# MULTIPLE TARGET DETECTION AND ESTIMATION BY EXPLOITING THE AMPLITUDE MODULATION INDUCED BY ANTENNA SCANNING. PART II: DETECTION*

*Fulvio Gini[1], Federica Bordoni[1], Maria Greco[1], and Alfonso Farina[2]*

[1] Dip. di Ingegneria dell'Informazione, Università di Pisa, via Diotisalvi, 2 - 56126 Pisa, Italia
Tel: +39-050-568550, Fax: +39-050-568522, e-mail: name.surname@ing.unipi.it

[2] Radar & Technology Division, Alenia-Marconi Systems, via Tiburtina Km 12.4, 00131 Roma, Italia
Tel: +39-06-41502279, Fax: +39-06-41502665, e-mail: afarina@amsjv.it

## ABSTRACT

*This work deals with the problem of detecting and estimating multiple radar targets present in the same range-azimuth resolution cell of a surveillance radar system with a mechanically rotating antenna. First, the target parameters are estimated assuming a maximum number of possible targets. To this purpose we use the asymptotic maximum likelihood (AML) RELAX estimator derived in the first part [3]. Subsequently, these estimates are used in a sequentially hypotheses test (SHT) procedure. The statistic of the test at each step of the SHT procedure is derived using an asymptotic expression of the generalized likelihood ratio test (GLRT) statistic. An upper bound of the false alarm probability is derived in closed form, whereas detection performance of the proposed SHT detector is investigated through Monte Carlo simulation.*

## 1. INTRODUCTION

In most modern radar systems, the target direction of arrival (DOA) is estimated by the monopulse technique [8]. When multiple targets are present in the range-azimuth resolution cell under test (CUT), the monopulse method provides an erroneous DOA measure [5]. This work describes a method to detect multiple targets present in the same CUT using only one receiving channel, exploiting knowledge of the antenna beam pattern and the amplitude modulation impressed on the received signal by mechanical scanning [1]. In the literature, the problem of estimating the number of components present in a multi-component signal is termed the "detection problem" or "model order selection". Many authors have investigated the model order selection problem by using information theoretic criteria (ITC) [9-11], maximum a posteriori probability (MAP) methods [4], or sequential hypotheses testing (SHT) methods [7,9].

The SHT approach is based on a sequential procedure which tests a set of mutually exclusive hypotheses $H_m$ and alternatives $K_m$. More precisely, at step $m$ the SHT procedure tests the hypothesis $H_{m-1}$, "`There are` $m-1$ `targets`", against the alternative $K_{m-1}$, "`There are` $m$ `targets`", by comparing the test statistic $S_m(\mathbf{z})$ with a proper threshold $\lambda_m$. Therefore, hypotheses $\{H_0, H_1, \cdots, H_{M_{\max}-1}\}$ are tested in sequence, going to the next one only if previous hypotheses have been rejected, and stopping when an hypothesis is accepted or eventually at step $M_{\max}$, where $M_{\max}$ is the maximum number of possible targets. The most relevant feature of SHT with respect to ITC methods is that they allow to control the probability of overestimating the number of targets, i.e. the probability of false alarm ($P_{FA}$). Since the possibility of controlling $P_{FA}$ is fundamental in surveillance radar applications, we focus our attention on the SHT procedure.

The rest of this paper is organized as follow. In Section 2, the data model and the problem statement are introduced. The SHT detection method is described in Section 3. Some numerical results of our performance analysis and concluding remarks are reported in Section 4.

## 2. DATA MODEL AND PROBLEM STATEMENT

Assume that $M$ point-like targets are present in the range-azimuth resolution cell under test with direction of arrivals (DOA) $\{\theta_{TGi}\}_{i=1}^{M}$ and Doppler frequencies $\{f_{Di}\}_{i=1}^{M}$. The $N \times 1$ complex data vector $\mathbf{z}$ is composed by the collection of the $N$ echoes received during the time on target (*ToT*). In vector notation, the data model for $M$ targets is given by

$$\mathbf{z} = \sum_{i=1}^{M} b_i \mathbf{a}(\theta_{TGi}, f_{Di}) + \mathbf{d} = \mathbf{A}(\boldsymbol{\theta})\mathbf{b} + \mathbf{d}, \quad (1)$$

where $\mathbf{b} = [b_1 \cdots b_M]^T$ is the $M \times 1$ vector of the unknown complex amplitudes, $^T$ denotes the transpose operation, $\mathbf{A}(\boldsymbol{\theta}) = [\mathbf{a}(\theta_{TG1}, f_{D1}) \cdots \mathbf{a}(\theta_{TGM}, f_{DM})]$ is the $N \times M$ steering

* This work has been funded by Alenia-Marconi Systems S.p.A.

matrix, $\boldsymbol{\theta} = [\theta_{TG1} \cdots \theta_{TGM}\ f_{D1} \cdots f_{DM}]^T$ is the $2M \times 1$ vector of the unknown DOAs and Doppler frequencies (normalized to the pulse repetition frequency, *PRF*), with $f_{Di} \in [-0.5, 0.5)$ and $\theta_{TGi} \in [0, \theta_B)$, where $\theta_B$ is the –3dB azimuth antenna beam width, $\mathbf{a}(\theta_{TGi}, f_{Di})$ is an $N \times 1$ vector which can be factored as the product of the spatial and temporal steering vectors: $\mathbf{a}(\theta_{TGi}, f_{Di}) = \mathbf{g}(\theta_{TGi}) \odot \mathbf{p}(f_{Di})$, where $\odot$ represents the Hadamard product, $[\mathbf{p}(f_{Di})]_n = \exp[j2\pi f_{Di}(n-1)]$, and $[\mathbf{g}(\theta_{TGi})]_n = G(n-1, \theta_{TGi})$, for $n = 1, \cdots, N$, $G(n, \theta_{TGi})$ is the two-way antenna gain for the *n*th pulse from the *i*th DOA. We assume that the antenna beam pattern $G(\cdot)$ has a Gaussian shape [1-3]. The elements of $\mathbf{b}$ and $\boldsymbol{\theta}$ are modeled as unknown deterministic parameters. The $N \times 1$ disturbance vector $\mathbf{d}$ is composed by thermal noise and clutter. It is modeled as a complex zero-mean Gaussian random vector with covariance matrix $\mathbf{M}_d = E\{\mathbf{d}\mathbf{d}^H\} = \sigma_d^2 \mathbf{M}$, where $\sigma_d^2$ is the total disturbance power and $\mathbf{M}$ is the normalized covariance matrix, i.e. $[\mathbf{M}]_{i,i} = 1$ for $i = 1, \cdots, N$. In this work, we assume that $\mathbf{M}_d$ is a priori known. In a realistic radar scenario it must be estimated from secondary data [2].

In [3] we assumed that the number *M* of targets is known and we tackled the problem of estimating $\mathbf{b}$ and $\boldsymbol{\theta}$. The "detection problem," which consists of determining the number *M* of targets, is the subject of this second part. In summary, the goal here is to jointly estimate *M*, $\mathbf{b}$, and $\boldsymbol{\theta}$, based on the observation of the complex data vector $\mathbf{z}$. The solution of the detection problem builds on the results obtained in [1] and [3]. We propose here a sequential hypotheses test approach, which employs the asymptotic maximum likelihood (AML) estimates of $\mathbf{b}$ and $\boldsymbol{\theta}$ derived as described in [3].

## 3. SEQUENTIAL HYPOTHESES TEST

To derive the test statistic $S_m(\mathbf{z})$, we adopt the generalized likelihood ratio test (GLRT) [6]. The test at step *m* is given by:

$$S_m(\mathbf{z}) = 2\ln L_{G,m}(\mathbf{z}) = 2\ln p_{\mathbf{z}|H_m}(\mathbf{z}|H_m; \hat{\boldsymbol{\vartheta}}_m) - 2\ln p_{\mathbf{z}|H_{m-1}}(\mathbf{z}|H_{m-1}; \hat{\boldsymbol{\vartheta}}_{m-1}) \underset{H_{m-1}}{\overset{K_{m-1}}{\gtrless}} \lambda_m,\ m = 1, \cdots, M_{\max}, \tag{2}$$

where $L_{G,m}(\mathbf{z})$ denotes the generalized likelihood ratio (GLR) for hypothesis $H_{m-1}$ and alternative $K_{m-1}$, and we used the fact that $K_{m-1} \equiv H_m \equiv$ "$m$ targets", $p_{\mathbf{z}|H_m}(\mathbf{z}|H_m; \boldsymbol{\vartheta}_m)$ is the data probability density function (pdf) under hypothesis $H_m$, and $\hat{\boldsymbol{\vartheta}}_m$ is the maximum likelihood (ML) estimate of $\boldsymbol{\vartheta}_m$, where $\boldsymbol{\vartheta}_m = [\boldsymbol{\theta}_m^T\ \mathbf{b}_m^T]^T$ is the vector of $2m$ real and $m$ complex target parameters, $\boldsymbol{\theta}_m$ and $\mathbf{b}_m$ are the parameter vectors previously defined for *m*=*M*. The procedure stops the first time the statistic does not exceed the threshold or when the number *m* of hypothesized targets reaches the maximum value $M_{\max}$. If the procedure stops at step *m* we estimate $\hat{M} = m-1$, otherwise $\hat{M} = M_{\max}$. The GLRT statistic (2) depends nonlinearly on the observed data and, unfortunately, its pdf is unknown for finite sample size. Therefore, we resort to asymptotic analysis. More precisely, to derive the test statistic, we use the large sample size expression of the compressed (with respect to $\mathbf{b}_m$) log-likelihood function (LLF) that was obtained in [3], that is given by

$$\ln p_{\mathbf{z}|H_m}(\mathbf{z}|H_m; \boldsymbol{\theta}_m, \hat{\mathbf{b}}_m) = -\left[\ln(\pi^N \sigma_d^{2N} |\mathbf{M}|) + \frac{\mathbf{z}^H \mathbf{M}^{-1} \mathbf{z}}{\sigma_d^2}\right] + \frac{1}{\sigma_d^2} \sum_{i=1}^{m} \frac{\left|\mathbf{z}^H \mathbf{M}^{-1} \mathbf{a}(\theta_i, f_i)\right|^2}{\mathbf{a}^H(\theta_i, f_i) \mathbf{M}^{-1} \mathbf{a}(\theta_i, f_i)} + o(N), \tag{3}$$

where $\hat{\mathbf{b}}_m$ is the ML estimate of $\mathbf{b}_m$ [1], and we used Landau's notation:

$$f(N) = o(N^k) \overset{def}{\Leftrightarrow} \lim_{N \to \infty} N^{-k} f(N) = 0, \tag{4}$$

i.e. the term $o(N)$ in (3) is negligible with respect to the sum of the *m* terms for large *N*. In the right-hand term of (3) the terms in square brackets are irrelevant, since they cancel out when we calculate the GLR. An efficient algorithm to derive the AML estimate of $\boldsymbol{\theta}_m$ based on the RELAX approach is described in [3]. The AML-RELAX estimator orders the elements of $\hat{\mathbf{b}}_m$ such that $|[\hat{\mathbf{b}}_m]_k| \geq |[\hat{\mathbf{b}}_m]_{k+1}|$, for $k = 1, 2, \cdots, m-1$, i.e. in decreasing order, and the elements of $\hat{\boldsymbol{\theta}}_m$ are ordered consequently. Inserting in equation (2) $\ln p_{\mathbf{z}|H_m}(\mathbf{z}|H_m; \hat{\boldsymbol{\theta}}_m, \hat{\mathbf{b}}_m)$ obtained from (3), after a few manipulations (not reported here for lack of space, all the details can be found in [2]), we obtain that asymptotically (large *N*) the GLRT statistic assumes the form:

$$S_m(\mathbf{z}) = 2\ln L_{G,m}(\mathbf{z}) = \frac{(2/\sigma_d^2)\left|\mathbf{z}^H \mathbf{M}^{-1} \mathbf{a}(\hat{\theta}_{TGm}, \hat{f}_{Dm})\right|^2}{\mathbf{a}^H(\hat{\theta}_{TGm}, \hat{f}_{Dm}) \mathbf{M}^{-1} \mathbf{a}(\hat{\theta}_{TGm}, \hat{f}_{Dm})}, \tag{5}$$

where $(\hat{\theta}_{TGm}, \hat{f}_{Dm})$ is the AML estimate of the *m*th target parameters under hypothesis $H_m$. An additional complication we encounter in the attempt to calculate $P_{FA}(m) = \Pr\{S_m(\mathbf{z}) > \lambda_m | H_{m-1}\}$, is that the target complex amplitude $b_m$ is zero under hypothesis $H_{m-1}$, therefore parameters $f_{Dm}$ and $\theta_{TGm}$ of the *m*th most powerful target signal are not observable. As a consequence, the standard asymptotic theory of GLRT is not applicable. However, conditioned on an arbitrary Doppler frequency-DOA pair, $(\hat{\theta}_{TGm}, \hat{f}_{Dm})$=$(\theta, f)$, GLRT theory states that the asymptotic distribution of the test statistic at step *m* is known under $H_{m-1}$. In particular, the conditional test statistic has asymptotically a Chi-Squared distribution, with number of degrees of freedom equal to the difference in the number of parameters under the two opposite hypotheses, $H_m$ and $H_{m-1}$ [6]. In our case, this difference is two, corresponding to the real and imaginary part of $b_m$. Therefore, $2\ln L_{G,m}(\mathbf{z};\theta,f)\overset{as}{\sim}\chi_2^2$ under $H_{m-1}$, where "*as*" stands for asymptotically ($N\to\infty$), and we rendered explicit the dependence on $\theta$ and $f$ to stress the fact that instead of replacing ($\theta_{TGm}, f_{Dm}$) with their ML estimates, we are conditioning the GLR on an arbitrary pair ($\theta, f$). Based on the GLRT approach, the statistic is given by the value of the *m*th highest peak of $\ln L_{G,m}(\mathbf{z};\theta,f)$, i.e. $S_m(\mathbf{z})$ is obtained by calculating $\ln L_{G,m}(\mathbf{z};\theta,f)$ at the location of the *m*th most powerful target component in the ($\theta, f$)-plane. To derive the pdf of $S_m(\mathbf{z})$, assume that we implement the 2-D nonlinear maximization required by (5) using a 2-D grid search approach:

$$S_m(\mathbf{z}) = \max_{(k,l)} S_m(\mathbf{z};k,l) = \max_{(k,l)} \frac{2}{\sigma_d^2} \frac{\left|\mathbf{z}_m^H \mathbf{M}^{-1}\mathbf{a}(\theta_k, f_l)\right|^2}{\mathbf{a}^H(\theta_k, f_l)\mathbf{M}^{-1}\mathbf{a}(\theta_k, f_l)} \quad (6)$$

for $k=1,\cdots,K$, and $l=1,\cdots,L$, where $K\times L$ is the size of the 2-D grid over which we search to derive the ML estimate of the two parameters of the *m*th most powerful target component. In (6) we defined $\mathbf{z}_m$ as the "new" data vector obtained by removing from **z** the first *m*-1 most powerful target components:

$$\mathbf{z}_m = \mathbf{z} - \sum_{i=1}^{m-1}\hat{b}_i \mathbf{a}(\hat{\theta}_{TGi}, \hat{f}_{Di}) \overset{as}{=} \sum_{i=m}^{M} b_i \mathbf{a}(\theta_{TGi}, f_{Di}) + \mathbf{d}. \quad (7)$$

To select the values of the thresholds $\{\lambda_m\}$ we need to derive the distribution of $S_{M+1}(\mathbf{z})$ under $H_M$, $M$ being the actual number of targets. The pdf of $S_{M+1}(\mathbf{z};k,l)\middle|H_M$ is a central Chi-Squared with two degrees of freedom (the proof is reported in [2]). It is worth stressing that we did not use large sample size analysis to derive the asymptotic distribution of the GLRT statistic, but we used an asymptotic expression of the LLF to derive a test statistic that has asymptotically the same form, and therefore also the same pdf, as the GLRT (2). The distribution of $S_{M+1}(\mathbf{z})|H_M$ could be easily found if the random variables $\{S_{M+1}(\mathbf{z};k,l)|H_M\}_{k,l}$ were independent. In this case, the local probability of false alarm at step $m=M+1$ is

$$P_{FA}(M+1) = \Pr\{S_{M+1}(\mathbf{z}) > \lambda_{M+1} \mid H_M\} = 1-(1-e^{-\frac{\lambda_{M+1}}{2}})^{KL}. \quad (8)$$

Therefore, assuming that we want $P_{FA}(M+1)=\alpha$, so that globally we get $P_{FA}\le\alpha$, we select the thresholds as

$$\lambda_m = -2\ln[1-(1-\alpha)^{1/KL}],\ m=1,\cdots,M+1,\cdots,M_{\max}. \quad (9)$$

Result (9) follows from the assumption that the random variables $\{S_{M+1}(\mathbf{z};k,l)|H_M\}_{k,l}$ are independent. Unfortunately, this assumption doesn't hold in our case. In fact, even if for $l_1\ne l_2$, $S_{M+1}(\mathbf{z};k,l_1)$ and $S_{M+1}(\mathbf{z};k,l_2)$ are conditionally independent, provided that *N* is large enough, $S_{M+1}(\mathbf{z};k_1,l)$ and $S_{M+1}(\mathbf{z};k_2,l)$ are not independent. However, the interesting fact about this approach is that even if $\{S_{M+1}(\mathbf{z};k,l)|H_M\}_{k,l}$ are correlated, still this choice guarantees that $P_{FA}<\alpha$ (the proof is outlined in [2]). Therefore, we use (9) to select $\{\lambda_m\}$. Another method to select the threshold is described in [2].

## 4. NUMERICAL PERFORMANCE ANALYSIS

We assumed that two targets are present: *M*=2. Performance has been evaluated in terms of the following conditional probabilities. *Probability of detection*: $P_D=\Pr\{\hat{M}=2\}=\Pr\{S_1(\mathbf{z})>\lambda_1, S_2(\mathbf{z})>\lambda_2, S_3(\mathbf{z})\le\lambda_3|H_2\}$; *probability of false alarm*: $P_{FA}=\Pr\{\hat{M}>2\}=\Pr\{S_1(\mathbf{z})>\lambda_1, S_2(\mathbf{z})>\lambda_2, S_3(\mathbf{z})>\lambda_3|H_2\}$; *probability of target missing*: $P_M=\Pr\{\hat{M}<2\}=\Pr\{(S_1(\mathbf{z})\le\lambda_1)\cup(S_1(\mathbf{z})>\lambda_1, S_2(\mathbf{z})\le\lambda_2)|H_2\}$.

They have been calculated by averaging over $10^4$ realizations of the SHT statistics $\{S_m(\mathbf{z})|H_M\}_{m=1}^{M_{\max}}$, with $M_{\max}=4$. The detection thresholds were selected to provide $P_{FA}(m)=10^{-2}$ for each *m*. The analysed scenarios were obtained by changing only one parameter at time, while keeping all the others constant: $SDR_1=SDR_2=20\ dB$, $\theta_B=2°$, $N=16$, $[\theta_{TG1}\ \theta_{TG2}]=[0.9°\ 1.5°]$, $[f_{D1}\ f_{D2}]=[-0.3\ 0.3]$. Performance have been investigated as a function of all target and disturbance parameters, however here we report only a small subset of the results. The results described here were derived assuming **M=I**, i.e. only thermal noise is present, whereas the case where also correlated clutter is present is investigated in [2].

In Figs. 1-4 we plot the performance of the SHT procedure. Note that in Fig. 1 and Fig. 3, the $P_M$-curve does not show up. This is because the curves were plotted in log-scale and out of $10^4$ Monte Carlo trials, the SHT procedure never produced at the output the decision $\hat{M}=0$ or $\hat{M}=1$ (in this cases we have $\hat{P}_D+\hat{P}_{FA}=1$). The results show that for $N\ge 8$ and $SDR_i\ge 5\ dB$ (*i*=1,2), the SHT algorithm performs very well ($P_D\cong 1$) and it always guarantees a $P_{FA}$ lower than $\alpha=10^{-2}$. Fig. 2 reveals that when $SDR_2<5\ dB$ (and $SDR_1=20\ dB$), the algorithm sometimes misses the least powerful target. Fig. 3 shows that the algorithm always resolves targets that have the same DOA, provided they have different Doppler frequencies. On the contrary, targets that have the same Doppler frequency but different DOAs cannot be resolved by the SHT procedure. In fact, Fig. 4 shows that when $f_{D1}=f_{D2}$, the algorithm always decides for the presence of a single target. However, this is not a problem of the SHT procedure, but a problem of the AML-RELAX algorithm [1,3]. To resolve multiple targets with the same Doppler frequency and different DOAs, we should use the true ML estimator in place of the AML [1].

These numerical results, and other not reported for lack of space, demonstrate the ability of the SHT method to correctly detect and estimate multiple targets present in the same range-azimuth cell under test, for a typical surveillance radar scenario.

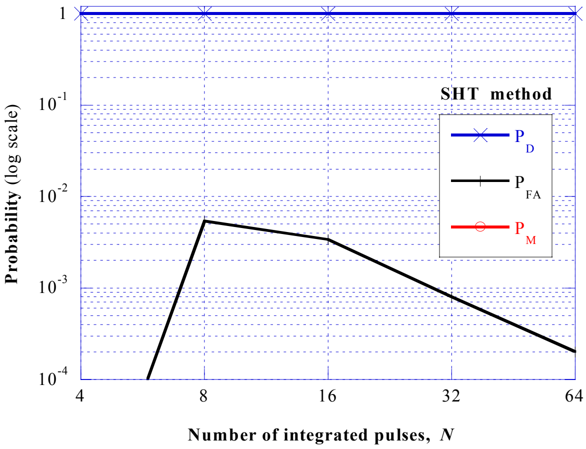


**Fig. 1.** *Plot of* $P_D$ , $P_{FA}$ , $P_M$ *versus N.*

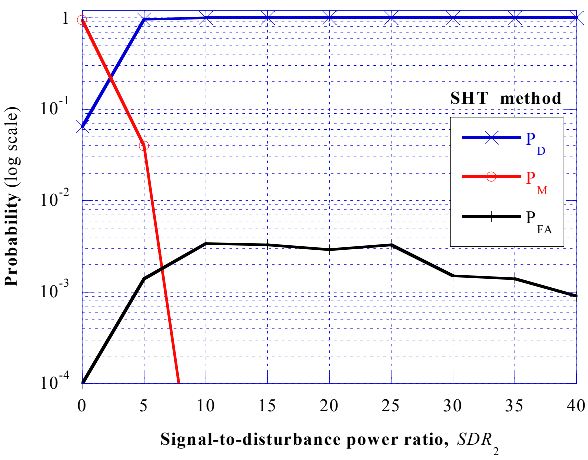


**Fig. 2.** *Plot of* $P_D$ , $P_{FA}$ , $P_M$ *versus* $SDR_2$ .

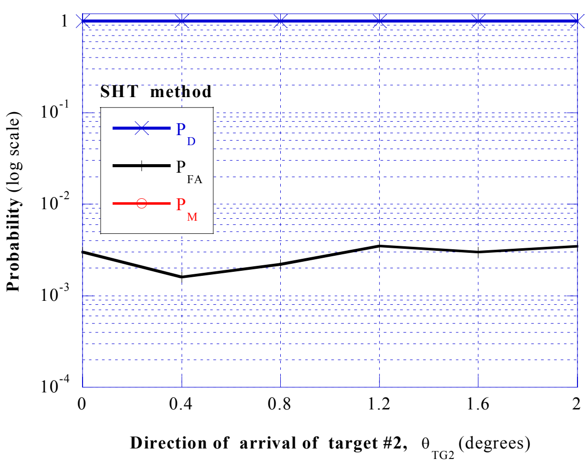


**Fig. 3.** *Plot of* $P_D$ , $P_{FA}$ , $P_M$ *versus the DOA of target #2.*

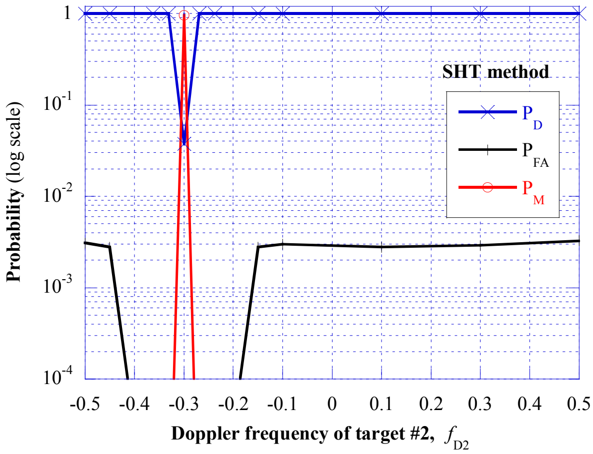


**Fig. 4.** *Plot of* $P_D$ , $P_{FA}$ , $P_M$ *versus* $f_{D2}$ .

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