay profile , α may not have a closed form expression, in which case we need to perform an numerical integration or Monte Carlo simulation. The latter approach is used to plot Fig. 3 (right graph) for a channel with $M = 16$ multipaths. When $\tau_{rms} = 1$, it degrades as if it has 3 multipaths when the power delay profile is uniform; when $\tau_{rms} = 5$, it degrades as if it has 10. Thus, a power delay profile close to a uniform one will also contribute better in multipath diversity. Besides multipath diversity, exploitation of transmit and receive antenna diversity can be carried out as well.

4. IMPLEMENTATION

As an example to implement estimator $\mathcal{M}$, we consider using one of the estimators described in [1] as estimator $\mathcal{E}$:

$$\mathcal{E}(\{y_n(m), n = 0, \cdots, N-1\}) = \sum_{n=0}^{N-2} w_n \arg(c_n(m)) \quad (18)$$

$$c_n(m) \triangleq y_{n+1}(m) y_n^*(m),$$

$$w_n \triangleq \frac{3N}{2(N^2-1)} \left[1 - \left(\frac{n - (N/2-1)^2}{N/2} \right) \right].$$

We separate the estimator as consisting of filter f_1:

$$f_1(y_n(m)) = c_n(m) = \exp(j\omega k) \times \left[|\widetilde{r}_m|^2 + \widetilde{r}_m u_n^*(m) + \widetilde{r}_m^* u_{n+1}(m) + u_n^*(m) u_{n+1}(m)\right], \quad (19)$$

which is evaluated using the definition in (12), and the subsequent operations as filter $f_2(c_n(m))$. From (19), the first term is the signal and the rest are noise terms and thus the SNR after f_1 is

$$\text{SNR}^o(m) = \frac{|\widetilde{r}_m|^4}{2|\widetilde{r}_m|^2\sigma^2 + \sigma^4} \approx \frac{|\widetilde{r}_m|^2}{2\sigma^2} \quad (20)$$

where the approximation is valid when $\sigma^2/|\widetilde{r}_m|^2 \ll 2$ or when SNR is high, i.e. approximately $\text{SNR}_m \gg 1/4$. Since the input SNR is $\text{SNR}^i(m) = |\widetilde{r}_m|^2/\sigma^2$, $\frac{\text{SNR}^o(m)}{\text{SNR}^i(m)} = 0.5$, a constant. Thus, the assumption of (13) is satisfied.

When we sum $c_{n,k}(m)$ for all m, the signal components $|\widetilde{r}_m|^2$ add coherently while the noise components do not, thus the effective SNR increases:

$$C_n \triangleq \sum_{m=0}^{M-1} c_n(m) = \exp(j\omega) \times \left[\sum_{m=0}^{M-1} |\widetilde{r}_m|^2 + \sum_{m=0}^{M-1} \widetilde{r}_m u_n^*(m) + \sum_{m=0}^{M-1} \widetilde{r}_m^* u_{n+1}(m) + \sum_{m=0}^{M-1} u_n^*(m) u_{n+1}(m)\right]. \quad (21)$$

with the effective SNR of C_n being

$$\text{SNR}_e = \frac{\left|\sum_{m=0}^{M-1} |\widetilde{r}_m|^2\right|^2}{2\sum_{m=0}^{M-1} |\widetilde{r}_m|^2\sigma^2 + M\sigma^4} \approx \frac{\sum_{m=0}^{M-1} |\widetilde{r}_m|^2}{2\sigma^2} \quad (22)$$

The approximation made is valid when $M\sigma^2 \ll 2\sum_{m=0}^{M-1} |\widetilde{r}_m|^2$, or when SNR is high, i.e. approximately $\text{SNR}_e \gg M/4$. Noting that SNR_e is a sum of all individual $\text{SNR}^o(m)$, we conclude that $C_{n,k}$ maximizes SNR when SNR is high, and is equivalent to performing MRC after f_1. Advantangeously, no additional information about the channel or even transmit sequence is required.

Finally, incorporating step $\mathcal{S}3$, the estimator $\mathcal{M}$ is:

$$\mathcal{M} = \frac{1}{M} \sum_{n=0}^{N-2} w_n \arg(C_n). \quad (23)$$

The simulation based on (23) is shown in Fig. 4 with $N = 9$, $M = 16$. It is seen that it indeed attains $\text{CRB}_{fade,p}(\omega)$. Because the channel is chosen so that $\tau_{rms} = 1$, a degradation of about 2 dB (see Fig. 3) separates the bound of the fading channel to the non-fading channel. It is also seen that the bound for AWGN channel is very close to that for a multipath channel.

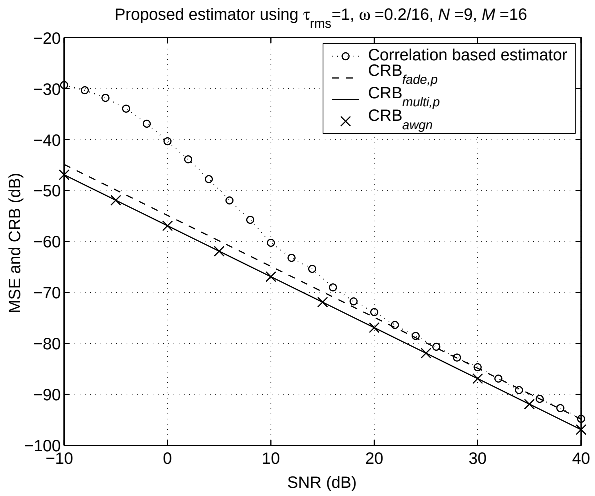

Fig. 4. Proposed estimator $\mathcal{M}$ using estimator $\mathcal{E}$ based on (23).

5. CONCLUSION

We have proposed a set of procedures to extend an estimator used in an AWGN channel to be used in a multipath fading channel by using a periodic transmit sequence. We have derived the sufficient conditions such that the extended estimator achieves the CRB under different channel conditions as well. We have also designed a robust transmit sequence such that a minimum CRB is guaranteed for arbitrary multipath channel, and illustrated that as multipath diversity increases, the CRB for multipath fading channel approach the CRB for a non-fading channel and quantified the degradation. Based on the sufficient conditions, we generated a practical estimator which indeed achieve the CRB for a multipath fading channel. Our future work would include proposing other estimators.

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