A Non-Line-of-Sight Equalization Scheme for Wireless Cellular Location

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ABSTRACT

Among the challenging issues that affect the performance of wireless location techniques is the temporal and spatial variations of the channel, and the distribution of the scatterers, which introduce non-line-of sight errors at the base station. This paper develops a technique for addressing this problem by exploiting the topology of the cellular network.

1. INTRODUCTION

The U.S. Federal Communications Commission (FCC) has made E911 a mandatary requirement for wireless communications services [1]. E911 requires all 911 calls from mobile telephones in the U.S. to be located within certain accuracy in order to route calls to the appropriate emergency service provider. Besides emergency assistance, this service will trigger many location-based services within the wireless network. One of the difficulties that affects the performance of location techniques is the temporal and spatial variations of the channel, and the distribution of scatterers, which introduce non-line-of-sight errors at the BSs. This paper presents an algorithm for equalizing the NLOS problem by using a constrained optimization formulation that exploits the topology of the cellular network. We assume we have measured the time-of-arrival (TOA) and the angle-of-arrival (AOA) of the mobile station (MS) at the base station (BS) using some known algorithms (e.g., [2, 3]). We then use the information from several BSs, and a data fusion scheme, to equalize these noisy measurements and to arrive at an improved location estimate.

2. PROBLEM FORMULATION

Fig. 1 shows a representation of a cellular system assuming four BSs. In the figure, we define the cellular system features as follows:

- $\binom{x_m}{y_m}$: mobile location.
- $\binom{x_i}{y_i}$: ith base station location.
- r_i: the distance from the MS to the ith BS.
- α_i: the angle of arrival from the MS to the ith BS.
- θ_i: angles due to the BSs topology.
- d_{ij}: the distance between ith and jth BSs.
- t_i: the time of arrival of the MS signal at the ith BS.
- t_m: the time of transmitting the signal from the MS.

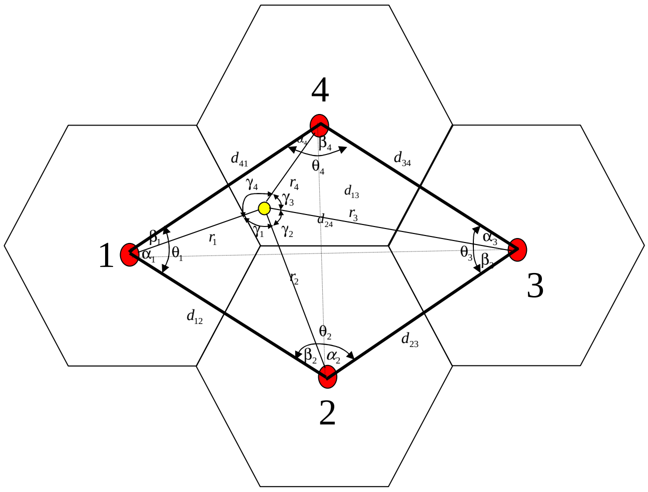

Fig. 1. A schematic of a cellular network topology with four BSs.

The location of a mobile station can be determined from knowledge of these features. For instance, we know that $r_i = (t_i - t_m)C$, where C is the speed of light ($3 \times 10^8 \mathrm{m/s}$) and, moreover,

$$r_i^2 = (x_i - x_m)^2 + (y_i - y_m)^2$$

If we take the first BS as the origin of the coordinate system (i.e., if we set $x_1 = y_1 = 0$), then the location of the MS can be estimated via the least-squares solution:

$$\begin{pmatrix} x_m \\ y_m \end{pmatrix} = (\mathbf{H}^{\mathbf{T}}\mathbf{H})^{-1}\mathbf{H}^{\mathbf{T}}\mathbf{b} \qquad (1)$$

where

$$\mathbf{H} = \begin{pmatrix} x_2 & y_2 \\ x_3 & y_3 \\ \vdots & \vdots \\ x_n & y_n \end{pmatrix}, \quad \mathbf{b} = \frac{1}{2}\begin{pmatrix} K_2^2 - r_2^2 + r_1^2 \\ K_3^2 - r_3^2 + r_1^2 \\ \vdots \\ K_n^2 - r_n^2 + r_1^2 \end{pmatrix}$$

$$K_i^2 = x_i^2 + y_i^2$$

In practice, we only have access to noisy measurements of $\{r_i, \alpha_i\}$, say

$$R_i = r_i + N_{r_i}, \quad \phi_i = \alpha_i + N_{\alpha_i} \qquad (2)$$

The noises $\{N_{r_i}, N_{\alpha_i}\}$ consist generally of two components each: a line-of-sight (LOS) term that arises from measurement noise, and a non-line-of-sight term that arises from the temporal and spatial variations of the channel, and the distribution of scatterers. We shall therefore write

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$$N_{r_i} = LOS_r + NLOS_r \quad (3)$$
$$N_{\alpha_i} = LOS_\alpha + NLOS_\alpha$$

and we will comment on the distribution of these noises in Secs. 4 and 5. Our scheme for enhanced location accuracy will be based on formulating a constrained optimization problem that reduces the effect of noises on location accuracy. The constraints will be a reflection of the topology of the cellular network. Thus consider again the cellular system shown in Fig. 1. The constraints are the distances between the BSs, which are given by

$$d_{12}^2 = r_1^2 + r_2^2 - 2r_1r_2\cos(\gamma_1) \quad (4)$$
$$\vdots$$
$$d_{41}^2 = r_4^2 + r_1^2 - 2r_4r_1\cos(\gamma_4)$$

and, for the cross nodes,

$$d_{13}^2 = r_1^2 + r_3^2 - 2r_1r_3\cos(\gamma_1+\gamma_2) \quad (5)$$
$$d_{24}^2 = r_2^2 + r_4^2 - 2r_4r_4\cos(\gamma_3+\gamma_4)$$

where the γ's are functions of the α's and θ's. This formulation is easily extendable to the case of n BSs. Then we can pose the problem of estimating the noises $\{N_{r_i}, N_{\alpha_i}\}$ by solving

$$\hat{\mathbf{N}} = \arg\min_{\mathbf{N}} \sum_{i=1}^{n} \left(\frac{N_{r_i}}{\sigma_{r_i}}\right)^2 + \left(\frac{N_{\alpha_i}}{\sigma_{\alpha_i}}\right)^2 \quad (6)$$

subject to

$$\begin{aligned} d_{12}^2 &= (R_1 - N_{r_1})^2 + (R_2 - N_{r_2})^2 - \\ &\quad 2(R_1 - N_{r_1})(R_2 - N_{r_2}) \\ &\quad \cos(\pi - (\phi_1 - N_{\alpha_1} + (\theta_2 - (\phi_2 - N_{\alpha_2})))) \\ &\vdots \\ d_{13}^2 &= (R_1 - N_{r_1})^2 + (R_3 - N_{r_3})^2 - \\ &\quad 2(R_1 - N_{r_1})(R_3 - N_{r_3}) \\ &\quad \cos((\phi_1 - N_{\alpha_1} + (\theta_2 - (\phi_2 - N_{\alpha_2}))) + \\ &\quad (\phi_2 - N_{\alpha_2} + (\theta_3 - (\phi_3 - N_{\alpha_3})))) \\ &\vdots \end{aligned}$$

where $\mathbf{N} = (N_{r_1}, N_{r_2}, \cdots, N_{r_n}, N_{\alpha_1}, N_{\alpha_2}, \cdots, N_{\alpha_n})^T$ is a vector of length $2n$, $\sigma_{r_i}^2$ is the variance of the distance error and $\sigma_{\alpha_i}^2$ is the variance of the angle error (both at the ith BS). If we consider N_{r_i} and N_{α_i} as Gaussian noises, then (6) is the maximum-likelihood estimation of N_{r_i} and N_{α_i}. There are some known methods for calculating the variances $\sigma_{r_i}^2$ and $\sigma_{\alpha_i}^2$ (see, e.g. [5, 6, 7]). These methods usually use the time history of errors, or the scattering model of the environment, to estimate the standard deviations. Specifically, the methods assume that the noises change faster than the MS distance from the BSs, so that the $\{r_i, \alpha_i\}$ in (2) can be assumed to be constants during estimation. The variance of the noises is then the same as the variances of the measurements R_i and ϕ_i. So assume we collect K measurements (say $K \approx 400$). Then, from [5],

$$\sigma_{r_i}^2 \approx \frac{1}{K}\sum_{n=0}^{K-1}(R_i(n) - \mu_{r_i})^2, \text{ where } \mu_{r_i} = \frac{1}{K}\sum_{n=0}^{K-1} R_i(n)$$

Likewise for $\sigma_{\alpha_i}^2$. Minimizing (6) results in estimates of distance and angle noises, which in turn lead to estimates for $\{r_i, \alpha_i\}$ as

$$\hat{r}_i = R_i - \hat{N}_{r_i}, \quad \hat{\alpha}_i = \phi_i - \hat{N}_{\alpha_i} \quad (7)$$

Using these equalized values in (1), will result in improved location accuracy.

3. OPTIMIZATION METHOD

For the solution of the constrained optimization problem (6), there are several well developed numerical algorithms. We shall use the SQP (Sequential Quadratic Programming) method [9], which essentially reduces a nonlinear optimization problem with nonlinear constraints to a sequence of constrained least-squares problems. The implementation consists of three main steps:

- Updating the Hessian matrix of the Lagrangian function.
- Solving a constrained least-squares problem.
- Line search and merit function calculation.

More specifically, using (6), we can denote the objective function and the constraints as:

$$\begin{aligned} f(\mathbf{N}) &: \text{ objective function in (6).} \\ g_i(\mathbf{N}) = 0 &: \text{ constraints in (6), say for } i = 1, \cdots, m_e. \\ \mathbf{N} &= (N_{r_1}, N_{r_2}, \cdots, N_{r_n}, N_{\alpha_1}, N_{\alpha_2}, \cdots, N_{\alpha_n}) \end{aligned}$$

The associated Lagrangian function is

$$L(\mathbf{N}, \lambda) = f(\mathbf{N}) + \sum_{i=1}^{m_e} \lambda_i g_i(\mathbf{N})$$

The solution is determined iteratively as

$$\mathbf{N}_{k+1} = \mathbf{N}_k + \mu_k d_k$$

where d_k is the search direction that is obtained by solving the constrained least-squares problem:

$$\begin{aligned} &\min_{d\in\Re^n} \frac{1}{2} d^T H_k d + \nabla f(\mathbf{N}_k)^T d \\ &\text{s.t. } \nabla g_i(\mathbf{N}_k)^T d + g_i(\mathbf{N}_k) = 0 \quad i = 1, ..., m_e \\ &\text{where } \quad H_k = \nabla_N^2 L(\mathbf{N}, \lambda)|_{N=N_k} \end{aligned}$$

or, equivalently,

$$\begin{aligned} &\min_{d\in\Re^n} \quad \frac{1}{2} d^T H d + c^T d \\ &\text{s.t.} \quad a_i d = b_i \quad i = 1, ..., m_e \end{aligned}$$

where $a_i = \nabla g_i(N_k)^T$, $b_i = -g_i(N_k)$ and $c = \nabla f(N_k)^T$. The solution d_k is be obtained from solving the linear system of equations:

$$\begin{pmatrix} H & A^T \\ A & 0 \end{pmatrix} \begin{pmatrix} d \\ \lambda \end{pmatrix} = \begin{pmatrix} -c \\ b \end{pmatrix} \quad (8)$$

where b is a vector containing the $\{b_i\}$ and A is a matrix of the $\{a_i\}$. The step size μ_k is chosen in a way that causes sufficient decrease in a merit function. There exist many different forms of merit functions, e.g., $\varphi(\mathbf{N}) = f(\mathbf{N}) + \sum_{i=1}^{m_e} g_i^2(\mathbf{N})$.

do

Update H_k

$d_k = arg\min_d\{\frac{1}{2}d^T H_k d + \nabla f(N_k)^T d :$

$\nabla g_i(N_k)^T d + g_i(N_k) = 0 \;\; i = 1, \cdots, m_e\}$

μ_k=linesearch$(\varphi(N_k, \lambda_k), d_k)$

$N_{k+1} = N_k + \mu_k d_k$

$k = k + 1$

until convergence

return(N_k, λ_k)

We have used the Matlab optimization toolbox for solving (6).

4. TOA ERROR DISTRIBUTION

We now comment on the distribution of the LOS and NLOS errors. The LOS error in distance measurements is modelled as zero mean Gaussian distribution with standard deviation between 30 and 60 meters. The NLOS error distribution can be deduced from the probability density function of the propagation delay between direct path and other paths. An exponential model has been investigated in [10],

$$P(\tau) = \begin{cases} \frac{1}{\tau_{rms}} e^{-\frac{\tau}{\tau_{rms}}} & \tau > 0 \\ 0 & \text{otherwise} \end{cases} \qquad (9)$$

where τ is the NLOS delay, τ_{rms} is the rms delay spread, which has a lognormal distribution and depends on the environment parameters. Using the model in [11], $\tau_{rms} = T_1 d^{\epsilon} \rho$, where T_1 is the median value of the rms delay spread at one kilometer, d is the distance between the MS and the BS, ϵ is the path loss exponent (PLE) with value between 0.5 and 1, ρ is a lognormal variable (such that $10 \log\ \rho$ is a zero mean Gaussin variable with standard deviation σ_ρ that lies between 4-6 dB). Typical parameters for different type of environments are given in the following table:

Enviroment	$T_1(\mu s)$	ϵ	σ_ρ(dB)
Bad Urban	1.0	0.5	4
Urban	0.4	0.5	4
Suburban	0.3	0.5	4
Rural	0.1	0.5	4

5. AOA ERROR DISTRIBUTION

The LOS error in AOA measurement can be modelled as zero mean Gaussian with standard deviation of approximately 3 degrees. The NLOS AOA error can be considered as a zero mean Gaussian random variable with standard deviation 5-10 degrees for different environments. An estimate for this standard deviation is given in [12] as $\sigma_\alpha = \frac{C\tau}{d}$, where C is speed of light, d is the distance between the MS and the BS, and τ is the TOA delay in (9).

6. SIMULATION RESULTS

For the simulation environment we consider four BSs, with 1 Km distance between them. We assume we have the measurements of AOA and TOA. For NLOS BSs, we add TOA noise according to (9) and a Gaussin noise with standard deviation of 45 meters as the LOS error. For AOA error of the NLOS BSs, we use a zero mean Gaussian noise with standard deviation given in Sec. 5, and a zero mean Gaussian noise with standard deviation 3 degrees. We consider the cases with 4 out of 4 NLOS BSs and 3 out of 4 NLOS BSs. To get the average over estimation errors, we choose 400 uniform random points in the plane and simulate 150 different NLOS and LOS noises for each point. The figures show the comparison between our equalization method, a traditional equalization scheme from [4], and using measured data without any equalization. The scheme from [4] uses the noisy measurements R_i to estimate the MS location by solving

$$\begin{pmatrix} \hat{x}_m \\ \hat{y}_m \end{pmatrix} = \underset{\begin{pmatrix} x \\ y \end{pmatrix}}{\arg\min} \sum_{i=1}^{n} \left(\frac{R_i - \left\| \begin{pmatrix} x \\ y \end{pmatrix} - \begin{pmatrix} x_i \\ y_i \end{pmatrix} \right\|}{\sigma_{r_i}} \right)^2 \qquad (10)$$

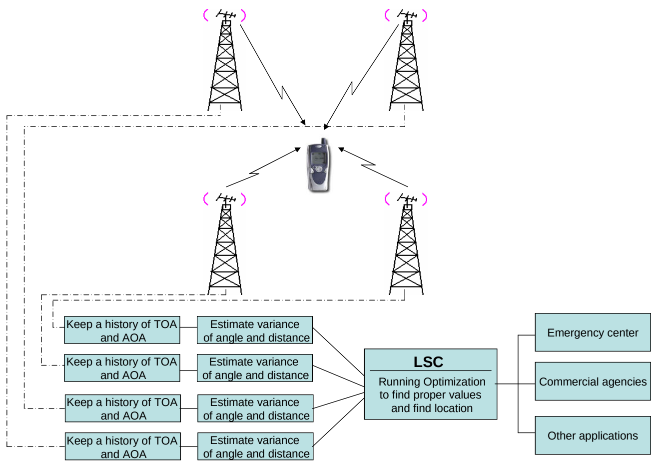

Fig. 2. block-diagram representation

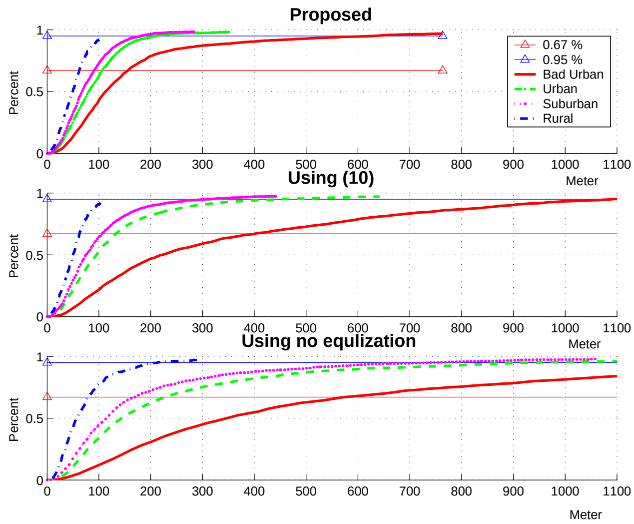

Fig. 3. The CDF of location error for NLOS noise models given in Secs.4 and 5 when there is 1 LOS BS and 3 NLOS BSs.

where $\| \ . \ \|$ denotes the Euclidean norm and σ_{r_i} is the standard deviation of the distance error at the ith BS. To initialize our simulation we find the MS location using raw data and (1), then we use the mobile location to initialize the least-squares equation using (10), and then use the obtained location to initialize the constrained optimization (6). We have also simulated some bad situations when the NLOS error is more than the assumed models (e.g., when the MS is inside a building). To model this error, we consider the given models in Secs. 4 and 5, but we amplify the AOA and TOA noises 3.3 more than before.

7. SUMMARY

As shown in the figures, the performance of the proposed constrained equalization scheme is superior to other schemes, especially in bad urban environments with higher NLOS noises. It also performs well in LOS situations, but we see dramatic performance improvement in NLOS cases.

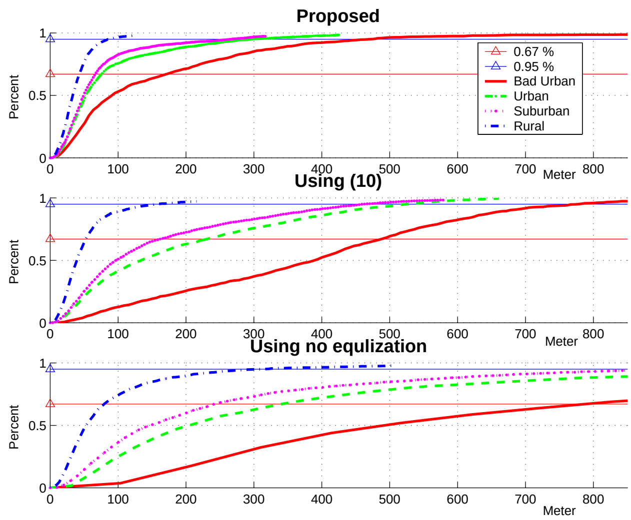

Fig. 4. The CDF of location error for three times higher than NLOS noise model given in Secs.4 and 5 when there is no LOS BS.

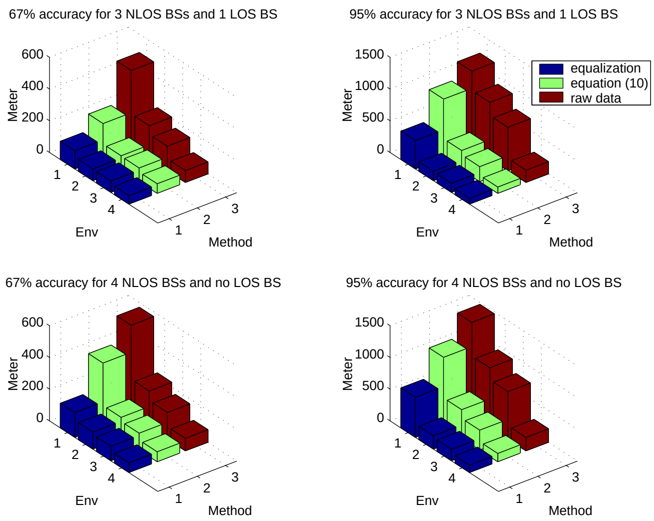

Fig. 5. 0.67 and 0.95 threshold point in location error for NLOS noise model given in Secs. 4 and 5. Methods from 3 to 1 are using raw data, using equation (10) and using constrained equalization. The environments 1-4 correspond to Bad Urban, Urban, Suburban and Rural.

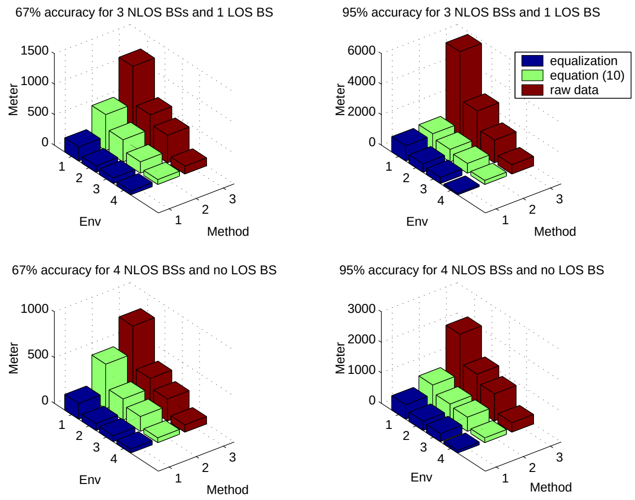

Fig. 6. 0.67 and 0.95 threshold point in location error for three times more than NLOS noise model given in Secs. 4 and 5. Methods from 3 to 1 are using raw data, using equation (10) and using constrained equalization. The environments 1-4 correspond to Bad Urban, Urban, Suburban and Rural.

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