PERFORMANCE ANALYSIS OF MEASUREMENT ERROR REGRESSION IN DIRECT-DETECTION LASER RADAR IMAGING

Christina Grönwall, Tomas Carlsson

Swedish Defence Research Agency (FOI)
Department of Laser Systems
P.O. Box 1165, SE-581 11 Linköping, Sweden
christina.gronwall,tomas.carlsson@foi.se

Fredrik Gustafsson

Linköping University
Department of Electrical Engineering
SE-581 83 Linköping, Sweden
fredrik@isy.liu.se

ABSTRACT

In this paper a tool for synthetic generation of scanning laser radar data is described and its performance is evaluated. By analyzing data from the system, we will recognize objects on ground. In the measurement system it is possible to add several design parameters, which make it possible to test an estimation scheme under different types of system design. The measurement system model includes laser characteristics, object geometry, reflection, speckles, atmospheric attenuation, turbulence and a direct detection receiver. A parametric method that estimates an object's size and orientation is described. There are measurement errors present and thus, the parameter estimation is based on a measurement error model. The parameter estimation accuracy is limited by the Cramer-Rao lower bound. Validations of both the measurement error model and the measurement system are shown. Data from both models generate parameter estimates that are close to the Cramer-Rao lower bound.

1. INTRODUCTION

1.1. The laser radar system

Laser radar systems have been investigated over several decades primarily for military applications, see for example [1]. As in microwave radar technology, the range of the object and background is often obtained by measuring the time of flight for a modulated laser beam from the transmitter to the object and back to the receiver. The high resolution makes 3D imaging possible and due to the short wavelength, in general 0.5-10 μm, detailed range images of objects and background can be obtained.

In this paper we will study the properties of data from a helicopter-carried, scanning laser radar system. The laser beam is swept from side to side and when the helicopter flies this results in a zigzag-shaped scanning pattern, see Fig. 1. The laser radar system is connected to a GPS and the laser samples are fused with the GPS coordinates. Each sample data point contains (x, y, z, I), where (x, y) is the position in north-south and east-west, respectively, z is the altitude, and I is the intensity in the returning pulse. This means that the set of data/image points corresponds to a 3D mapping of the terrain. An example of data from a scanning laser radar system is shown in Fig. 2. The scanning is usually performed in another direction than (x, y). Let us call the direction of the scanning ξ and the flight direction of the helicopter η. Thus, the data collection is performed in the coordinate system (ξ, η).

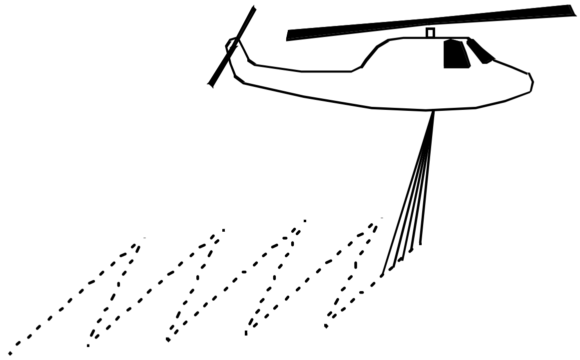

Fig. 1. Typical scanning pattern for a helicopter-carried scanning laser radar system.

1.2. Object recognition

In [2] methods for estimation of object's size and orientation from scanning laser radar data are proposed. The work is based on the assumption that from a top view most vehicles are approximately of rectangular shape. When the object is detected a rectangle that with minimal area contains the convex hull of the ob-

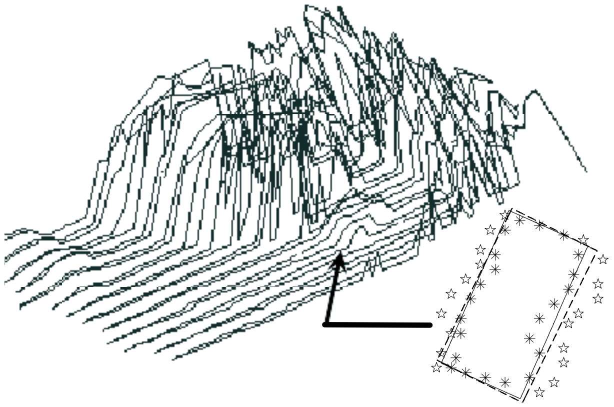

Fig. 2. Example of (raw) data from a helicopter-carried scanning laser radar system. The arrow marks a vehicle placed in an opening of the forest. The folded image shows the rectangle estimation process. $*$, $\star$: object and ground data, respectively, solid: the minimum rectangle estimate and dashed: TLS estimate. (Data is from the TopEye system, see http://www.topeye.com.)

ject's data is estimated. This rectangle estimate is then improved using least squares methods based on both object and ground data. Both linear (total least squares, TLS, see e.g. [3]) and nonlinear least squares methods (Newton-type) were evaluated. These improved rectangle estimates are less biased compared to the initial estimates. An example of the method is shown in Fig. 2.

In this paper we will study the estimation properties of the TLS formulation, in terms of correctness in parameter estimates. In the rectangle estimation method described above the analysis was performed in the (ξ, η) dimension. Below we will study a simplified problem; estimation of a slope instead of a rectangle, and in the (ξ, z) dimension. In Section 2 the system, the measurement error (ME) model and the Cramer-Rao lower bound (CRLB) of the ME model are described. In Section 3 both the estimation model and the measurement system are validated. Finally, in Section 4 conclusions are found.

2. THE MEASUREMENT ERROR MODEL

2.1. System description

Let us define a regressor

$$\varphi = (\xi, \eta, z)^T,$$

which contains data used for the size and orientation estimation. The regressor φ is a function of the measurement system

$$\varphi = f(r, \alpha_{scan}, \alpha_{pitch}),$$

where r is the slant range distance measured by the laser range finder in a certain scan angle, α_{scan}, and pitch angle, α_{pitch}. α_{scan} is considered parallel with ξ and α_{pitch} is considered parallel with η. Implicitly, φ is also a function of the object's shape, atmosphere, receiver, detector properties etc.

Gauss approximation formula gives that the covariance of the regressor can be approximated using a first order Taylor expansion by

$$R = Cov(\varphi) = f' Cov\left((r, \alpha_{scan}, \alpha_{pitch})^T\right)(f')^T + \mathcal{O}(\|f''\|), \quad (1)$$

where f' and f'' are the first and second order derivative in r, α_{scan} and α_{pitch}, respectively, and $\mathcal{O}$ is the ordo operator.

2.2. The slope model

Let us derive a model for one scan over a slope (i.e., a tilted plane) in the (ξ, z) dimension. As only one scan is studied, the η axis is constant and can be ignored. Note that the regressor now is redefined to $\varphi = (\xi, z)^T$. In each sample m we retrieve

$$\begin{pmatrix} \xi_m \\ z_m \end{pmatrix} = \begin{pmatrix} \xi^0 \\ z^0 \end{pmatrix} + \begin{pmatrix} e_\xi \\ e_z \end{pmatrix}$$

or

$$\varphi_m = \varphi^0 + e_\varphi, \quad (2)$$

where φ_m is the measured coordinate, φ^0 is the unobservable, true coordinate and e_φ is the noise in (ξ, z), respectively. Note that we have error in both coordinates and thus, we have a measurement error (ME) regression problem. For the estimation of a slope we use the following equation for a straight line

$$\begin{pmatrix} \varphi^0 \\ 1 \end{pmatrix}^T \theta = 0,$$

where the parameters are

$$\theta = (\theta_1, \theta_2, \theta_3)^T$$

with the constraint

$$g(\theta) = \theta_1^2 + \theta_2^2 - 1 = 0$$

to guarantee a unique solution. The covariance matrix of $\begin{pmatrix} \varphi_m^T & 1 \end{pmatrix}^T$ is

$$S = Cov\begin{pmatrix} \varphi_m \\ 1 \end{pmatrix} = \begin{pmatrix} R & 0 \\ 0 & 0 \end{pmatrix}.$$

The (total) model error can now be defined as

$$\varepsilon = \begin{pmatrix} \varphi_m^T - (\varphi^0)^T & 1 \end{pmatrix} \theta.$$

2.3. Pre-whitening of the ME model

The errors in the TLS estimates have minimum variance when the error covariance matrix is proportional to the identity matrix, i.e., $R \sim I$. If this is not the case, as in this example, the error covariance matrix R must be scaled. When R is a symmetric, positive definite matrix the regressor and parameter vector can be scaled as

$$\begin{aligned} \bar{\varphi}_m &= R^{-\frac{1}{2}} \varphi_m, \\ \left(\bar{\theta}_1, \bar{\theta}_2\right)^T &= R^{-\frac{1}{2}} \left(\theta_1, \theta_2\right)^T. \end{aligned}$$

This gives a new estimation problem with

$$\begin{pmatrix} \bar{\varphi} \\ 1 \end{pmatrix}^T \bar{\theta} = 0$$

subject to

$$g\left(\bar{\theta}\right) = 1,$$

where

$$\bar{R} = Cov\left(\bar{\varphi}\right) = I.$$

The TLS will now give minimum variance estimates of $\widehat{\bar{\theta}}$, and the estimates in the "true" coordinates can be retrieved from $\theta = R^{\frac{1}{2}} \bar{\theta}$.

2.4. The Cramer-Rao lower bound of the ME model

It is always desirable to understand how a particular estimation scheme performs under a certain model. If we assume some distribution of the perturbations, a lower bound on the error covariance of the estimated parameters can be calculated by the Cramer-Rao lower bound (CRLB).

Consider the measurement error model (2) and assume that e_ξ and e_z are Gaussian distributed with zero mean and variance $\sigma^2_{e_\xi}$ and $\sigma^2_{e_z}$, respectively. Further, e_ξ and e_z are assumed to be independent. The CRLB states that the quality of any estimator in terms of its mean-square error (MSE) is bounded from below as

$$E\left(\left(\theta^0 - \hat{\theta}\right)\left(\theta^0 - \hat{\theta}\right)^T\right) \geq J^{-1},$$

where J is the Fisher information matrix. Inserting $\theta = R^{\frac{1}{2}} \bar{\theta}$, we get the CRLB expression

$$E\left(\left(\bar{\theta}^0 - \widehat{\bar{\theta}}\right)\left(\bar{\theta}^0 - \widehat{\bar{\theta}}\right)^T\right) \geq R^{-\frac{1}{2}} J^{-1} R^{-\frac{T}{2}} = \bar{J}^{-1}.$$

Following the calculations in [4] for $\bar{J}^{-1}$ we get the following expression of CRLB

$$\begin{aligned} Var\left(\bar{\theta}^0_1 - \widehat{\bar{\theta}}_1\right) &= \frac{1}{N}\frac{1}{D} \\ Var\left(\bar{\theta}^0_3 - \widehat{\bar{\theta}}_3\right) &= \frac{1}{N}\frac{\overline{E}\left(\xi - \frac{n_1}{\sqrt{1-n_1^2}} z\right)^2}{D} \\ D &= \overline{Var}\left(\xi - \frac{n_1}{\sqrt{1-n_1^2}} z\right), \end{aligned}$$

where $\bar{\theta}^0$ are the true parameters, $\widehat{\bar{\theta}}$ are the estimated parameters, N is the number of samples, $\overline{Var}\left(x\right)$ is the approximation $\overline{Var}\left(x\right) = \overline{E}\left(x^2\right) - \overline{E}^2\left(x\right)$, where $\overline{E}\left(x^2\right) = \frac{1}{N}\sum_{i=1}^{N} x_i^2$ and $\overline{E}\left(x\right) = \frac{1}{N}\sum_{i=1}^{N} x_i$. Note that θ_2 is a function of θ_1, and not included in the CRLB expression.

3. VALIDATION

3.1. Validation of the ME model

The performance of the estimation method is investigated in Monte Carlo simulations, in Matlab. The performance is evaluated in terms of correctness in estimates of θ. We start with an exact (error free) description of a slope. True values of the parameters are $\theta_1 = 0.5$, $\theta_2 = \sqrt{1-\theta_1^2} \approx 0.87$ and $\theta_3 = 0$, respectively, and $\xi = -5 : 1/(N-1) : 5$. From this, z is calculated. Random errors, Gaussian distributed with zero mean and variance $\sigma^2_{e_\xi} = \sigma^2_{e_z} = 0.01$ are added to the coordinates ξ and z, respectively. The noise is generated separately for ξ and z. Then the parameters are estimated using TLS on the perturbed data set. The simulations are repeated for an increasing number of samples N. The statistical properties of the estimates are studied by the mean square error (MSE), which is averaged over 100 sets. The CRLB (theoretical limit) is plotted together with the MSE for each case, see Fig. 3. We can see that the MSE of the parameter estimate follows the theoretical bound.

3.2. Validation of the system

In [5] and [6] models of how the object's shape, atmospheric, receiver and detector properties affect the laser beam are described and simulations are shown. Using these models we can simulate a scanning laser radar system and generate synthetic images of objects on the ground.

For simple object surfaces the laser impulse response, i.e., the returning laser pulse, can be calculated analytically. In that case it is assumed that the impulse

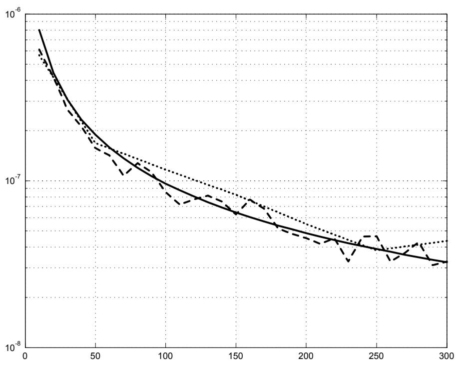

Fig. 3. Validation of the models. $Var\left(\theta_1^0 - \hat{\theta}_1\right)$ as a function of the number of samples N is shown. Solid: CRLB, dashed: MSE from a simulation of the ME model, dotted: MSE from a simulation of the measurement error model.

response is symmetric around the mean time of the two-way propagation and that the laser beam has Gaussian energy distribution. This is, however not suitable for simulations of complex surfaces.

In the simulation method described in [6] the area of the laser beam is divided into a large number of subareas. In this way we can assume that each subarea of the laser beam is reflected on a plane surface and that complex object surfaces consist of a number of plane surfaces. Therefore, no theories of reflection for other objects than plane surfaces are needed. The assumption that the energy distribution of the laser beam inside each subarea is constant is also made.

In our simulations we let the object's surface be a tilted plane and we make one scan over the surface. From the simulated laser radar data we estimate the parameters θ and compare them with the properties of the tilted plane (i.e., θ^0). The number of subareas of the laser beam is 20×20. The reflection properties of the object's surface are set to represent a metal surface painted with a subdued color. The uncertainties in the slant range estimates and measurement angles are included. When the regressor φ_m is retrieved, $\hat{\theta}$ is calculated using TLS and then the estimate is weighted with $R^{-1/2}$ to retrieve $\hat{\hat{\theta}}$. The parameter estimation process is repeated in the same manner as described in Section 3.1, but $\sigma^2_{e_\xi}$ and $\sigma^2_{e_z}$ are given by the uncertainties in φ_m. In Fig. 3 the MSE of a parameter estimate from this simulated measurement is shown. Thus, by using a proper pre-whitening matrix R the MSE of the parameter estimates originating from the laser radar system is also close to the CRLB.

It is worth mentioning that the approximation formula (1) is applicable when $f\left(r, \alpha_{scan}, \alpha_{pitch}\right)$ is approximately linear. This is fulfilled in this case as α_{scan} and α_{pitch} typically is rather small, e.g., $\alpha_{scan} = \pm 10 - \pm 20$ degrees.

4. CONCLUSIONS

Our goal with this work is to connect the design parameters of the measurement system with the parametric description of the object. This gives us a tool for evaluation of the effects of the system's design parameters on the result of the recognition algorithm. For example, it will be possible to study what maximum scan angle that is allowed if a certain estimation error shall be accomplished. The system model includes laser characteristics, object geometry, reflection, speckles, atmospheric attenuation, turbulence and a direct detection receiver. Thus, the data generation is based on sound calculations of physical system properties.

In this paper a tool for synthetic generation of scanning laser radar data is described. The performance of the simulated measurement system and the measurement error model is evaluated. For both models the error variances of the parameter estimates are close to the Cramer-Rao lower bound.

5. REFERENCES

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